

1 Big/little- $\{\text{oh}, \text{omega}, \text{theta}\}$ notation

Relationships btw the sets $O(g(n))$, $o(g(n))$, $\Omega(g(n))$, $\omega(g(n))$ and $\Theta(g(n))$.

$$\begin{aligned}\Theta(g(n)) &= \Omega(g(n)) \cap O(g(n)) \\ o(g(n)) &\subsetneq O(g(n)) \setminus \Omega(g(n))\end{aligned}$$

3.1-1 Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ -notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

If we allow us to reframe the question. Define

$$\begin{aligned}h: n &\mapsto \max(f(n), g(n)) \\ l: n &\mapsto f(n) + g(n)\end{aligned}$$

Then we are supposed to show that $h(n) = \Theta(l(n))$.
Since f and g are asymptotically nonnegative, we have

$$\begin{aligned}\exists n_1 \quad \text{s.t.} \quad f(n) &\geq 0 \quad \forall n \geq n_1 \\ \exists n_2 \quad \text{s.t.} \quad g(n) &\geq 0 \quad \forall n \geq n_2\end{aligned}$$

Let $n_3 = \max(n_1, n_2)$. Then for the constants $c_1 = \frac{1}{2}$, $c_2 = 1$, n_3 , we see that

$$\begin{aligned}c_1 l(n) &\leq h(n) \leq c_2 l(n) \quad \forall n \geq n_3, \quad \text{i.e.} \\ \frac{1}{2}(f(n) + g(n)) &\leq \max(f(n), g(n)) \leq f(n) + g(n) \quad \forall n \geq n_3.\end{aligned}$$

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