COMPUTER GRAPHICS

Lecture 3:

Line-Drawing Method

Associate Professor Lý Quốc Ngọc

KHOA CÔNG NGHỆ THÔNG TIN
TRƯỜNG ĐẠI HỌC KHOA HỌC TỰ NHIÊN

Content

3.1. Problem statement

3.2. Line-Drawing Method

3.3. Line-Drawing Algorithm

3.1. Problem statement

Input: coordinates of **two ends** of line segment.

Output: Draw a line based on the **determination of points** in **the line equation** on the **integer coordinate grid.**

The **necessary requirements** must be met:

- The drawing points must be **the best approximation of the line**.
- The drawing points must satisfy **the spatial continuity in the neighborhood of 8 pixels**.
- **Low** computational complexity**.**

3.1. Problem statement

Assumptions on the problem

- Suppose the line has the equation $y = mx + b$ through two points A (xa, ya) and B (xb, yb).
- Investigate the method of drawing a straight segment with $0 < m < 1$, $xa < xb$.
- The remaining cases are easily deduced from the above case.

3.2. Line-Drawing Method

- Find the **cost function** to determine the **next drawing point** from the **current drawing point**.

Assuming at the step k, we have determined the pixel at (x_k, y_k).

At **step k+1**, we need to decide which pixel to draw at positions:

$$(x_k+1, y_k), \quad (x_k+1, y_k+1).$$

3.2. Line-Drawing Method

- Find the **cost function** to determine the **next drawing point** from the **current drawing point**.

Consider the difference between two pixel separations:

$d_1 – d_2= (y- y_k) – (y_k+1 - y)$

$= [m . (x_k+1)+b – y_k] – [y_k+1 - m . (x_k+1)+b]$

$= 2. m . (x_k+1) + 2b – 1 - 2. y_k \qquad (1)$

3.2. Line-Drawing Method

- Find the **cost function** to determine the **next drawing point** from the **current drawing point**.

Substitute $m = \Delta y / \Delta x$ and define :

$p_k = \Delta x \, . \, (d_1 - d_2) = 2. \, \Delta y. \, x_k - 2. \, \Delta x. \, y_k + C, \qquad (2)$

$C = 2. \, \Delta y + \Delta x. \, (2b - 1).$

If $p_k < 0$ $\Rightarrow$ Select the next drawing point as (x_k+1, y_k).

If $p_k >= 0$ $\Rightarrow$ Select the next drawing point as (x_k+1, y_k+1).

3.2. Line-Drawing Method

- Find the **cost function** to determine the **next drawing point** from the **current drawing point**.

Define the inductive formula to calculate p_k,

$p_{k+1} = 2.\ \Delta y.\ x_{k+1} - 2.\ \Delta x.\ y_{k+1} + C,$ (3)

From (2) and (3), we have:

$p_{k+1} - p_k = 2.\ \Delta y.\ (x_{k+1} - x_k) - 2.\ \Delta x.\ (y_{k+1} - y_k)$

$p_{k+1} = p_k + 2.\ \Delta y - 2.\ \Delta x.\ (y_{k+1} - y_k)$ (4)

3.2. Line-Drawing Method

- Find the **cost function** to determine the **next drawing point** from the **current drawing point**.

To complete the inductive formula for p_k, $\mathbf{p_0}$ must be calculated,

$\mathbf{p_0} = 2.\ \Delta y - \ \Delta x$ (5)

In summary, from (4) and (5) we have **the inductive formula to calculate p_k**

Depend on p_k, we can select the **next point** at **step (k+1)th**

3.2. Line-Drawing Method

- **Discussion**.

Since Δx, Δy is an integer $\Rightarrow$ **p_0 is an integer**

From (4) we have:

If $\mathbf{p_k < 0} \quad \Rightarrow \quad p_{k+1} = p_k \ \mathbf{+ 2. \Delta y}$

If $\mathbf{p_k >= 0} \Rightarrow \quad p_{k+1} = p_k \ \mathbf{+ 2. \Delta y - 2. \Delta x}$

Since **p_k is integer for all k**, so arithmetic involves only **integer addition and subtraction with these two constants**.

3.3. Line-Drawing Algorithm

Bresenham's Line-Drawing Algorithm for 0<m<1

B1. Input the two line endpoints, store the left endpoint in(x_0,y_0).

B2. Plot the first point $\mathbf{(x_0,y_0)}$.

B3. Calculate the constants: $\mathbf{\Delta x, \Delta y, 2.\Delta y, 2.\Delta y- 2\Delta x}$ and obtain the starting value for the cost function as

$$\mathbf{p_0 = 2.\,\Delta y - \Delta x}.$$

B4. k = 0

B5. **Loop**

B6. If $\mathbf{p_k < 0}$ then Next point to plot is $\mathbf{(x_k+1, y_k)}$

Update $\mathbf{p_{k+1}= p_k + 2.\Delta y}$

B7. If $\mathbf{p_k >= 0}$ then Next point to plot is $\mathbf{(x_k+1, y_k+1)}$

Update $\mathbf{p_{k+1}= p_k + 2.\Delta y - 2.\Delta x}$

B8. k = k + 1

B9. **Until** $k = \Delta x$