A priority of preferences for teacher assignment problem

ADVANCED ALGORITHMS

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Content

Problem statement

The problem of assigning lecturers based on priority is described as the following:

- Each training program has a list of subjects, and each subject has a list of full-time lecturers.
- Teachers register the subjects in the curriculum according to the priority that is not the same for each subject, numbered from 1 to n, with 1 being the most desired to teach.
- Based on the maximum number of classes the teacher can take in the semester and the subjects that the teacher has registered, the head of the department assigns the teacher to the list of subject classes to satisfy two main goals:
  - O1. Maximum number of classes that can be assigned to teachers.
  - O2. Minimum total priority of all assigned teachers.

Proposed method

- Step 1: Transform the original problem into a standard assignment problem.
- Step 2: Reformulate mathematical model.
- Step 3: Solve the problem using Genetic Algorithm.

Step 1 - Transform

Constraints:

- Each class is assigned with maximum one teacher:

$$\sum_{i=1}^{n} x_{i,j} \leq 1, \quad \forall j = 1, \ldots, m \tag{1}$$

Tabela 1: Priority matrix ($n \times m$)

Teacher ID \ Class ID	C03-1	C04-1	C04-2	C05-1	C05-2	C05-3	C06-1	C06-2	C07-1
T01	3	4	4	999	999	999	999	999	999
T02	1	2	2	3	3	3	4	4	999
T03	999	999	999	1	1	1	2	2	3
T05	999	999	999	999	999	999	999	999	999

Step 1 - Transform

Constraints:

- Each teacher is assigned with a maximum of *mc* of classes, where *mc* is the maximum number of classes that the teacher registers to teach in the semester:

$$\sum_{j=1}^{m} x_{i,j} \leq mc_i, \quad \forall i = 1, \ldots, n \tag{2}$$

Tabela 2: Replicated priority matrix ($n_1 \times m_1$)

Teacher ID \ Class ID	C03-1	C04-1	C04-2	C05-1	C05-2	C05-3	C06-1	C06-2	C07-1
T01-1	3	4	4	999	999	999	999	999	999
T01-2	3	4	4	999	999	999	999	999	999
T02-1	1	2	2	3	3	3	4	4	999
T02-2	1	2	2	3	3	3	4	4	999
T02-3	1	2	2	3	3	3	4	4	999
T03-1	999	999	999	1	1	1	2	2	3
T03-2	999	999	999	1	1	1	2	2	3
T05-1	999	999	999	999	999	999	999	999	999

Step 1 - Transform

Constraints:

- Either all classes are assigned with 1 teacher, or all teachers are assigned with *mc* classes that they registered:

$$(\sum_{i=1}^{n} x_{i,j} - 1) \times (\sum_{j=1}^{m} x_{i,j} - mc_i) = 0, \quad \forall i = 1, \ldots, n, \quad \forall j = 1, \ldots, m \quad (3)$$

Tabela 3: Padded priority matrix ($n_2 \times n_2$)

Teacher ID \ Class ID	C03-1	C04-1	C04-2	C05-1	C05-2	C05-3	C06-1	C06-2	C07-1
T01-1	3	4	4	999	999	999	999	999	999
T01-2	3	4	4	999	999	999	999	999	999
T02-1	1	2	2	3	3	3	4	4	999
T02-2	1	2	2	3	3	3	4	4	999
T02-3	1	2	2	3	3	3	4	4	999
T03-1	999	999	999	1	1	1	2	2	3
T03-2	999	999	999	1	1	1	2	2	3
T05-1	999	999	999	999	999	999	999	999	999
Txxx	0	0	0	0	0	0	0	0	0

Step 1 - Transform

Constraints:

- Basic-subject classes must be opened:

$$\sum_{i=1}^{n} x_{i,j} = 1, \quad \forall j \in C_B \tag{4}$$

Tabela 4: Added priority matrix ($n_2 \times n_2$). Bold and italic numbers are added to the max priority

Teacher ID n Class ID	C03-1	C04-1	C04-2	C05-1	C05-2	C05-3	C06-1	C06-2	C07-1
T01-1	3	4	4	999	999	999	999	999	999
T01-2	3	4	4	999	999	999	999	999	999
T02-1	1	2	2	3	3	3	***8***	***8***	999
T02-2	1	2	2	3	3	3	***8***	***8***	999
T02-3	1	2	2	3	3	3	***8***	***8***	999
T03-1	999	999	999	1	1	1	***6***	***6***	***7***
T03-2	999	999	999	1	1	1	***6***	***6***	***7***
T05-1	999	999	999	999	999	999	999	999	999
Txxx	0	0	0	0	0	0	0	0	0

Step 2 - Reformulate mathematical model

The mathematical model is reformulated as the following:

- The cost matrix $C_2 = \{c_{ij}\}$ $(\forall i = 1, \ldots, n_2; \forall j = 1, \ldots, n_2)$ is the latest transformed priority matrix provided shown in the Tab. 4.
- Decision variable: $x_{i,j} \in \{0, 1\}$ has value 1 if the teacher i $(i = 1, \ldots, n_2)$ is assigned to the class j $(j = 1, \ldots, n_2)$, and 0 otherwise.
- Objective function: minimum sum of priority of assigned teachers, where $c_{i,j}$ is the priority at the cell (i, j).

$$\min\left(\sum_{i=1}^{n_2} \sum_{j=1}^{n_2} c_{i,j} \times x_{i,j}\right) \tag{5}$$

Step 2 - Solve With Hungarian Method

This transformed problem can be directly solved by the Hungarian algorithm. After matching, the assignment is then post-processed to obtain a final semantic result. There are following cases:

- If the assignment pair has value of 999, this pair would be canceled because the teacher does not register the class.
- In terms of basic-subject classes, if the number of matched teachers are not enough, an alert is notified so that the department head can contact part-time lectures in order to ensure unassigned classes must be opened.

Step 3 - Solve with GA

Transform the problem with changing some representations:

- Vector for classes: $O = [1, 2, 3, ..., \#classes]$
- Vector for Teacher: $P = [1, 2, 3, ..., \#teachers]$
- With some constrains:

$$|O| = |P| = n_2 \tag{6}$$

- So, we can have the result that matching:

$$(O_i, P_i), \ \ \forall i = 1, \ldots, n_2 \tag{7}$$

Step 3 - Solve with GA

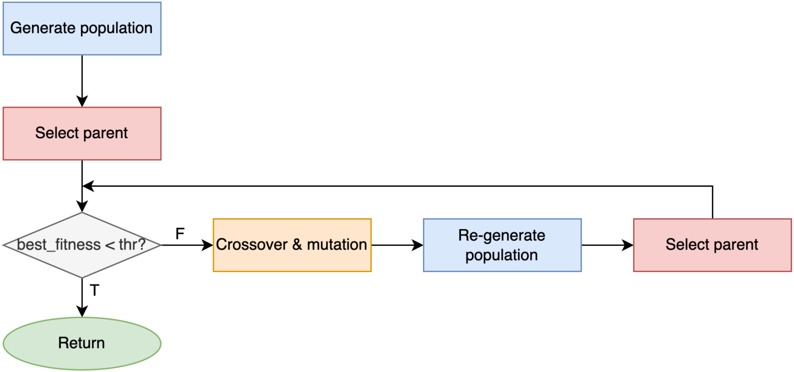

Figura 1: Genetic algorithm pipeline

Experimental results

- For case default: The number of classes is larger than the number of teachers.

Tabela 5: (Case 3) Registration of courses with the priority of the teacher

Course\Teacher	T01	T02	T03	T04	T05	T06
C01	1	3	1	0	2	1
C02	2	2	2	3	3	0
C03	3	1	3	2	0	0
C04	4	0	0	1	1	0
C05	0	4	0	4	0	2

Tabela 6: (Case 3) Classes to be opened

Course ID	Basic	No. Classes
C01	1	3
C02	0	2
C03	1	2
C04	0	4
C05	1	4
Total		**15**

Tabela 7: (Case 3) Registered teachers

Teacher ID	Max. Classes
T01	3
T02	2
T03	2
T04	3
T05	1
T06	1
Total	**12**

Experimental results

For case default: Both the methods have a comparable solution in term of all metrics, excepting run-time. The Hungarian-based one is faster the Genetic-based one.

Tabela 8: (Case default) Results.

Course	Hungarian		Genetic	
	Teacher	Priority	Teacher	Priority
C01	T01	1	T01	1
C01	T01	1	T01	1
C01	T01	1	T03	1
C02	T03	2	T03	2
C02	T03	2	T01	2
C03	T02	1	T02	1
C03	T02	1	T02	1
C04	n/a	n/a	T05	1
C04	n/a	n/a	n/a	n/a
C04	n/a	n/a	n/a	n/a
C04	T05	1	n/a	n/a
C05	T06	2	T06	2
C05	T04	4	T04	4
C05	T04	4	T04	4
C05	T04	4	T04	4
Count	12		12	
Sum		24		24
Std		1.28		1.28
Mean		2.0		2.0
Time(ms)		8		596

Experimental results

Case new: There is a basic subject but no one want to teach it.

Tabela 9: (Case 5) Registered teachers

Teacher ID	Max. Classes
T01	2
T02	3
T03	2
T04	0
T05	1
Total	**8**

Tabela 10: (Case 5) Classes

Course ID	Basic	No. Classes
C01	1	0
C02	1	0
C03	1	1
C04	1	2
C05	1	3
C06	0	2
C07	0	1
C08	0	0
C09	0	0
C10	0	0
Total		**9**

Tabela 11: (Case 5) Priority

Course\Teacher	T01	T02	T03	T04	T05
C01	1	0	0	0	3
C02	2	0	0	0	4
C03	3	1	0	0	0
C04	4	2	0	0	0
C05	0	3	1	0	0
C06	0	4	2	0	0
C07	0	0	3	1	0
C08	0	0	4	2	0
C09	0	0	0	3	1
C10	0	0	0	4	2

Experimental results

For new case:

- The Hungarian-based one is totally better than the Genetic-based one because the former can exploit more teacher than the latter
- The course C05 is a basic-subject but no one is assigned to teach this class (highlight in the red color)
- The head of department must contact part-time lectures in order to host these basic-subject classes.

Tabela 12: (Case new) Assignment results of the Hungarian-based and Genetic-based methods

Course	Hungarian		Genetic	
	Teacher	Priority	Teacher	Priority
C03	T02	1	T02	1
C04	T02	2	T02	2
C04	T01	4	T02	2
C05	n/a	n/a	T01	n/a
C05	T01	n/a	T01	n/a
C05	T05	n/a	T05	n/a
C06	T02	4	T03	2
C06	T03	2	T03	2
C07	T03	3	n/a	n/a
Count	6		5	
Sum		16		9
Std		1.21		0.45
Mean		2.67		1.80
Time(ms)		9		280

Conclusion

- Transform the original assignment problem with constraints to solve by using another algorithm like Hungarian Algorithm.
- Exploit the Genetic Algorithm to solve this problem and compare with Hungarian Algorithm:
    - The Genetic Algorithm does not always achieve a comparable output as Hungarian Algorithm.
    - Genetic algorithm is more flexible than the proposed method in term of constraints.
- In the future, we will investigate other conditional assignment problems to verify this.

THANKS FOR YOUR ATTENTION!