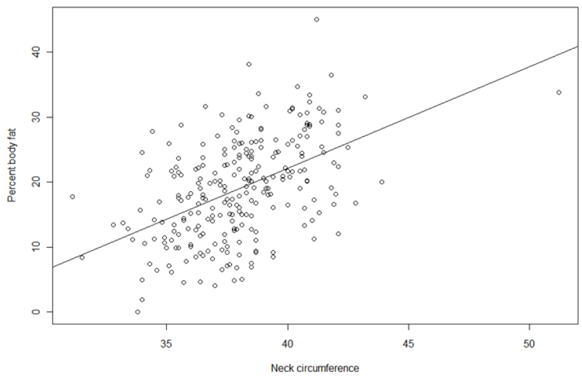
Simple Linear Regression (SLR)



Simple linear regression model

1. **<u>Notation:</u>**
   - m: number of training examples
   - x: input variable
   - y: output variable
   - $x^{(i)}$: input of i^{th} training example
   - $y^{(i)}$: output of i^{th} training example
   - $(x^{(i)}, y^{(i)})$: i^{th} training example
2. **<u>Hypothesis:</u>**

$$h_\theta(x) = \theta_0 + \theta_1 x$$

3. **<u>Cost (Loss) Functions:</u>**
   - Squared error (SE) cost function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

   - Sum squared error (SSE) cost function:

$$J(\theta_0, \theta_1) = \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

   - Mean squared error (MSE) cost function:

$$J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

   - Root mean squared error (RMSE) cost function:
     - For classification problem, RMSE is not a good choice for loss function

$$J(\theta_0, \theta_1) = \sqrt{\frac{\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2}{m}}$$

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4. General Gradient Descent Algorithm for Simple Linear Regression:

Repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)\ ;\ (for\ j = 0, 1)$$

}

5. Gradient Descent Algorithm Using SE Loss Function in SLR:

Repeat until convergence {

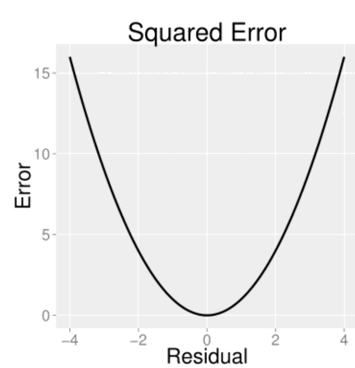
$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left[(1) \times \left(h_\theta\left(x^{(i)}\right) - y^{(i)} \right) \right]$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left[\left(x^{i}\right) \times \left(h_\theta\left(x^{(i)}\right) - y^{(i)} \right) \right]$$

}

6. Note:

- We have a cost function and we want to minimize the cost function to obtain an optimal solution for the hypothesis function → Use gradient descent algorithm to continuously finding θ_0, θ_1 values that are closest to the $min\, J(\theta_0, \theta_1)$
- The α value is called: learning rate.
  - It determines the aggressiveness of the learning algorithm
  - If α is too small → gradient descent can be slow
  - If α is too big → gradient descent can overshoot the minimum. It may fail to converge, or even diverge



Example of the squared-error loss function

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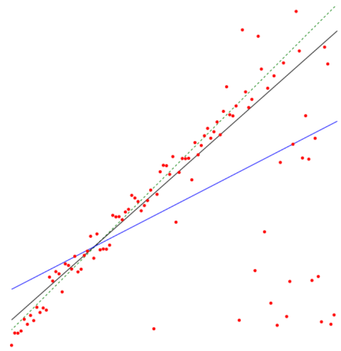

Example of different hypothesis base on different set of (θ_0, θ_1)

- For SLR, when we plot the cost function $J(\theta_0, \theta_1)$, the resulting graph is a convex function (has only 1 global minimum). Because of this, another approach from using gradient descent is to set the partial derivative with respect to θ_0, θ_1 to be 0 in order to find the minimum.
- Another approach to find the hypothesis function would be using Pearson's R correlation coefficient (PCC):

$$\theta_0 = \overline{y} - \theta_1 \overline{x}$$

$$\theta_1 = r\left(\frac{s_y}{s_x}\right)$$

$$r = \frac{\sum_{i=1}^{m}\left(z_{x^{(i)}}\right)\left(z_{y^{(i)}}\right)}{m-1}$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{m}(x^{(i)} - \overline{x})^2}{m-1}} \qquad s_y = \sqrt{\frac{\sum_{i=1}^{m}(y^{(i)} - \overline{y})^2}{m-1}}$$

$$z_{x^{(i)}} = \frac{x^{(i)} - \overline{x}}{s_x} \qquad z_{y^{(i)}} = \frac{y^{(i)} - \overline{y}}{s_y}$$

$$\overline{x} = \frac{\sum_{i=1}^{m} x^{(i)}}{m} \qquad \overline{y} = \frac{\sum_{i=1}^{m} y^{(i)}}{m}$$

- Before deciding whether we should apply SLR or not, if we compute the Pearson's R correlation coefficient and find that $|r| \geq \frac{2}{\sqrt{m}}$, then we can say that a linear relationship exists