Exam 1 Review

Exam 1

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Exam structure.

- 8 Questions
- 90 minutes
- May use one-page notes front/back can be typed if wanted upload after test in Test 1 notes
- Can also write any written solutions (please be neat to get credit) to be uploaded with the notes.
- Test is open Thursday October 7, 9:00 am - 1:00 pm
- Once started needed to be complete in 1 hour 30 minutes plus 15 minutes after of uploading notes and written solutions.

Three problems will present you with a data example and ask you an array of modeling/interpretation questions about that data. (Short answer questions)

Other problems will just be a mix of single questions on general knowledge of the class material. Will be a mixture of multiple choice and short answer questions.

Topics Covered

- Types of statistical learning
- Simple linear regression
- Multiple linear regression
- Polynomial regression
- Best subsets
- Logistic Regression
- Test/Training data
- Confusion Matrix

Type of Statistical Learning

In many data problems we are faced with one of two tasks:

- Prediction
- Inference

Are the following problems a) Prediction or b) Inference?

1. Explain what factors cause cancer $\rightarrow$? b
2. Forecast the weather $\rightarrow$? a
3. Predict freshman's final college GPA $\rightarrow$? a
4. Explain what factors affect college GPA $\rightarrow$? b

Prediction Versus Inference

In prediction, our sole and primary goal is to predict well at all costs, no matter the interpretation of underlying mechanism.

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In inference, our goal is to infer the relationship between variables and response, to estimate population parameters (μ, or β_0, β_1 etc). Interpretation is king for inference, most times at a cost of a worse prediction performance.

Response and predictor, or explanatory, variables.

Having mentioned prediction and inference tasks, we have to ask:

- What are we predicting? $\Longrightarrow$

Response and predictor, or explanatory, variables.

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- What are we predicting? $\Longrightarrow$ response variable
- With help of what are we predicting the response variable? $\Longrightarrow$

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- In inference, we are inferring the relationships between...? $\Longrightarrow$

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- What are we predicting? $\Longrightarrow$ response variable
- With help of what are we predicting the response variable? $\Longrightarrow$ predictor, or explanatory, variables
- In inference, we are inferring the relationships between...? $\Longrightarrow$ Response and predictor variables.

QuaNTitative or QuaLitative?

Is the variable a) QuaNTitative or b) quaLitative variable? (Numerical or factor? Continuous or categorical?)

5. Person's height → ? a
6. Eye color → ? b
7. Test score → ? a
8. County → ? b

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QuaLitative - something that takes on a value of a category or class. Can't perform mathematical operations on them directly (e.g. for color $Color \in \{Red, Green, Blue\}$, what's $3 + 0.2 * Color$?)

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Question: how do we incorporate qualitative variables to perform math operations on them? $\Longrightarrow$

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Question: how do we incorporate qualitative variables to perform math operations on them? $\Longrightarrow$ **Dummy Variables**. 0/1

1 = "TRUE" 0 = "FALSE"

Regression and Classification.

As far as the response variable is concerned, we have:

- Regression task $\rightarrow$ response variable is...

Regression and Classification.

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- Classification task $\rightarrow$ response variable is...

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In this course, which of the covered methods corresponds to:

9. Regression?

a) Linear Regression

b) Logistic Regression

Regression and Classification.

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- Regression task $\rightarrow$ response variable is... qua**NT**itative (continuous, numeric)
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In this course, which of the covered methods corresponds to:

9. Regression?

   a) Linear Regression   b) Logistic Regression

10. Classification?

   a) Linear Regression   b) Logistic Regression

Simple Linear Regression Example

You're given data on movies' total gross, opening gross, the # of weeks and # of theaters where movie was shown.

Task #1. Assume you are asked to use movies' opening gross to predict their total gross.

- Is it classification or regression? What model do we use?

  Response = total gross ⇒ Regression
  model ⇒ Linear regression

Simple Linear Regression Example

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Task #1. Assume you are asked to use movies' opening gross to predict their total gross.

- Is it classification or regression? What model do we use?
- What is the model formula for our problem?

assumptions

$$Gross = \beta_0 + \beta_1 Opening + \epsilon, \ \epsilon \sim N(0, \sigma^2)$$

Summary(lm(Gross ~ Opening))

Movies data: Task #1.

```
> summary(lm(Gross~Opening))

Call:
lm(formula = Gross ~ Opening)

Residuals:
    Min      1Q  Median      3Q     Max
-69.567  -4.829  -1.265   4.019 113.830

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.44585    2.31326   0.193    0.848
Opening      2.89977    0.04918  58.957   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 20.33 on 98 degrees of freedom
Multiple R-squared:  0.9726, Adjusted R-squared:
0.9723
F-statistic:  3476 on 1 and 98 DF,  p-value: < 2.2e-16
```

$\widehat{Gross} = 2.89977 * Opening + 0.44585$

```
> predict(movie.lm,newdata=data.frame(Opening=100),interval="c",level = 0.95)
fit       lwr       upr
1 290.423 281.8443 299.0016
```

$(281.8443, 299.0016)$ $281.8443 \le \mu_y \le 299.00$

```
> predict(movie.lm,newdata=data.frame(Opening=100),interval="p",level = 0.95)
fit       lwr       upr
1 290.423 249.1752 331.6707
```

$(249.1752, 331.6707)$

$249.1752 \le y \le 331.6707$

What does a $(281m\$, 299m\$)$ 95% **confidence** interval tell us here?

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(249.1752, 331.6707)

$249.1752 \leq y \leq 331.6707$

What does a $(281m\$, 299m\$)$ 95% **confidence** interval tell us here?

We predict the **average** (μ_y) gross of all movies with an opening gross of 100k\$ to end up in $(281k\$, 299k\$)$ with 95% confidence.

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> predict(movie.lm,newdata=data.frame(Opening=100),interval="c",level = 0.95)
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```

(281.8443, 299.0016) $281.8443 \leq \mu_y \leq 299.0016$

```
> predict(movie.lm,newdata=data.frame(Opening=100),interval="p",level = 0.95)
fit      lwr      upr
1 290.423 249.1752 331.6707
```

(249.1752, 331.6707)

$249.1752 \leq y \leq 331.6707$

What does a $(281m\$, 299m\$)$ 95% **confidence** interval tell us here?

We predict the **average** (μ_y) gross of all movies with an opening gross of $100k\$$ to end up in $(281k\$, 299k\$)$ with 95% confidence.

What does a $(249m\$, 331m\$)$ 95% **prediction** interval tell us here?

We predict the gross of **any single movie** (y) with an opening gross of $100m\$$ to end up in $(249m\$, 331m\$)$ with 95% confidence.

Movies data: Task #2.

Task # 2 (still *Movies* data): Assume you are asked to use movies' opening gross, # of weeks and # of theaters, to predict their total gross.

- Is it classification or regression? What model do we use?

Response = Gross , regression
multiple linear regression

Movies data: Task #2.

Task # 2 (still *Movies* data): Assume you are asked to use movies' opening gross, # of weeks and # of theaters, to predict their total gross.

- Is it classification or regression? What model do we use?
- What is the model formula for our problem?

Movies data: Task #2.

Task # 2 (still *Movies* data): Assume you are asked to use movies' opening gross, # of weeks and # of theaters, to predict their total gross.

- Is it classification or regression? What model do we use?
- What is the model formula for our problem?

$$Gross = \beta_0 + \beta_1 Theaters + \beta_2 Opening + \beta_3 Weeks + \epsilon, \ \epsilon \sim N(0, \sigma^2)$$

Movies data: Task #2.

```
movieall.lm=lm(Gross~Theaters+Opening+Weeks)
summary(movieall.lm)

Call:
lm(formula = Gross ~ Theaters + Opening + Weeks)

Residuals:
Min       1Q  Median      3Q     Max
-73.513  -7.733   0.363   4.634  95.983

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.101133   5.403947  -1.314 0.191956
Theaters    -0.002171   0.001850  -1.173 0.243576
Opening      2.904524   0.057292  50.697  < 2e-16 ***
Weeks        1.331971   0.364575   3.653 0.000422 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

                                          n-3-1
Residual standard error: 19.15 on 96 degrees of freedom
Multiple R-squared:  0.9762,Adjusted R-squared:  0.9754
F-statistic:  1310 on 3 and 96 DF,  p-value: < 2.2e-16
```

estimates $\Rightarrow \hat{\text{Gross}} = -7.10113 - 0.002171 \times \text{Theaters} + 2.904524 \times \text{Opening} + 1.331971 \times \text{Weeks}$

We interpret β_j as the average effect of Y (the predictor) of a one unit increase in X_j, **holding all othe prectictors fixed**.

- $\hat{\beta}_2 = 2.905$ This means that for one added million dollars that the movie makes during opening weekend the total gross is predicted to increase on average by $2.9 million dollars. For a fixed value of the number of theaters and the number of weeks.
- $\hat{\beta}_3 = 1.332$, So for one additional week, the total gross will increase by $1.33 million dollars for a fixed value of the number of theaters and the opening gross.

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```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.101133   5.403947  -1.314 0.191956
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```

$H_0: \beta_0 = 0$
$H_0: \beta_1 = 0$
$H_0: \beta_2 = 0$
$H_0: \beta_3 = 0$

- Notice for tesing $H_0 : \beta_1 = 0$, *P*-value = 0.24357.
- This is testing if we need the variable Theaters if Opening and Weeks are in the model.
- Thus Theaters is not needed to predict the total Gross for movies.

Movies data: Task #2.

- Confidence interval

```
> predict(moviet.lm,newdata = data.frame(Opening=90,Weeks=15),interval="c",level=0.95)
       fit      lwr      upr
1 266.3005 258.5255 274.0755
```

This means we predict the **average** total gross for a movie that has a opeining weekend gross of $90 million dollars and has been in the theaters for 15 weeks to be in [266.31, 258.52] with 95% confidence.

- Prediction interval

```
> predict(moviet.lm,newdata = data.frame(Opening=90,Weeks=15),interval="p",level=0.95)
       fit      lwr      upr
1 266.3005 227.4304 305.1705
```

This means we predict the total gross for a movie that has a opeining weekend gross of $90 million dollars and has been in the theaters for 15 weeks to be in [227.43, 305.17] with 95% confidence.

Movies data: Task #3.

Example: Assume you are asked to describe a relationship between movies' opening gross and the # of theaters.

Question: If asked to describe a relationship between two quaNTitative variables, what do we, as extremely promising data scientists, do **first**?

Movies data: Task #3.

Example: Assume you are asked to describe a relationship between movies' opening gross and the # of theaters.

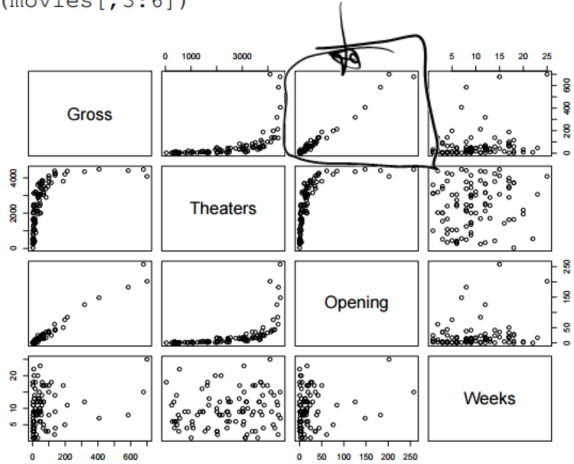
Question: If asked to describe a relationship between two quaNTitative variables, what do we, as extremely promising data scientists, do **first**?

Answer:

P-L-O-T (or V-I-S-U-A-L-I-Z-E).
T-H-E D-A-T-A

Movies data: Task #3.

```
pairs(movies[,3:6])
```



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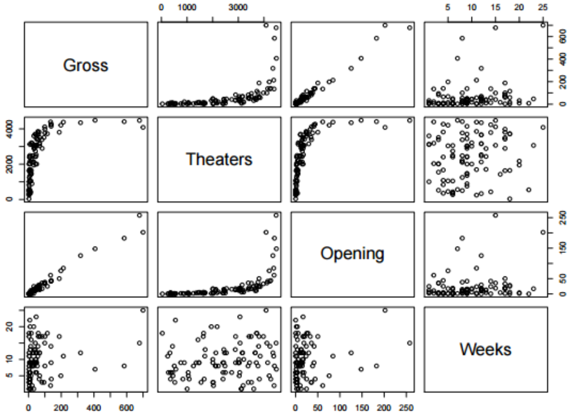

Gross and theaters show a clear non-linear pattern. How do we deal with that? →

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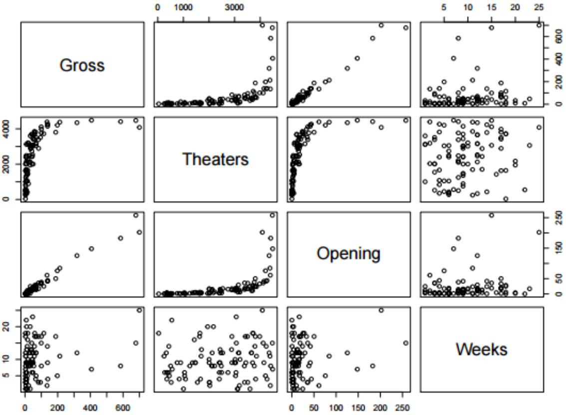

Gross and theaters show a clear non-linear pattern. How do we deal with that? → **Polynomial regression**.

Movies data: Task #3.

Model for quadratic polynomial regression of *Gross* on *Theaters* is

$$Gross = \beta_0 + \beta_1 Theaters + \beta_2 Theaters^2, \ \epsilon \sim N(0, \sigma^2)$$

In *R* it can be carried out as:

```
> lm.polynom <- lm(Gross ~ poly(Theaters,2), data=movies)
> summary(lm.polynom)
...
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)            65.51       8.69   7.538 2.54e-11 ***
poly(Theaters, 2)1    675.15      86.90   7.769 8.27e-12 ***
poly(Theaters, 2)2    537.51      86.90   6.186 1.47e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
...
```

Is the quadratic relationship significant? Yes

F-statistic:

Hypotheses $H_0: \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_p = 0$

H_A: At least one $\beta_i \neq 0$

T-test:

Hypotheses: $H_0: \beta_j = 0$, given $\beta_i \neq 0$ for $i \neq j$

$H_A: \beta_j \neq 0$

If p-value ≤ 0.1 then the term with β_j is significant for the model.

BreastCancer data.

Example. You are given multiple breast cancer tissue samples that are labeled as either malignant (*Class* $= 1$) or benign (*Class* $= 0$). We want to use cell shape (*Cell.shape*) to predict class labels.

- Is it classification or regression? What model do we use?

Response : malignant or benign
classification
Logistic Regression

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- What is the model formula for our problem?

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  While in linear regression we can model the response Y directly:

  $$Y = \beta_0 + \beta_1 X + ...$$

  in logistic regression ($Y = 0/1$)...

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  - First, let $p(X) = P(Y = 1|X)$ - we will model probability of $Y = 1$.

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  - Can we do $p(X) = \beta_0 + \beta_1 X + ..$?

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  - Can we do $p(X) = \beta_0 + \beta_1 X + ..$? No, the left side is stuck $\in [0, 1]$.
  - $p(X) \in [0, 1] \implies$

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  - Can we do $p(X) = \beta_0 + \beta_1 X + ..$? No, the left side is stuck $\in [0, 1]$.
  - $p(X) \in [0, 1] \implies \frac{p(X)}{1-p(X)} \in (0, \infty) \implies$

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  - First, let $p(X) = P(Y = 1|X)$ - we will model probability of $Y = 1$.
  - Can we do $p(X) = \beta_0 + \beta_1 X + ..$? No, the left side is stuck $\in [0, 1]$.
  - $p(X) \in [0, 1] \implies \frac{p(X)}{1-p(X)} \in (0, \infty) \implies \log\left(\frac{p(X)}{1-p(X)}\right) \in (-\infty, \infty)$

BreastCancer data.

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  While in linear regression we can model the response Y directly:

  $$Y = \beta_0 + \beta_1 X + ...$$

  in logistic regression ($Y = 0/1$)... we need to do transformations:

  - First, let $p(X) = P(Y = 1|X)$ - we will model probability of $Y = 1$.
  - Can we do $p(X) = \beta_0 + \beta_1 X + ..$? No, the left side is stuck $\in [0, 1]$.
  - $p(X) \in [0, 1] \implies \frac{p(X)}{1-p(X)} \in (0, \infty) \implies \log\left(\frac{p(X)}{1-p(X)}\right) \in (-\infty, \infty)$
  - formula $\log(\frac{p(X)}{1-p(X)})$ is also known as logit.

BreastCancer data.

Denoting $p(Class = Malignant \mid X) = p(X)$, the model formula is:

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 Cell.shape$$

$$p(X) = \frac{\exp(\beta_0 + \beta_1 \, Cell.Shape)}{1 + \exp(\beta_0 + \beta_1 \, Cell.Shape)}$$

$$\cancel{p(X) = \beta_0 + \beta_1 \, Cell.Shape}$$

Denoting $p(Class = Malignant \mid X) = p(X)$, the model formula is:

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 Cell.shape$$

```
summary(glm(Class ~ Cell.shape, family="binomial", data = bc))

Call:
glm(formula = Class ~ Cell.shape, family = "binomial", data = bc)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.6383  -0.2219  -0.2219   0.0517   2.7263

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -5.1645     0.3865  -13.36   <2e-16 ***
Cell.shape    1.4727     0.1205   12.22   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$H_0: \beta_1 = 0$

Interpretation: $\hat{\beta}_1 = 1.47 \implies$ as cell shape increases, the probability of tissue being *Malignant* also **increases**.

Titanic data.

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  Response: Survival (Yes/No), logistic

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- What is the model formula? Letting $p(Surv = Yes \mid X) = p(X)$:

$$\log\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 x_{Sex}$$

$$p(x) = \begin{cases} \dfrac{\exp(\beta_0)}{1 + \exp^{\beta_0}} & \text{Female} \\ \dfrac{\exp(\beta_0 + \beta_1)}{1 + \exp(\beta_0 + \beta_1)} & \text{Male} \end{cases}$$

Titanic data.

```
> summary(glm(ifelse(Survived=="Yes",1,0)~Sex,
                  family="binomial",
                   data=Titanic_tab,weights=Count))
...
Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)   0.8127     0.1070   7.592 3.14e-14 ***
SexMale      -2.1255     0.1221 -17.404  < 2e-16 ***
...
```

$$P(\text{Survival} \mid \text{Female}) = e^{0.8127} / (1 + \exp^{0.8127})$$

Interpretation: there is a significant gender effect on the survival outcome. Males had lower survival probability (or "the logit of survival probability for males was on average 2.1255 lower than for females").

$$P(\text{Survival} \mid \text{Male}) = e^{(0.8127 - 2.1255)} / (1 + \exp(0.8127 - 2.125$$

NOTE: For FACTOR VARIABLES, **DON'T** SAY "Per 1 unit increase in [factor variable], the [response or logit probability] on average decreases by 2.12..". It applies to both **linear** and **logistic** regression.

Confusion Matrix

	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	Correct true negatives	Incorrect false positives
$Y = 1$	Incorrect false negative	Correct true positives

Accuracy rate: $\dfrac{\text{true neg} + \text{true post}}{\text{total}}$

Sensitivity: $\dfrac{\text{correct positive}}{\text{Total actual positives}}$

error rate: $\dfrac{\text{false neg} + \text{false pos}}{\text{total}}$

Specifict: $\dfrac{\text{correct neg}}{\text{total actual neg.}}$

Discriminant Analysis

- Let K be number of classes of a response variable, $K \geq 2$.
- Let π_k represent the overall or *prior* probability that a randomly chosen observation comes from the kth class of Y.
- Let $f_k(x) = P(X = x | Y = k)$ denote the density function of X for an observation that comes from the kth class of Y.
- Then the **Bayes' Theorem** states that

$$P(Y = k | X = x) = p_k(x) = \frac{\pi_k f_k(x)}{\sum_{i=1}^{K} \pi_i f_i(x)}$$

- To estimate π_k we can compute the fraction of the training observations that belong to the kth class.
- The problem is how we estimate $f_k(x)$?

The LDA Classifier

The LDA classifier plugs the estimates given for $\hat{\mu}_k$, $\hat{\sigma}^2$, and $\hat{\pi}_k$ in the Bayes classifier and assigns and observation $X = x$ to the class for which

$$\hat{\delta}_k(x) = x\frac{\hat{\mu}_k}{\hat{\sigma}^2} - \frac{\hat{\mu}_k^2}{2\hat{\sigma}^2} + \log(\hat{\pi}_k)$$

is the largest.

Breast Cancer with 3 variables

```
> bc.lda2 = lda(Class ~ Cell.size + Cl.thickness + Cell.shape,data = train)
> bc.lda2
Prior probabilities of groups:
0         1
0.6542969 0.3457031

Group means:
   Cell.size Cl.thickness Cell.shape
0  1.340299     3.020896   1.429851
1  6.581921     7.158192   6.519774

Coefficients of linear discriminants:
                   LD1
Cell.size    0.2884712
Cl.thickness 0.2226278
Cell.shape   0.2320740
> lda.pred2 = predict(bc.lda2,test)
> table(test$Class,lda.pred2$class)

    0   1
0 109   0
1  11  51
```