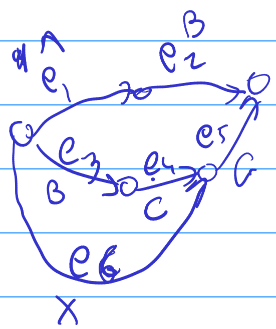


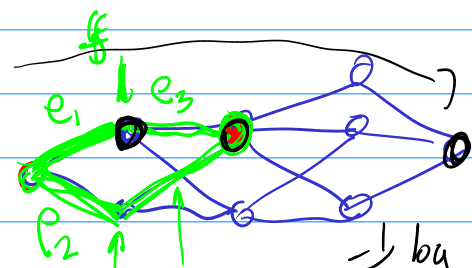
$$F_\infty(x,y) = \min_z E(x,y,z)$$

$$F_\beta(x,y) = -\frac{1}{\beta}\log\sum_{z\in \text{Paths}} e^{-\beta E(x,y,z)}$$

Y: AB

BCG

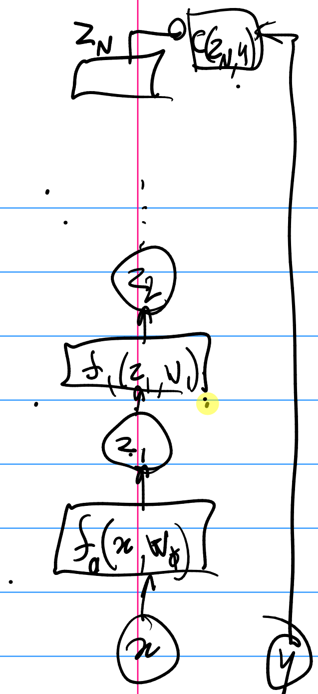
~~X~~G



$$-\frac{1}{\beta}\log\left[e^{-\beta(e_1+e_3)} + e^{-\beta(e_2+e_4)}\right]$$

$$\alpha_i = -\log\sum_{k\in UP(i)} e^{-\beta(\alpha_k+e_{ki})}$$ forward algorithm

Belief Propagation



$$z_{k+1} = f_k(z_k, w_k) \qquad z_0 = x$$

$$C(z_N, y)$$ minimize C such that

$$\mathcal{L}(x, y, \lambda, z, w) = C(z_N, y) + \sum_{k=0}^{N-1} \lambda_{k+1}^T \left(z_{k+1} - f_k(z_k, w_k)\right)$$

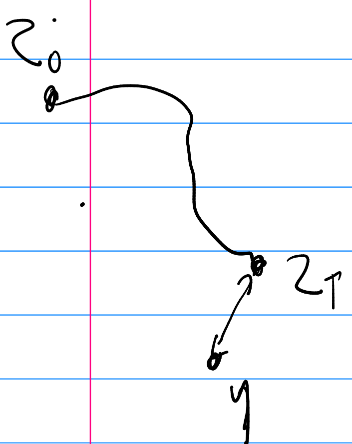
$$\frac{\partial \mathcal{L}}{\partial \lambda_{k+1}} = 0 \qquad z_{k+1} = f_k(z_k, w_k)$$

$$\frac{\partial \mathcal{L}}{\partial z_k} = 0 \qquad \lambda_k^T - \lambda_{k+1}^T \frac{\partial f_k(z_k, w)}{\partial z_k} = 0$$

$$\lambda_k = \frac{\partial f_k(z_k, w)}{\partial z_k}^T \lambda_{k+1}$$

$$\mathcal{L} = \sum_k \left[C_k(z_k, y_k) + \lambda_{k+1}^T \left(z_{k+1} - f_k(z_k, w_k)\right)\right]$$

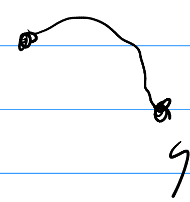
Neural ordinary Differential Equation



$$z_{t+dt} = z_t + dt\, f(z_t, w)$$

$$\frac{\partial z(t)}{\partial t} = f(z_t, w)$$

$$\left(f(y, w) = 0 \right)$$

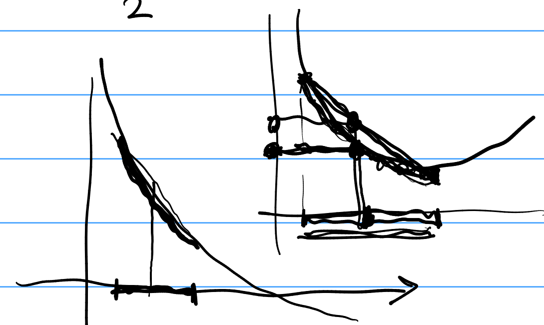


a theoretical framework for backpropagation 1988

$$P(z) = \frac{e^{-\beta' E(x, y, z)}}{\int_{z'} e^{-\beta' E(x, y, z')}}$$

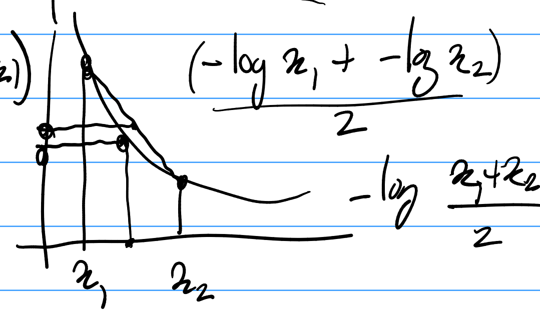
$$L(x,y) = -\frac{1}{\beta}\log\int_z e^{-\beta L(x,y,z)} \qquad F_\beta(x,y) = -\frac{1}{\beta}\log\int_z e^{-\beta E(x,y,z)}$$

$$L(x,y) = -\frac{1}{\beta}\log\int_z q(z)\,\frac{e^{-\beta L(x,y,z)}}{q(z)}$$



Jensen's inequality

$$\text{Convex}\left[\int_z q(z)\, h(z)\right] \le \int_z q(z)\,\text{Convex}(h(z))$$



$$\le \int_z q(z)\left[-\frac{1}{\beta}\log \frac{e^{-\beta L(x,y,z)}}{q(z)}\right]$$

$$\le \int_z q(z)\left[L(x,y,z) + \frac{1}{\beta}\log q(z)\right]$$

$$\le \underbrace{\int_z q(z)\,L(x,y,z)}_{\text{average energy}} + \underbrace{\frac{1}{\beta}\int_z q(z)\log q(z)}_{-\frac{1}{\beta}\,\text{Entropy}(q)}$$

$$F = \langle E \rangle - \underbrace{T}_{\frac{1}{\beta}}\, S$$