1 Little-{oh,omega} notation

$$o(g(n)) \cup \Omega(g(n)) = O(g(n))$$
$$o(g(n)) \cap \Omega(g(n)) = \emptyset$$

$$o(g(n)) \subseteq O(g(n))$$
$$\Omega(g(n)) \subseteq O(g(n))$$

3.1-1 Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

If we allow us to reframe the question. Define

$$\begin{aligned} h\colon \quad n &\mapsto \quad \max(f(n), g(n)) \\ l\colon \quad n &\mapsto \quad f(n) + g(n) \end{aligned}$$

Then we are supposed to show that $h(n) = \Theta(l(n))$.
Since f and g are asymtotically nonnegative, we have

$$\begin{aligned} &\exists\, n_1 \quad \text{s.t.} \quad f(n) \geq 0 \quad \forall\, n \geq n_1 \\ &\exists\, n_2 \quad \text{s.t.} \quad g(n) \geq 0 \quad \forall\, n \geq n_2 \end{aligned}$$

Let $n_3 = \max(n_1, n_2)$. Then for the constants $c_1 = \frac{1}{2}, c_2 = 1, n_3$, we see that

$$c_1 l(n) \leq h(n) \leq c_2 l(n) \quad \forall\, n \geq n_3\,, \quad \text{i.e.}$$
$$\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n)) \leq f(n) + g(n) \quad \forall\, n \geq n_3\,.$$

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