# CS 5751: Assignment #1

Due on January 23, 2018

*Eleazar 11:00am*

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# Exercise #5

Suppose we have an unfair die such that Pr(1) = 0.2, Pr(2) = 0.3, Pr(3) = 0.1, Pr(4) = 0.1, Pr(5) = 0.1, Pr(6) = 0.2. Now do the following tasks. Note that you cannot simply write the value; you need to write the complete formula and then solve it step by step:

## (1)

What is the expected value of the face on which the die will land?

We have the expected value of a discrete random variables defined as follows:

$$E(\mathbf{X}) = \sum_{i=1}^{n} x_i Pr(x_i)$$

Given an unfair die with probabilities defined above, we can compute the expected value of the random variable as follows:

$$\begin{aligned}
E(\mathbf{X}) &= \sum_{i=1}^{n} x_i Pr(x_i) \\
&= \sum_{i=1}^{6} x_i Pr(x_i) \\
&= (1)Pr(1) + (2)Pr(2) + (3)Pr(3) + (4)Pr(4) + (5)Pr(5) + (6)Pr(6) \\
&= 0.2 + (2)(0.3) + (3)(0.1) + (4)(0.1) + (5)(0.1) + (6)(0.2) \\
&= 0.2 + 0.6 + 0.3 + 0.4 + 0.5 + 1.2 \\
&= 3.2
\end{aligned}$$

## (2)

What is the variance of the value of the face on which the die will land?

We have the variance of a discrete random variables defined as follows:

$$Var(\mathbf{X}) = \sum_{i=1}^{k}[x_i - E(\mathbf{X})]^2 Pr(x_i)$$

Given an unfair die with probabilities defined above, we can compute the variance of the random variables as follows:

$$\begin{aligned}
Var(\mathbf{X}) &= \sum_{i=1}^{k}[x_i - E(\mathbf{X})]^2 Pr(x_i) \\
&= \sum_{i=1}^{6}[x_i - E(\mathbf{X})]^2 Pr(x_i) \\
&= (1-3.2)^2(0.2) + (2-3.2)^2(0.3) + (3-3.2)^2(0.1) + (4-3.2)^2(0.1)+ \\
&\quad (5-3.2)^2(0.1) + (6-3.2)^2(0.2) \\
&= 0.968 + 0.432 + 0.004 + 0.064 + 0.324 + 1.568 \\
&= 3.36
\end{aligned}$$

## (3)

Suppose that you throw that same die, but you can't see where it landed. Someone else tells you that the top face of the die is even number. What is the probability that the top face is "2"? And the probability that it's "4"? And the probability that it's "2" or "4"?

The probability that the top face is 2 given that the top face of the die is even number can be computed as follows:

$$
\begin{aligned}
Pr(\mathbf{X} = 2 | \mathbf{X}\ is\ even) &= \frac{Pr(\mathbf{X}\ is\ even | \mathbf{X} = 2) Pr(\mathbf{X} = 2)}{Pr(\mathbf{X}\ is\ even)} \\
&= \frac{Pr(\mathbf{X}\ is\ even | \mathbf{X} = 2) Pr(\mathbf{X} = 2)}{Pr(\mathbf{X} = 2) + Pr(\mathbf{X} = 4) + Pr(\mathbf{X} = 6)} \\
&= \frac{(1)(0.3)}{0.3 + 0.1 + 0.2} \\
&= 0.5
\end{aligned}
$$

The probability that the top face is 4 given that the top face of the die is even number can be computed as follows:

$$
\begin{aligned}
Pr(\mathbf{X} = 4 | \mathbf{X}\ is\ even) &= \frac{Pr(\mathbf{X}\ is\ even | \mathbf{X} = 4) Pr(\mathbf{X} = 4)}{Pr(\mathbf{X}\ is\ even)} \\
&= \frac{(1)(0.1)}{0.6} \\
&= 0.167
\end{aligned}
$$

The probability that the top face is 4 or 2 given that it's even number is:

$$
\begin{aligned}
Pr(\mathbf{X} = 4\ or\ 2 | \mathbf{X}\ is\ even) &= \frac{Pr(\mathbf{X}\ is\ even | \mathbf{X} = 4\ or\ 2) Pr(\mathbf{X} = 4\ or\ 2)}{Pr(\mathbf{X}\ is\ even)} \\
&= \frac{(1)(0.1 + 0.3)}{0.6} \\
&= 0.667
\end{aligned}
$$