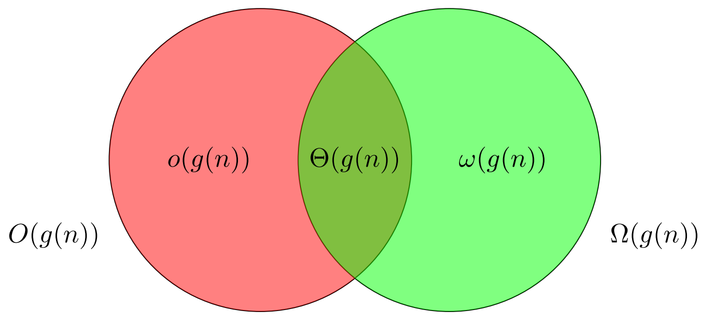
1 The Big/Little-{`oh`,`omega`,`theta`} Notations

Relationships btw the sets $O(g(n)),\ o(g(n)),\ \Omega(g(n)),\ \omega(g(n))$ and $\Theta(g(n))$:



$$\begin{aligned}
\Theta(g(n)) &= \ \Omega(g(n)) \cap O(g(n)) \\
O(g(n)) &= o(g(n)) \sqcup \Theta(g(n)) \\
\Omega(g(n)) &= \omega(g(n)) \sqcup \Theta(g(n))
\end{aligned}$$

where $\sqcup$ denotes disjoint union.

3.1-1 Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

If we allow us to reframe the question. Define

$$\begin{aligned}
h\colon \quad n &\mapsto \quad \max(f(n), g(n)) \\
l\colon \quad n &\mapsto \quad f(n) + g(n)
\end{aligned}$$

Then we are supposed to show that $h(n) = \Theta(l(n))$.
Since f and g are asymtotically nonnegative, we have

$$\begin{aligned}
&\exists\, n_1 \quad \text{s.t.} \quad f(n) \geq 0 \quad \forall\, n \geq n_1 \\
&\exists\, n_2 \quad \text{s.t.} \quad g(n) \geq 0 \quad \forall\, n \geq n_2
\end{aligned}$$

Let $n_3 = \max(n_1, n_2)$. Then for the constants $c_1 = \frac{1}{2}, c_2 = 1, n_3$, we see that

$$c_1 l(n) \leq h(n) \leq c_2 l(n) \quad \forall\, n \geq n_3\,, \quad \text{i.e.}$$
$$\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n)) \leq f(n) + g(n) \quad \forall\, n \geq n_3\,.$$

$\blacksquare$