# 1 Little-{oh,omega,theta} notation

Relationships btw the sets $O(g(n)), \Omega(g(n)), \Theta(g(n)), o(g(n)), \omega(g(n))$ and $\theta(g(n))$.

$$\begin{aligned} \Theta(g(n)) &= \Omega(g(n)) \cap O(g(n)) \\ o(g(n)) &\subsetneq O(g(n)) \setminus \Omega\,(g(n)) \end{aligned}$$

***3.1-1*** Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of $\Theta$-notation, prove that $\max\left(f(n), g(n)\right) = \Theta(f(n) + g(n))$.

If we allow us to reframe the question. Define

$$\begin{aligned} h\colon \quad n &\mapsto \quad \max(f(n), g(n)) \\ l\colon \quad n &\mapsto \quad f(n) + g(n) \end{aligned}$$

Then we are supposed to show that $h(n) = \Theta(l(n))$.
Since $f$ and $g$ are asymtotically nonnegative, we have

$$\begin{aligned} \exists\, n_1 \quad \text{s.t.} \quad f(n) \geq 0 \quad \forall\, n \geq n_1 \\ \exists\, n_2 \quad \text{s.t.} \quad g(n) \geq 0 \quad \forall\, n \geq n_2 \end{aligned}$$

Let $n_3 = \max(n_1, n_2)$. Then for the constants $c_1 = \frac{1}{2}, c_2 = 1, n_3$, we see that

$$\begin{gathered} c_1 l(n) \leq h(n) \leq c_2 l(n) \quad \forall\, n \geq n_3\,, \quad \text{i.e.} \\ \frac{1}{2}(f(n) + g(n)) \leq \max\left(f(n), g(n)\right) \leq f(n) + g(n) \quad \forall\, n \geq n_3\,. \end{gathered}$$

$\blacksquare$