Music 421a Winter 2012
Homework #6 Solutions
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Question 1

Because we know $S(f) \propto 1/f$ (that is, $S(f) = K/f$), and because the average power within a frequency band is obtained by integrating the sample power spectral density across that band,

$$\begin{aligned} P &= \int_{\omega}^{2\omega} S(f)\, df \\ &= K \int_{\omega}^{2\omega} 1/f\, df \\ &= K[\log 2\omega - \log \omega] \\ &= K \log 2 \end{aligned}$$

Question 2

a) Here $s(t) = 2\delta(t)$, so:

$$\begin{aligned} r(t) &= (s * h)(t) + n(t) \\ &= (2\delta * h)(t) + n(t) \\ &= 2(\delta * h)(t) + n(t). \end{aligned}$$

Normalizing (multiplying by 1/2), we get:

$$= (\delta * h)(t) + \frac{n(t)}{2}.$$

We know the mean of $n(t)$ is zero and the variance of $n(t)$ is σ^2 so the variance of $\frac{n(t)}{2}$ will be given by:

$$E\left\{\left|\frac{n(t)}{2} - 0\right|^2\right\} = E\left\{\left|\frac{n(t)}{2}\right|^2\right\} = \frac{\sigma^2}{4} \tag{1}$$

b) Now we have $s(t) = 2\delta(t)$ and two responses, $r_1(t)$ and $r_2(t)$:

$$\begin{aligned} r_1(t) &= (s * h)(t) + n_1(t), \\ r_2(t) &= (s * h)(t) + n_2(t), \\ r_1(t) + r_2(t) &= 2(s * h)(t) + \frac{n_1(t)}{2} + \frac{n_2(t)}{2} \end{aligned}$$

Because we know that the variance of $\frac{n(t)}{2}$ is $\frac{\sigma^2}{4}$ (from above), and because $n_1(t)$ and $n_2(t)$ will not add coherently, our new variance is $2\frac{\sigma^2}{4} = \frac{\sigma^2}{2}$.

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c) We have $a(t)$ and $b(t)$ such that $(a \star b)(t) + (b \star b)(t) = 2L\delta(t)$. We have

$$\begin{aligned}
r_a(t) &= (a * h)(t) + n_a(t), \\
r_b(t) &= (b * h)(t) + n_b(t), \\
(a \star r_a)(t) &= a \star ((a * h)(t) + n_a(t)), \\
(b \star r_b)(t) &= b \star ((b * h)(t) + n_b(t)), \\
(a \star r_a)(t) + (b \star r_b)(t) &= a \star (a * h)(t) + a \star n_a(t) + b \star (b * h)(t) + b \star n_b(t) \\
&= ((a \star b)(t) + (b \star b)(t)) * h(t) + (n_a \star a)(t) + (n_b \star b)(t) \\
&= 2L\delta * h(t) + (n_a \star a)(t) + (n_b \star b)(t)
\end{aligned}$$

Question 3

a)

b)

c)