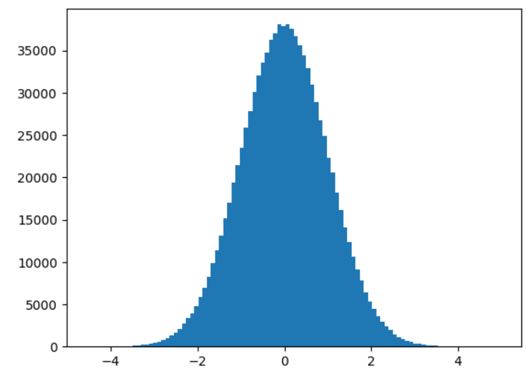
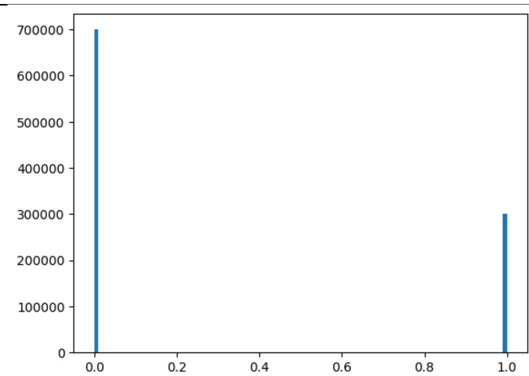
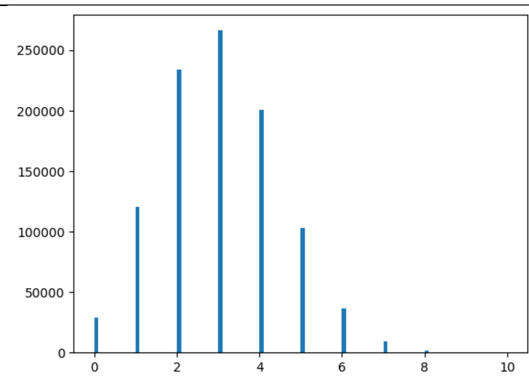
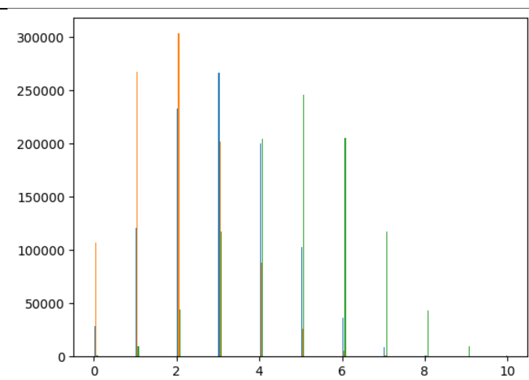
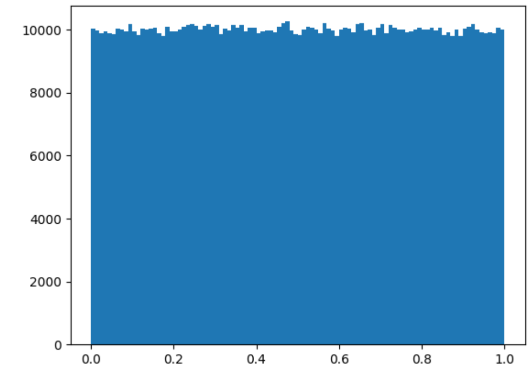
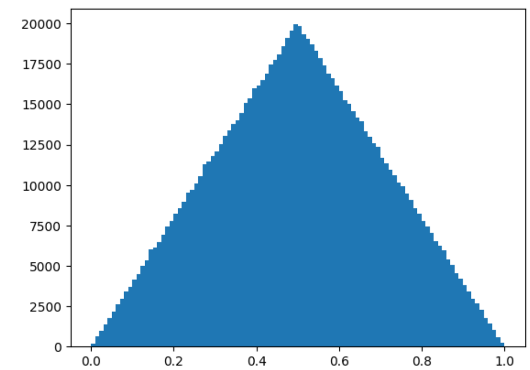
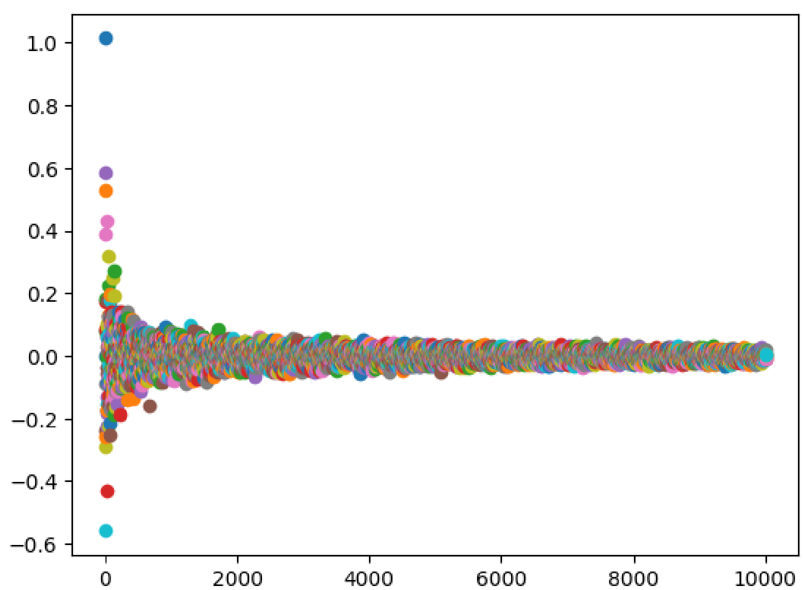
TODO 1

Distribution	Histogram
$\mathcal{N}(0,1)$	
$Bernoulli(0.3)$	
$B(10,0.3)$	
$Multinomial(n=10, p=[0.3,0.2,0.5])$	

$U(0,1)$	
$T(0,1)$	

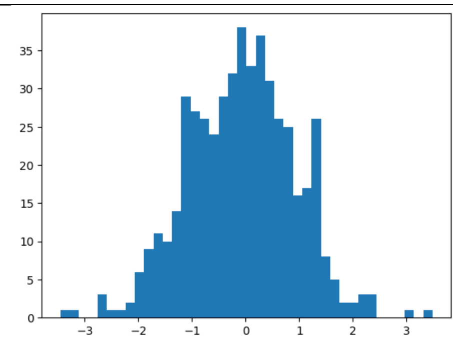
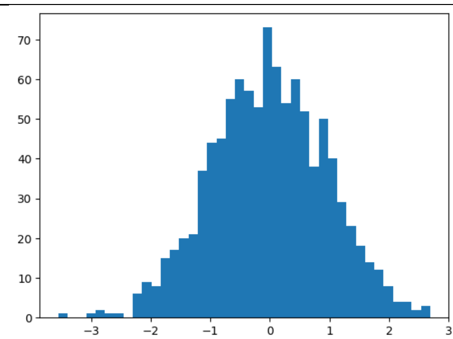
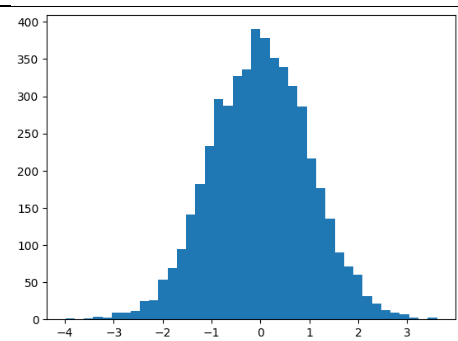
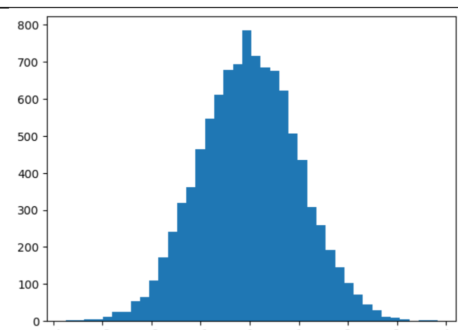
TODO 2

From the graph, we can see that the empirical mean is getting closer to 0 as the sample size increases. This implies that the empirical mean is converging to the theoretical mean, which is 0.

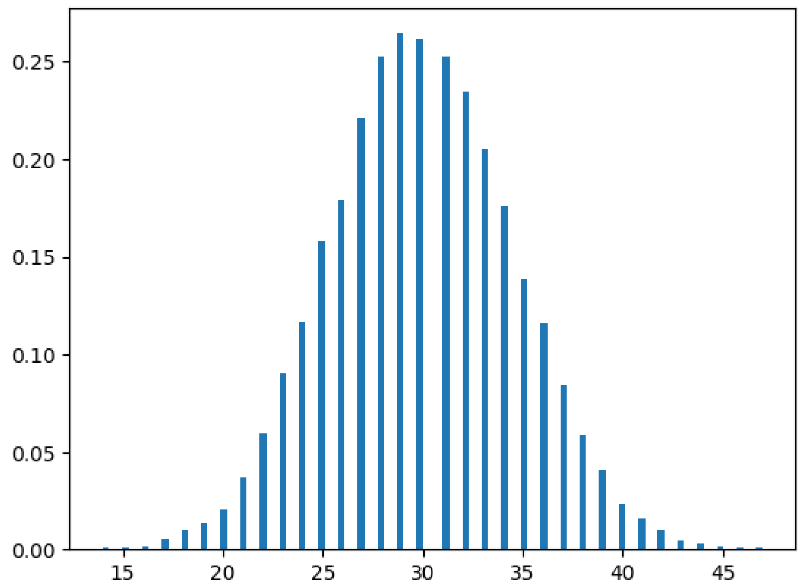


TODO 3

From the graph, we can see that with the increase of sample size, the approximation given by the histogram is more similar to the true PDF.

$\mathcal{N}(n,0,1)$	Histogram
With $n=500$	
With $n=1000$	
With $n=5000$	
With $n=10000$	

TODO 4



The resulting histogram looks like a normal distribution due to the Central Limit Theorem, which states that the sum of a large number of independent and identically distributed random variables tends towards a normal distribution.

TODO 5

```python
n = 100
p = 0.3
mu = n*p
std = np.sqrt(n*p*(1-p))

# calculate z-score
z = (40 - mu) / std

prob = 1 - norm.cdf(z)

print("Probability of getting more than 40 heads is: ", prob)
```

✓ 0.0s

```
Probability of getting more than 40 heads is:  0.014548165870626129
```

TODO 6

```
# Compare with the result from binomial distribution

n = 100
p = 0.3
prob_binom = 1 - binom.cdf(40, n, p)

print("Probability of getting more than 40 heads is: ", prob_binom)
print("Difference: ", prob - prob_binom)
```

✓ 0.0s

```
Probability of getting more than 40 heads is:  0.012498407166438241
Difference:  0.0020497587041878873
```

TODO 7

To find $P(3 < Z < 5)$, we need to calculate the following.

$$\int_{z=3}^{z=5} (X * Y) dz$$

```
Probability of 3 < Z < 5 is:  0.49999592328729675
```

TODO 8

```
X = sample_uniform(sample_size=sample_size, from_x=-1, to_x=1)

# A = 10
Y = X + 10
print("A=10\t\t", np.corrcoef(X, Y)[0][1])

# A ~ U(-1,1)
A = sample_uniform(sample_size=sample_size, from_x=-1, to_x=1)
Y = X + A
print("A~U(-1,1)\t", np.corrcoef(X, Y)[0][1])

# A ~ U(-10,10)
A = sample_uniform(sample_size=sample_size, from_x=-10, to_x=10)
Y = X + A
print("A~U(-10,10)\t", np.corrcoef(X, Y)[0][1])

# A ~ U(-100,100)
A = sample_uniform(sample_size=sample_size, from_x=-100, to_x=100)
Y = X + A
print("A~U(-100,100)\t", np.corrcoef(X, Y)[0][1])

```

✓ 0.0s

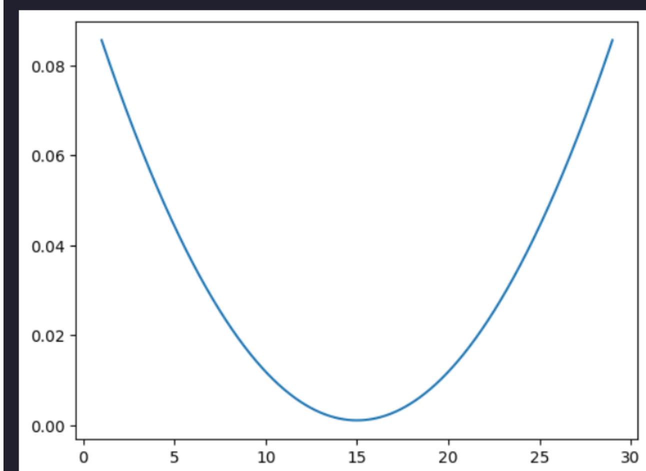
```
A=10            1.0
A~U(-1,1)       0.7066350679768639
A~U(-10,10)     0.09573616266085563
A~U(-100,100)   0.006663491638389606 5
```

TODO 9

1. Yes, the correlation decreases as we increase the randomness of A
2. Changing A from $U(-10,10)$ to $U(9990,10010)$ is equivalent to adding 10000 to each sample of A. This shifts the distribution of A but does not change its shape or spread. Therefore, it should not affect the correlation between X and Y, because correlation is not affected by shifts in the mean.

TODO 10

```
The minimum of P_fail_mu is:  0.001143704548850501
The corresponding mu is:  15.001400140014
The minimum number of disks is:  2
The probability of having more than 1 failure in 10k requests is:  8.480033392543884e-05
```



1. To minimize the failure, The probability of failure is

$$f(\mu)=\int_{t=\mu-1}^{t=\mu+1}(\frac{1}{2}\cdot(\frac{97}{2250}(t-15)^2+0.001)dt)$$

The minimum probability is when $\mu=15$ by analyzing parabola graph.

2. The minimum probability is around 0.001
3. The probability of failure is around 0.001^n when n is number of disks. That means if $n=2$

TODO 10 (Continuous)

```
The minimum of P_fail_mu is:  0.004879939517563487
The corresponding mu is:  15
The minimum number of disks is:  3
The probability of having more than 1 failure in 10k requests is:  6.746473749652537e-07
```

4.1. Since the temperature distribution is normal distribution around μ, then the minimum failure chance must be at 15 Celsius.

4.2 The probability of failure is

$$f(t)=\int_{t=0}^{t=30}(\frac{1}{3\sqrt{2\pi}}\cdot e^{-\frac{1}{2}(\frac{t-15}{3})^2}\cdot(\frac{97}{2250}(t-15)^2+0.001)dt)$$

After calculating, the result is around 0.0049

4.3 The probability of failure can be calculated similarly to 10.3 with changing of failure probability to 0.0049. After calculating the probability, the result is $n=3$

TODO 11

1. $(A,B),(B,C),(B,D)$ are independent since their covariance is equal to 0.
2.

Coin 1 with T = 30 days Expected return: 0.788591023012626 Variance of return: 37.33150828811413 Probability of positive return: 0.4541 Coin 1 with T = 180 days Expected return: 6.989121958995691 Variance of return: 1488.0225645322646 Probability of positive return: 0.3855 Coin 2 with T = 30 days Expected return: 0.5711986404057319 Variance of return: 10.647493032441334 Probability of positive return: 0.5113 Coin 2 with T = 180 days Expected return: 4.135178757234925 Variance of return: 143.45224159301893 Probability of positive return: 0.5434	Coin 3 with T = 30 days Expected return: 1.2805438479523459 Variance of return: 53.44962528466268 Probability of positive return: 0.4664 Coin 3 with T = 180 days Expected return: 10.403009789431337 Variance of return: 2452.5090258819705 Probability of positive return: 0.4115 Coin 4 with T = 30 days Expected return: 1.3983325826759785 Variance of return: 72.59187385301968 Probability of positive return: 0.4532 Coin 4 with T = 180 days Expected return: 10.315242809333652 Variance of return: 3967.5783970609846 Probability of positive return: 0.3535

3. Coin B has the highest probability

```
Coin 1 with T = 30 days
Expected return: 0.788591023012626
Variance of return: 37.33150828811413
Probability of positive return: 0.4541

Coin 1 with T = 180 days
Expected return: 6.989121958995691
Variance of return: 1488.0225645322646
Probability of positive return: 0.3855

Coin 2 with T = 30 days
Expected return: 0.5711986404057319
Variance of return: 10.647493032441334
Probability of positive return: 0.5113

Coin 2 with T = 180 days
Expected return: 4.135178757234925
Variance of return: 143.45224159301893
Probability of positive return: 0.5434
```

```
Coin 3 with T = 30 days
Expected return: 1.2805438479523459
Variance of return: 53.44962528466268
Probability of positive return: 0.4664

Coin 3 with T = 180 days
Expected return: 10.403009789431337
Variance of return: 2452.5090258819705
Probability of positive return: 0.4115

Coin 4 with T = 30 days
Expected return: 1.3983325826759785
Variance of return: 72.59187385301968
Probability of positive return: 0.4532

Coin 4 with T = 180 days
Expected return: 10.315242809333652
Variance of return: 3967.5783970609846
Probability of positive return: 0.3535
```

4. Because of the law of large number, The mean will converge into true mean that >1 which make the expected return positive.
5. 30 Days

Strategy	Buy A	Buy B	Buy C	Buy D	Expected	Variance	Prob
1	100	0	0	0	0.79	37.33	0.4541
2	0	100	0	0	0.57	10.64	0.5113
3	0	0	100	0	1.28	53.44	0.4664
4	0	0	0	100	1.40	72.59	0.4532
5	50	50	0	0	0.78	12.77	0.5273
6	50	0	50	0	1.22	32.26	0.5015
7	50	0	0	50	1.04	36.12	0.4816

180 Days

Strategy	Buy A	Buy B	Buy C	Buy D	Expected	Variance	Prob
1	100	0	0	0	6.99	1488.02	0.3855
2	0	100	0	0	4.14	143.45	0.5434
3	0	0	100	0	10.40	2452.51	0.4115
4	0	0	0	100	10.32	3967.58	0.3535
5	50	50	0	0	5.99	404.65	0.5521
6	50	0	50	0	8.90	1148.74	0.4674
7	50	0	0	50	8.46	1492.10	0.4282

6. 30 Days = strategy 4 180 Days = strategy 3

7. 30 Days = strategy 2 180 Days = strategy 2
8. Strategy has higher variance than Strategy 6 as same as $\text{cov}(r_a, r_c) > \text{cov}(r_a, r_d)$
9. The “good” plan can be defined with many objectives. In my opinion, the good plan must be > 50% Chance of getting expected return and the return must not be too low. That why I think strategy 6 is the optimal choice.