Entry Test VNPAY - Part 1: Probability & Statistics

Question 1:

Question:

The probability of passing three candidates A, B, and C in one interview is 60%, 10%, and 20% respectively. Find the probability of success of at least two

Answer:

Given:

- Probability that candidate A passes: $P(A) = 0.6$
- Probability that candidate B passes: $P(B) = 0.1$
- Probability that candidate C passes: $P(C) = 0.2$

Probability that exactly two candidates pass:

$$P(two\ passing) = P(A) \cdot P(B) \cdot (1 - P(C)) + P(A) \cdot P(C) \cdot (1 - P(B)) + P(B) \cdot P(C) \cdot (1 - P(A))$$
$$= 0.6 \times 0.1 \times 0.8 + 0.6 \times 0.2 \times 0.9 + 0.1 \times 0.2 \times 0.4 = 0.164$$

Probability that all three candidates pass:

$$P(three\ passing) = P(A) \cdot P(B) \cdot P(C) = 0.6 \times 0.1 \times 0.2 = 0.012$$

Thus, probability that at least two candidates pass:

$$P = P(two\ passing) + P(three\ passing) = 0.164 + 0.012 = 0.176$$

Result: 0.176

Question 2:

Question:

Nine cards are drawn from a deck of 52 cards. What is the probability that 5 cards are diamonds and 2 are hearts and 2 are kings?

Answer:

The total ways to choose any 9 cards from 52 cards:

$$Total\ ways = C_{52}^{9} = 3,679,075,400$$

Case 1: King of Diamonds and King of Hearts are both present

- Choose 5 Diamonds with 1 King from the Diamond suit: C_{12}^{4}
- Choose 2 Hearts with 1 King from the Heart suit: C_{12}^{1}

- Choose 2 random cards from $52 - 13 - 13 - 2 = 24$ remaining non-King, non-Diamond, non-Heart cards: C_{24}^2
- The total ways of case 1: $C_{12}^4 \cdot C_{12}^1 \cdot C_{24}^2 = 495 \cdot 12 \cdot 276 = 1,639,440$

Case 2: Only King of Diamonds is present

- Choose 5 Diamonds with 1 King from the Diamond suit: C_{12}^4
- Choose 2 Hearts without King from the Heart suit: C_{12}^2
- Choose 1 King from the remaining 2 King: C_2^1
- Choose 1 random cards from $52 - 13 - 13 - 2 = 24$ remaining non-King, non-Diamond, non-Heart cards: C_{24}^1
- The total ways of case 2: $C_{12}^4 \cdot C_{12}^2 \cdot C_2^1 \cdot C_{24}^1 = 495 \cdot 66 \cdot 2 \cdot 24 = 1,568,160$

Case 3: Only King of Hearts is present

- Choose 5 Diamonds without King from the Diamond suit: C_{12}^5
- Choose 2 Hearts with 1 King from the Heart suit: C_{12}^1
- Choose 1 King from the remaining 2 King: C_2^1
- Choose 1 random cards from $52 - 13 - 13 - 2 = 24$ remaining non-King, non-Diamond, non-Heart cards: C_{24}^1
- The total ways of case 3: $C_{12}^5 \cdot C_{12}^1 \cdot C_2^1 \cdot C_{24}^1 = 792 \cdot 12 \cdot 2 \cdot 24 = 456,192$

Case 4: Neither King of Diamonds nor King of Hearts is present

- Choose 5 Diamonds without King: C_{12}^5
- Choose 2 Hearts without King: C_{12}^2
- Choose 2 Kings from the remaining 2 King: C_2^2
- The total ways of case 4: $C_{12}^5 \cdot C_{12}^2 \cdot C_2^2 = 792 \cdot 66 \cdot 1 = 52,272$

Final result:

$$\frac{Case1 + Case2 + Case3 + Case4}{Totalways} = 0.00101$$

Result: 0.00101

Question 3:

Question:

There are transactions from banks A, B, and C at your company. About 30% of the population comes from A, 20% from B, and 50% from C. Knowing that there is a chance of fraud in the transaction history. The percentages for banks A, B, and C are 3%, 5%, and 4%, respectively.
Which bank will most likely belong in the event of fraud detection?

Answer:

Given:

- $P(A) = 0.3;\ P(B) = 0.2;\ P(C) = 0.5$
- $P(F|A) = 0.03;\ P(F|B) = 0.05;\ P(F|C) = 0.04$

Total propability of Fraud:

$$P(F) = P(A) \cdot P(F|A) + P(B) \cdot P(F|B) + P(C) \cdot P(F|C)$$
$$= 0.039$$

The probability that the transaction came from bank A, given that Fraud has been detected:

$$P(A|F) = \frac{P(F|A) \cdot P(A)}{P(F)} = \frac{3}{13}$$

The probability that the transaction came from bank B, given that Fraud has been detected:

$$P(B|F) = \frac{P(F|B) \cdot P(B)}{P(F)} = \frac{10}{39}$$

The probability that the transaction came from bank C, given that Fraud has been detected:

$$P(C|F) = \frac{P(F|C) \cdot P(C)}{P(F)} = \frac{20}{39}$$

Conclusion: Since $P(C|F)$ is the highest, the most likely bank involved in a fraud case is bank C.

Question 4:

Question:

Assume that the transaction amount in Service X in your company follows the Normal distribution with mean μ. A random sample of 25 transactions yielded an average amount of $\bar{x}$= 525 000 VND, and the standard deviation of the 25 transactions is found to be s = 28 340. A 90% confidence interval for μ is *(Hint: The t critical value for 90% CI for Degrees of freedom = 24 is 1.711)*

Answer:

We will calculate the 90% confidence interval (CI) for the population mean μ using the **t-distribution**, since the population standard deviation σ is unknown and the sample size is small ($n = 25$).

Given:

- Sample mean: $\bar{x} = 525,000$
- Sample size: $n = 25$
- Sample standard deviation: $s = 28,340$
- Degree of freedom: $df = n - 1 = 24$
- t-critical value for 90% CI for $df = 24$: $t_{\alpha/2} = 1.711$

Compute CI:

$$\bar{x} \pm t_{\alpha/2} \times \frac{s}{\sqrt{n}}$$

Lower bound: $525,000 - 1.711 \times \frac{28,340}{\sqrt{25}} = 515,302.052$

Upper bound: $525,000 + 1.711 \times \frac{28,340}{\sqrt{25}} = 534,697.948$

Result: $CI = (515\ 302\ VND; 534\ 698\ VND)$