Statistical Learning: Lab Chapter 3

```r
library(tidyverse)
library(dplyr)
library(ggplot2)
library(ggthemes)
library(gridExtra)
```

3.6.2 Simple Linear Regression:

```r
library(MASS)
```

```
##
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##     select
```

```r
## fix(Boston)
names(Boston)
```

```
##  [1] "crim"    "zn"      "indus"   "chas"    "nox"     "rm"      "age"
##  [8] "dis"     "rad"     "tax"     "ptratio" "black"   "lstat"   "medv"
```

- We start using the lm() function to fit a simple linear regression model with `medv` as the response and `lstat` as the predictor:

```r
lm.fit = lm(medv ~ lstat, data = Boston)

attach(Boston)
lm.fit2 = lm(medv ~lstat)

print(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Coefficients:
## (Intercept)        lstat
##       34.55        -0.95
```

```r
summary(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.168  -3.990  -1.318   2.034  24.500
##
```

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.55384    0.56263   61.41   <2e-16 ***
## lstat       -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

- We use the names() function to find out order pieces of informnation stored in lm.fit:

```r
names(lm.fit)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

```r
coef(lm.fit)
```

```
## (Intercept)       lstat
##  34.5538409  -0.9500494
```

- To obtain the confidence interval for the coefficient estimates, we use `confint()` command:

```r
confint(lm.fit)
```

```
##                 2.5 %     97.5 %
## (Intercept) 33.448457 35.6592247
## lstat       -1.026148 -0.8739505
```

- the `predict()` function is to produce confidence intervals and prediction intervals for the prediction of medv for a given value of lstat:

```r
## confidence interval of a given value of lstat:
predict(lm.fit, data.frame(lstat = c(5,10,15)), interval = "confidence")
```

```
##        fit      lwr      upr
## 1 29.80359 29.00741 30.59978
## 2 25.05335 24.47413 25.63256
## 3 20.30310 19.73159 20.87461
```

```r
### prediction interval for a given value of lstat:
predict(lm.fit, data.frame(lstat = c(5,10,15)), interval = "prediction")
```

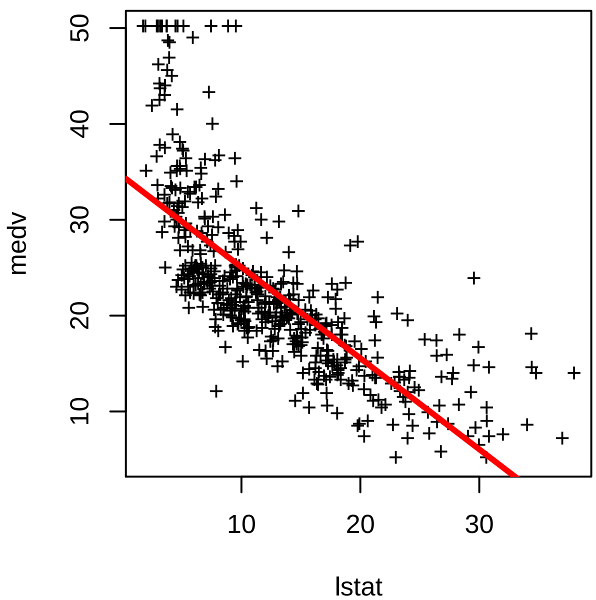
```
##        fit       lwr      upr
## 1 29.80359 17.565675 42.04151
## 2 25.05335 12.827626 37.27907
## 3 20.30310  8.077742 32.52846
```

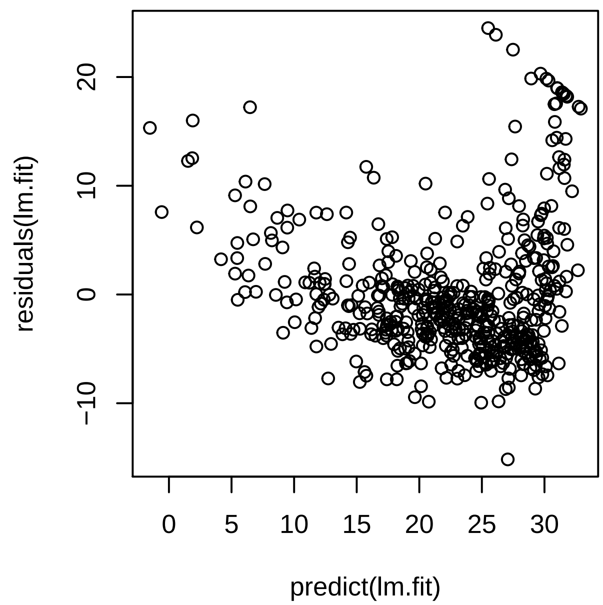
- We will now plot the medv and lstat along with the least squares regression line using the `plot()` and `abline()` function:

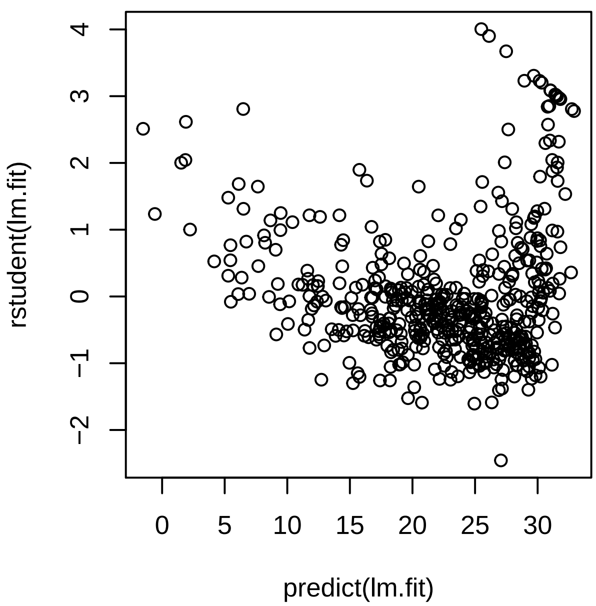
```r
par(mfrow = c(2,2))
plot(lstat, medv, pch = "+")
abline(lm.fit, lwd = 3, col = "red")

## We plot the residuals versus fitted values:
plot(predict(lm.fit), residuals(lm.fit))
```

```r
### or with student residuals:
plot(predict(lm.fit), rstudent(lm.fit))
```

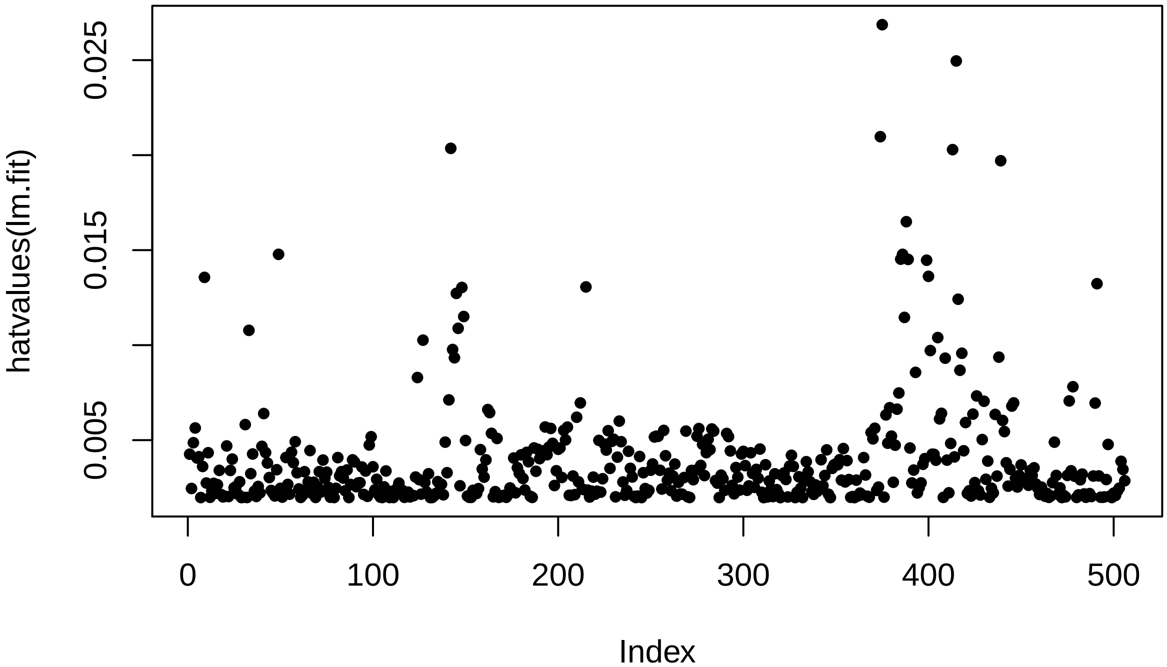






- There is some evidence of non-linearity. Leverage statsitics can be computed using the `hatvalues()` function:
- The `which.max()` function identifies the index of the largest element of a vector.

```r
plot(hatvalues(lm.fit), pch = 20)
```




```r
which.max(hatvalues(lm.fit))
```

```
## 375
## 375
```

3.6.3 Multiple linear regression:

- We can fit a multiple linear regression:

```r
lm.fit = lm(medv ~ lstat + age, data = Boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat + age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.981  -3.978  -1.283   1.968  23.158
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.22276    0.73085  45.458  < 2e-16 ***
## lstat       -1.03207    0.04819 -21.416  < 2e-16 ***
## age          0.03454    0.01223   2.826  0.00491 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.173 on 503 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5495
```

```
## F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

- The dataset `Boston` contains 13 variables and so it would be cumbersom to have to type all of these in order to perform a regression using all of the predictors.

```r
lm.fit = lm(medv ~., data = Boston)

summary(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ ., data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.595  -2.730  -0.518   1.777  26.199
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.646e+01  5.103e+00   7.144 3.28e-12 ***
## crim        -1.080e-01  3.286e-02  -3.287 0.001087 **
## zn           4.642e-02  1.373e-02   3.382 0.000778 ***
## indus        2.056e-02  6.150e-02   0.334 0.738288
## chas         2.687e+00  8.616e-01   3.118 0.001925 **
## nox         -1.777e+01  3.820e+00  -4.651 4.25e-06 ***
## rm           3.810e+00  4.179e-01   9.116  < 2e-16 ***
## age          6.922e-04  1.321e-02   0.052 0.958229
## dis         -1.476e+00  1.995e-01  -7.398 6.01e-13 ***
## rad          3.060e-01  6.635e-02   4.613 5.07e-06 ***
## tax         -1.233e-02  3.760e-03  -3.280 0.001112 **
## ptratio     -9.527e-01  1.308e-01  -7.283 1.31e-12 ***
## black        9.312e-03  2.686e-03   3.467 0.000573 ***
## lstat       -5.248e-01  5.072e-02 -10.347  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.745 on 492 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7338
## F-statistic: 108.1 on 13 and 492 DF,  p-value: < 2.2e-16
```

- To compute the VIF's Variance Inflation Factor using the library `car` in `R`:

```r
library(car)
vif(mod = lm.fit)
```

```
##     crim       zn    indus     chas      nox       rm      age      dis
## 1.792192 2.298758 3.991596 1.073995 4.393720 1.933744 3.100826 3.955945
##      rad      tax  ptratio    black    lstat
## 7.484496 9.008554 1.799084 1.348521 2.941491
```

- `age` has a high p-value so we may wish to run a regression excluding this predictor. The following syntax results in a regression using all predictors except `age`:

```r
lm.fit1 = lm(medv ~. -age, data = Boston)
summary(lm.fit1)
```

```
##
## Call:
```

```
## lm(formula = medv ~ . - age, data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.6054  -2.7313  -0.5188   1.7601  26.2243
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.436927   5.080119   7.172 2.72e-12 ***
## crim         -0.108006   0.032832  -3.290 0.001075 **
## zn            0.046334   0.013613   3.404 0.000719 ***
## indus         0.020562   0.061433   0.335 0.737989
## chas          2.689026   0.859598   3.128 0.001863 **
## nox         -17.713540   3.679308  -4.814 1.97e-06 ***
## rm            3.814394   0.408480   9.338  < 2e-16 ***
## dis          -1.478612   0.190611  -7.757 5.03e-14 ***
## rad           0.305786   0.066089   4.627 4.75e-06 ***
## tax          -0.012329   0.003755  -3.283 0.001099 **
## ptratio      -0.952211   0.130294  -7.308 1.10e-12 ***
## black         0.009321   0.002678   3.481 0.000544 ***
## lstat        -0.523852   0.047625 -10.999  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.74 on 493 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7343
## F-statistic: 117.3 on 12 and 493 DF,  p-value: < 2.2e-16
```

- Alternatively the `update()` function can be used:

```r
lm.fit1 = update(lm.fit, ~.-age)
summary(lm.fit1)
```

```
##
## Call:
## lm(formula = medv ~ crim + zn + indus + chas + nox + rm + dis +
##     rad + tax + ptratio + black + lstat, data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.6054  -2.7313  -0.5188   1.7601  26.2243
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.436927   5.080119   7.172 2.72e-12 ***
## crim         -0.108006   0.032832  -3.290 0.001075 **
## zn            0.046334   0.013613   3.404 0.000719 ***
## indus         0.020562   0.061433   0.335 0.737989
## chas          2.689026   0.859598   3.128 0.001863 **
## nox         -17.713540   3.679308  -4.814 1.97e-06 ***
## rm            3.814394   0.408480   9.338  < 2e-16 ***
## dis          -1.478612   0.190611  -7.757 5.03e-14 ***
## rad           0.305786   0.066089   4.627 4.75e-06 ***
## tax          -0.012329   0.003755  -3.283 0.001099 **
## ptratio      -0.952211   0.130294  -7.308 1.10e-12 ***
```

```
## black         0.009321   0.002678   3.481 0.000544 ***
## lstat        -0.523852   0.047625 -10.999  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.74 on 493 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7343
## F-statistic: 117.3 on 12 and 493 DF,  p-value: < 2.2e-16
```

3.6.4 Interaction Terms:

- The syntax : tells R`~ to include an interaction term between 2 variables. The syntax`*simultaneously `includeslstatage‘ and the interaction term.

```
lm.fit = lm(medv~lstat*age, data = Boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat * age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.806  -4.045  -1.333   2.085  27.552
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) 36.0885359  1.4698355  24.553  < 2e-16 ***
## lstat       -1.3921168  0.1674555  -8.313 8.78e-16 ***
## age         -0.0007209  0.0198792  -0.036   0.9711
## lstat:age    0.0041560  0.0018518   2.244   0.0252 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.149 on 502 degrees of freedom
## Multiple R-squared:  0.5557, Adjusted R-squared:  0.5531
## F-statistic: 209.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

3.6.5 Nonlinear transformation of the predictors:

- We perform a regression of `medv` onto `lstat` and `lstat^2`:

```
lm.fit = lm(medv ~ lstat + I(lstat^2), data = Boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat + I(lstat^2), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.2834  -3.8313  -0.5295   2.3095  25.4148
```

```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 42.862007   0.872084   49.15   <2e-16 ***
## lstat       -2.332821   0.123803  -18.84   <2e-16 ***
## I(lstat^2)   0.043547   0.003745   11.63   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.524 on 503 degrees of freedom
## Multiple R-squared:  0.6407, Adjusted R-squared:  0.6393
## F-statistic: 448.5 on 2 and 503 DF,  p-value: < 2.2e-16
```

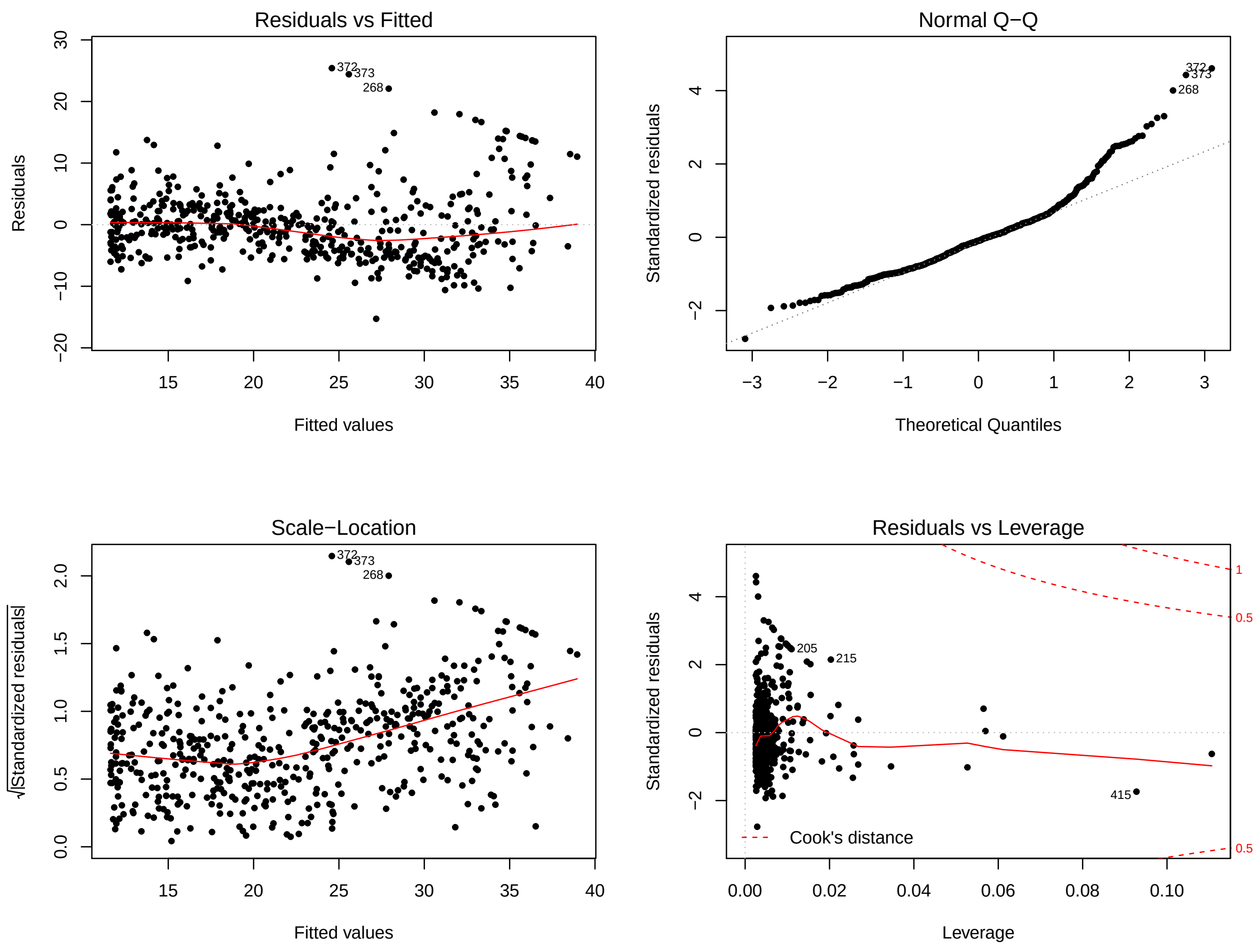
- We use the `anova()` function to quantify the extent to which the quadratic fit is superior to the linear fit.

```
lm.fit2 = lm(medv ~ lstat, data = Boston)
anova(lm.fit2, lm.fit)
```

```
## Analysis of Variance Table
##
## Model 1: medv ~ lstat
## Model 2: medv ~ lstat + I(lstat^2)
##   Res.Df   RSS Df Sum of Sq     F    Pr(>F)
## 1    504 19472
## 2    503 15347  1    4125.1 135.2 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Clear evidence that the model containing the predictors `lstat` and `lstat^2` is far superior to the model that only contains the predictor `lstat`.

```
par(mfrow = c(2,2))
plot(lm.fit, pch = 20)
```



- An alternative way is to use `poly()` function to create the polynomial within `lm()` function.

```
lm.fit5 = lm(medv~poly(lstat, degree = 5), data = Boston)
summary(lm.fit5)
```

```
##
## Call:
## lm(formula = medv ~ poly(lstat, degree = 5), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5433  -3.1039  -0.7052   2.0844  27.1153
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)                22.5328     0.2318  97.197  < 2e-16 ***
## poly(lstat, degree = 5)1 -152.4595     5.2148 -29.236  < 2e-16 ***
## poly(lstat, degree = 5)2   64.2272     5.2148  12.316  < 2e-16 ***
## poly(lstat, degree = 5)3  -27.0511     5.2148  -5.187 3.10e-07 ***
## poly(lstat, degree = 5)4   25.4517     5.2148   4.881 1.42e-06 ***
## poly(lstat, degree = 5)5  -19.2524     5.2148  -3.692 0.000247 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.215 on 500 degrees of freedom
```

```
## Multiple R-squared:  0.6817, Adjusted R-squared:  0.6785
## F-statistic: 214.2 on 5 and 500 DF,  p-value: < 2.2e-16
```

- Of course, we are in no way restrcited to using polynomial transformations of the predictors. We try `log` transformation:

```r
lm.log = lm(medv~log(lstat), data = Boston)
summary(lm.log)
```

```
##
## Call:
## lm(formula = medv ~ log(lstat), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.4599  -3.5006  -0.6686   2.1688  26.0129
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  52.1248     0.9652   54.00   <2e-16 ***
## log(lstat)  -12.4810     0.3946  -31.63   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.329 on 504 degrees of freedom
## Multiple R-squared:  0.6649, Adjusted R-squared:  0.6643
## F-statistic:  1000 on 1 and 504 DF,  p-value: < 2.2e-16
```

3.6.6 Qualitative Predictors:

- We examine the data set `Carseats` part of the `ISLR` library. We predict `Sales` (child car seat sales) in 400 locations based on the number of predictors:

```r
library(ISLR)

names(Carseats)
```

```
##  [1] "Sales"       "CompPrice"   "Income"      "Advertising" "Population"
##  [6] "Price"       "ShelveLoc"   "Age"         "Education"   "Urban"
## [11] "US"
```

- The data set includes qualitative predictors such as `Shelveloc` an indicator of the quality of the shelving location - the space within a store in which the car seat is displayed at each location. The predictor `Shelveloc` takes on 3 values: Bad, Medium and Good.

```r
lm.fit = lm(Sales ~. + Income:Advertising +Price:Age, data = Carseats)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Sales ~ . + Income:Advertising + Price:Age, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9208 -0.7503  0.0177  0.6754  3.3413
##
```

```
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)    
## (Intercept)         6.5755654  1.0087470   6.519 2.22e-10 ***
## CompPrice           0.0929371  0.0041183  22.567  < 2e-16 ***
## Income              0.0108940  0.0026044   4.183 3.57e-05 ***
## Advertising         0.0702462  0.0226091   3.107 0.002030 ** 
## Population          0.0001592  0.0003679   0.433 0.665330    
## Price              -0.1008064  0.0074399 -13.549  < 2e-16 ***
## ShelveLocGood       4.8486762  0.1528378  31.724  < 2e-16 ***
## ShelveLocMedium     1.9532620  0.1257682  15.531  < 2e-16 ***
## Age                -0.0579466  0.0159506  -3.633 0.000318 ***
## Education          -0.0208525  0.0196131  -1.063 0.288361    
## UrbanYes            0.1401597  0.1124019   1.247 0.213171    
## USYes              -0.1575571  0.1489234  -1.058 0.290729    
## Income:Advertising  0.0007510  0.0002784   2.698 0.007290 ** 
## Price:Age           0.0001068  0.0001333   0.801 0.423812    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 1.011 on 386 degrees of freedom
## Multiple R-squared:  0.8761, Adjusted R-squared:  0.8719 
## F-statistic:   210 on 13 and 386 DF,  p-value: < 2.2e-16
```

- The `contrasts()` function returns the coding that `R` uses for the dummy variables.

```r
attach(Carseats)

contrasts(ShelveLoc)
```

```
##        Good Medium
## Bad       0      0
## Good      1      0
## Medium    0      1
```

```r
str(Carseats)
```

```
## 'data.frame':    400 obs. of  11 variables:
##  $ Sales      : num  9.5 11.22 10.06 7.4 4.15 ...
##  $ CompPrice  : num  138 111 113 117 141 124 115 136 132 132 ...
##  $ Income     : num  73 48 35 100 64 113 105 81 110 113 ...
##  $ Advertising: num  11 16 10 4 3 13 0 15 0 0 ...
##  $ Population : num  276 260 269 466 340 501 45 425 108 131 ...
##  $ Price      : num  120 83 80 97 128 72 108 120 124 124 ...
##  $ ShelveLoc  : Factor w/ 3 levels "Bad","Good","Medium": 1 2 3 3 1 1 3 2 3 3 ...
##  $ Age        : num  42 65 59 55 38 78 71 67 76 76 ...
##  $ Education  : num  17 10 12 14 13 16 15 10 10 17 ...
##  $ Urban      : Factor w/ 2 levels "No","Yes": 2 2 2 2 2 1 2 2 1 1 ...
##  $ US         : Factor w/ 2 levels "No","Yes": 2 2 2 2 1 2 1 2 1 2 ...
```

- ShelveLocGood dummary takes on value of 1 if the location is good, and 0 otherwise. ShelvelocMedium is 1 if the shelving location is medium and 0 otherwise. A bad shelving location corresponds to the zero for each of the two dummy variable.