Recurrent neurons (p.381)

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Abstract

Personally, I found the equations on p.381 were not to the point, at least not enough for me to grasp the exact meaning and shapes of the input and output of a layer of recurrent neurons. Every time I re-read these paragraphs, I cannot recall the understanding I reached at the previous reading. Thus, I decided to make a note here to try to change the situation, hoping to make things clearer.

1 A Single Recurrent Neuron

The output of a single recurrent neuron of a single input instance $x_{(t)}$ equals

$$y_{(t)} = \phi\Big(\langle x_{(t)}, w_x\rangle + y_{(t-1)}w_y + b\Big),$$

where

- $x_{(t)}, w_x \in \mathbb{R}^k$ (k can be any positive integer the user consider suitable)
- $\langle \cdot\,, \cdot\rangle$ denotes the inner product in $\mathbb{R}^k$
- $y_{(t-1)} \in \mathbb{R}$ is the output of the previous time step
- w_y, b are both in $\mathbb{R}$

2 A Layer of Recurrent Neurons

To illustrate, let's take Figure 14-2 for example, in which we have a layer of **5** recurrent neurons. So, the output of this particular layer equals

$$\begin{pmatrix} y_{(t),1} \\ y_{(t),2} \\ y_{(t),3} \\ y_{(t),4} \\ y_{(t),5} \end{pmatrix} = \phi \begin{pmatrix} \langle x_{(t)}, w_{x,1} \rangle + \langle y_{(t-1)}, w_{y,1} \rangle + b_1 \\ \langle x_{(t)}, w_{x,2} \rangle + \langle y_{(t-1)}, w_{y,2} \rangle + b_2 \\ \langle x_{(t)}, w_{x,3} \rangle + \langle y_{(t-1)}, w_{y,3} \rangle + b_3 \\ \langle x_{(t)}, w_{x,4} \rangle + \langle y_{(t-1)}, w_{y,4} \rangle + b_4 \\ \langle x_{(t)}, w_{x,5} \rangle + \langle y_{(t-1)}, w_{y,5} \rangle + b_5 \end{pmatrix}, \text{ i.e.}$$

$$y_{(t)} = \phi\Big({W_x}^T \, x_{(t)} + {W_y}^T \, y_{(t-1)} + b \Big),$$

where

- $y_{(t),j} \in \mathbb{R}$ for all $j = 1, 2, 3, 4, 5$
- $x_{(t)} \in \mathbb{R}^k$ as before, and $y_{(t-1)} \in \mathbb{R}^l$, where $l = \#(\text{recurrent neurons})$, i.e. number of recurrent neurons in the layer, here, in particular, $l = 5$
- $w_{x,j} \in \mathbb{R}^k, w_{y,j} \in \mathbb{R}^l, b_j \in \mathbb{R}$ are the weights and bias associated with the j-th neuron (for all $j = 1, 2, 3, 4, 5$)
- We define

$$ {W_x}^T = \begin{pmatrix} — {w_{x,1}}^T — \\ — {w_{x,2}}^T — \\ — {w_{x,3}}^T — \\ — {w_{x,4}}^T — \\ — {w_{x,5}}^T — \end{pmatrix}$$

or equivalently

$$W_x = \begin{pmatrix} | & | & | & | & | \\ w_{x,1} & w_{x,2} & w_{x,3} & w_{x,4} & w_{x,5} \\ | & | & | & | & | \end{pmatrix}.$$

- We define similarly

$$
{W_y}^T = \begin{pmatrix} \text{---}\ {w_{y,1}}^T\ \text{---} \\ \text{---}\ {w_{y,2}}^T\ \text{---} \\ \text{---}\ {w_{y,3}}^T\ \text{---} \\ \text{---}\ {w_{y,4}}^T\ \text{---} \\ \text{---}\ {w_{y,5}}^T\ \text{---} \end{pmatrix}
$$

or equivalently

$$
W_y = \begin{pmatrix} | & | & | & | & | \\ w_{y,1} & w_{y,2} & w_{y,3} & w_{y,4} & w_{y,5} \\ | & | & | & | & | \end{pmatrix}.
$$

- It is understood that

$$
b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}
$$

3 A Layer of Recurrent Neurons on A Batch of Instances

Like in the case of a fully connected neural network, we can have an expression (using matrices) for an entire batch of input instances, say, m instances

$$
Y_{(t)} = \phi\Bigg(X_{(t)}W_x + Y_{(t-1)}W_y + b\Bigg)
$$

where

- `shape(`$X_{(t)}$`)` $= (m, k)$
- `shape(`W_x`)` $= (k, l)$
- `shape(`$Y_{(t-1)}$`)` $= (m, l)$
- `shape(`W_y`)` $= (l, l)$
- `shape(`b`)` $= (1, l)$