New Framework for Structure-Aware PSI From Distributed Function Secret Sharing

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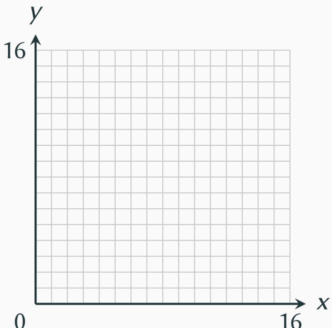
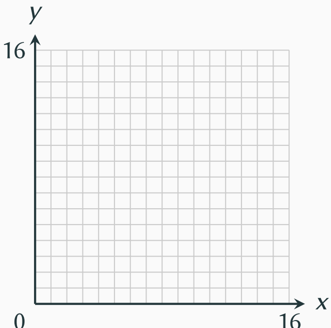
Private Set Intersection

- Alice has a set of points $x \in (\{0,1\}^u)^d$ in d-dimensional space.
- Bob has a set of points $y \in (\{0,1\}^u)^d$ in d-dimensional space.
- Each ball has *diameter* δ.





Structure-Aware Private Set Intersection

- Alice has a set of L_∞ balls $\mathcal{B}_x \subseteq (\{0,1\}^u)^d$ in d-dimensional space.
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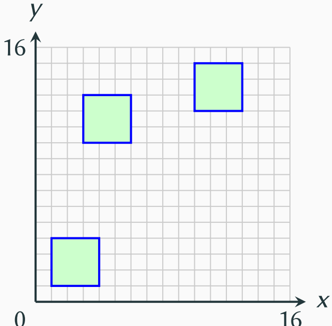

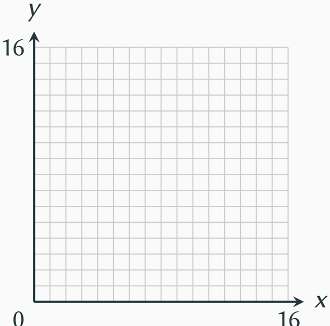

Related Works

- Structure-aware PSI was proposed by [GRS22] for L_∞ metric, introducing the framework:

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- [GGM24] improve spatial hashing, and introduces trade-offs between computation and communication.

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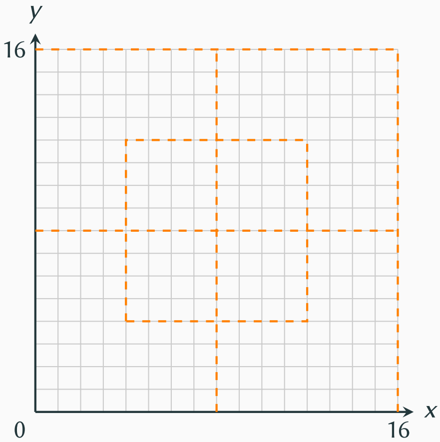
- [GRS23] upgrades the protocol to malicous security.
- [GGM24] improve spatial hashing, and introduces trade-offs between computation and communication.
- [vBP24, GQL+24] generalizes the approach into *fuzzy map*, using Homomorphic Encryption, and support more distance metrics: L_p and Hamming distance.
- [ZCC+25, vBP25] replaces HE with symmetric key techniques, [PST+25] introduces distance-aware OT, to improve fuzzy maps efficiency.

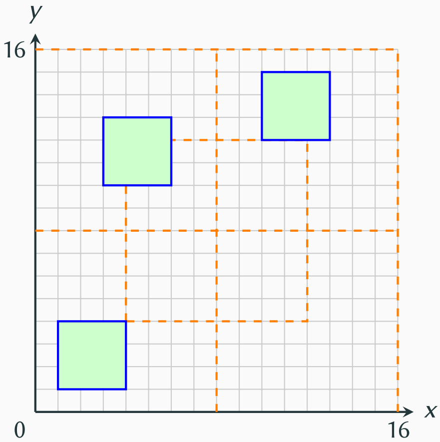
Spatial Hashing

- The space contains (can be overlapped) *mini-universes*.
- Each mini-universe has side length $u \geq 2\delta$.



Spatial Hashing and Input Assumption

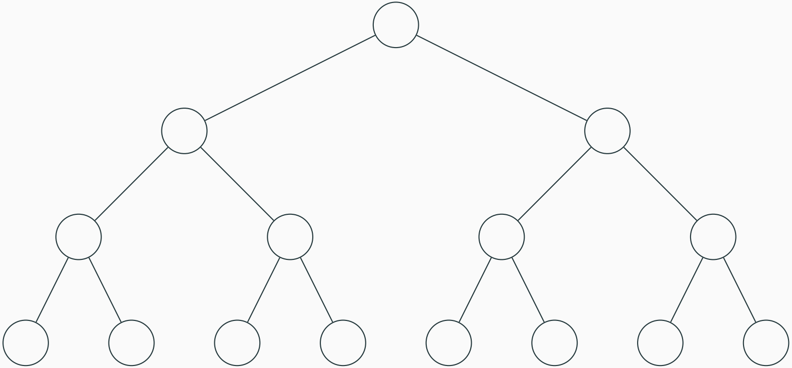
- The space contains (can be overlapped) *mini-universes*.
- Each mini-universe has side length $u \geq 2\delta$.
- Each L_∞ ball is fully contained in a unique mini-universe.



Results Analysis

Matching Phase:

- All related works (including ours) use tree-searching technique to find matching elements.
- Total sender communication - receiver computation cost is $O(u^d)$.



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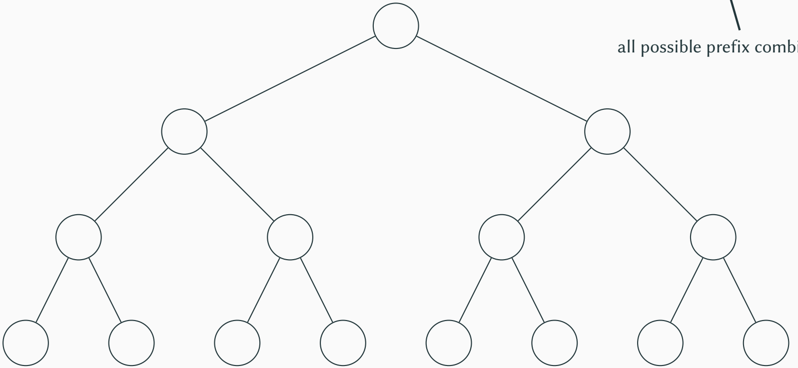
all possible prefix combinations!



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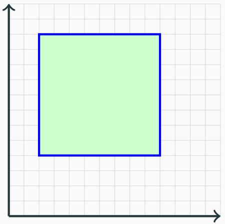
Fuzzy Map:

- The fuzzy map step in [vBP25, ZCC⁺25, PST⁺25] has communication complexity depends *linearly* on $u \approx \delta$ (the distance).
- Function Secret Sharing in [GGM24] has comm. $O(\kappa^2 \cdot N_A \cdot \log \delta \cdot d)$.
- This work: Communication $O(\kappa \cdot N_A \cdot \log \delta \cdot d)$.

Secret Sharing L_∞ ball

- For center $(x_1, \ldots, x_d)$, we want to secret share this huge L_∞ ball:

$$[x_1 - \delta, x_1 + \delta] \times [x_2 - \delta, x_2 + \delta] \times \ldots \times [x_n - \delta, x_n + \delta]$$



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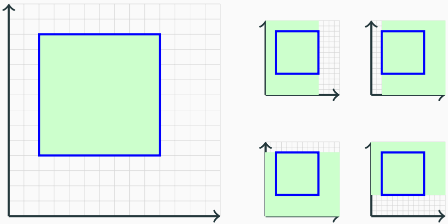
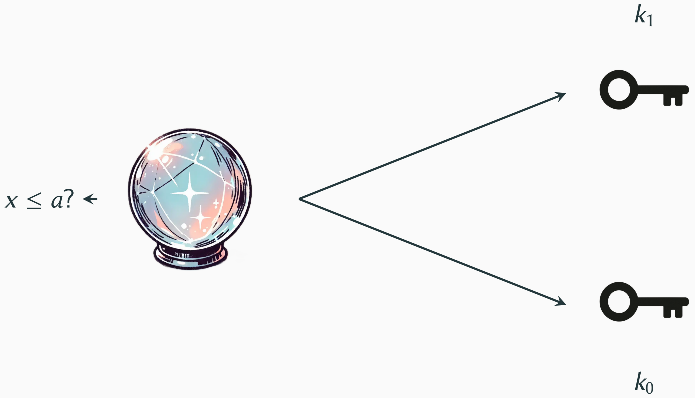
- We can secret share each dimension separately, then AND the results.

$$y_1 \geq x_1 - \delta \quad \wedge \quad y_1 \leq x_1 + \delta \quad \wedge \quad \ldots \quad \wedge \quad y_d \geq x_d - \delta \quad \wedge \quad y_d \leq x_d + \delta$$



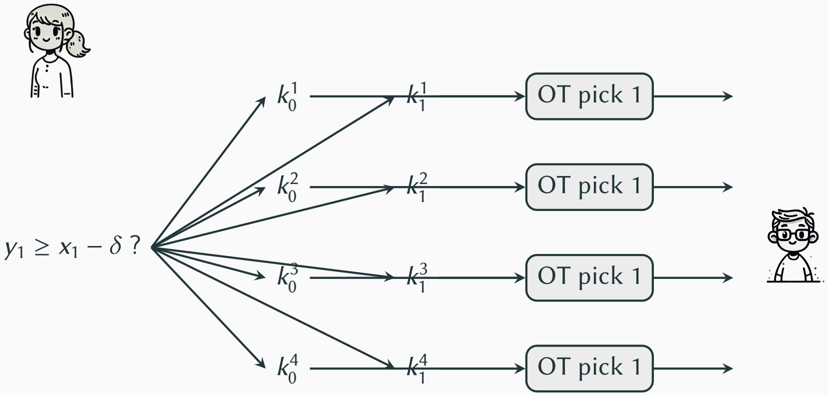
Primitive: Function Secret Sharing (FSS)

- FSS was introduced in [BGI15], allowing to secret share a function f between multiple parties.
- For domain bit length u, security parameter κ, payload bit length v, the key size is $O(u \cdot (\kappa + v))$ bits.



Function Secret Sharing in [GGM24]

- Alice prepares $O(\kappa)$ FSS keys pairs with *binary output* (*in* / *out*?) for each inequality check.
- Alice and Bob runs 1-out-of-2 OTs, for Bob to learn either k_0^i or k_1^i for each inequality check.
- If the inequality check succeeds, for any of κ key pairs, the evaluations would be equal.



Can we use only one key pair?

- Alice can prepare only one FSS key pair for each inequality check, with *longer payload*.
- This will also avoid dictionary attack from Bob.
- However, Alice can prepare malicious payload, which is undesirable.
- Nonetheless, the key size for one inequality check is improved:

$$O(\kappa \cdot u \cdot (\kappa + 1))$$

$$\downarrow$$

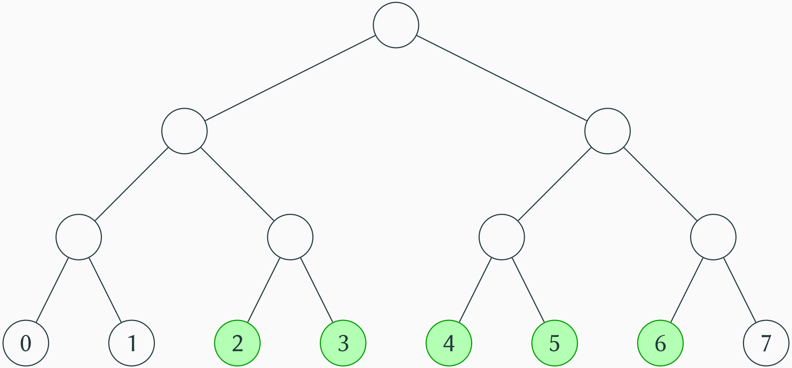
$$O(u \cdot (\kappa + \kappa))$$

Distributed DCF

Critical Prefix

- Each interval $\mathcal{I}$ can be represented as set of *critical prefixes* $\text{pref}_{\mathcal{I}}$.

$$y \in \mathcal{I} \quad \leftrightarrow \quad \text{exists prefix } p \prec y, p \in \text{pref}_{\mathcal{I}}$$



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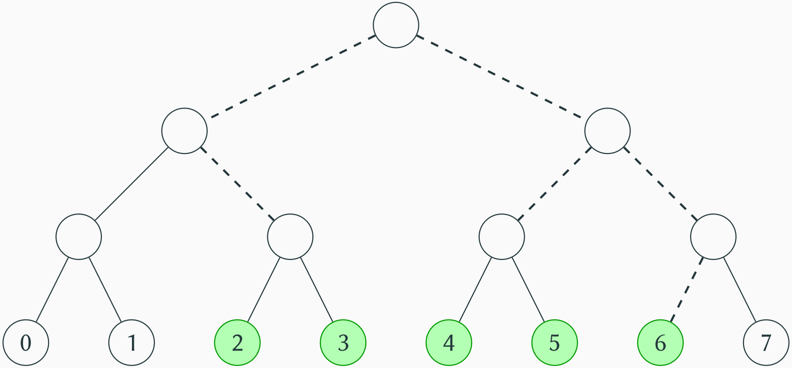

Trade-off in Tree Search

- One way to make trade-off is by slicing the dimensions.

Analysis

Input Assumptions

- Disjoints balls
- Disjoint projection
- At least one dimension being 2δ far away from others

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In this paper, we also assume that the distance is a power of 2, so $\delta = 2^a$.

Experimental Results

- Only include our own results, don't need to compare with other works.
- Highlight the bottleneck is still the tree search.

Open Questions

- Is there better, "flattened" tree search? The goal is to remove exponential dependency on d.

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