Relative Error L_2 Table: Classical Method for kdV Equation

0.1 kdV Equation: Zabursky-Kruskal finite difference

- The relative error L_2 with initial condition $u_0(x) = -\sin\left(\pi \frac{x}{8}\right)$

Time step Δt \ Relative Error L_2 at T	$T = 5.0$	$T = 4.0$	$T = 3.5$	$T = 2.5$
1.5×10^{-3}	1.422×10^{-2}	7.201×10^{-3}	5.195×10^{-3}	3.567×10^{-3}
1.3×10^{-3}	7.305×10^{-3}	5.362×10^{-3}	2.694×10^{-3}	2.694×10^{-3}
1.2×10^{-3}	1.426×10^{-2}	7.292×10^{-3}	5.420×10^{-3}	3.436×10^{-3}

- The relative error L_2 with initial condition $u_0(x) = \cos\left(-\pi \frac{x}{8}\right)$

Time step Δt \ Relative Error L_2 at T	$T = 5.0$	$T = 4.0$	$T = 3.5$	$T = 2.5$
1.5×10^{-3}	5.602×10^{-2}	4.387×10^{-2}	3.758×10^{-2}	2.971×10^{-2}
1.3×10^{-3}	5.604×10^{-2}	4.395×10^{-2}	3.771×10^{-2}	2.907×10^{-2}
1.2×10^{-3}	5.615×10^{-2}	4.395×10^{-2}	3.776×10^{-2}	2.963×10^{-2}

- Remark: (from course notes: Lax Convergence Theorem) We use Zabusky-Kruskal finite difference method. The method has a stability requirement of:

$$\frac{\Delta t}{\Delta x}| - 2u_{max} + \frac{1}{(\Delta x)^2}| \leq \frac{2}{3\sqrt{3}} \tag{1}$$

- The method ha a truncation error of $O((\Delta t)^2) + O((\Delta x)^2)$