

**BNAD 276. STATISTICAL INFERENCE IN MANAGEMENT****Presession. Summer 2017.****FINAL EXAM. DATE: FRIDAY 2 JUNE 2017****Time: 120 minutes (2 hours)**

NOTE:

1. There are total 33 questions of which there are 81 sub-questions.
2. You can use calculator. Your calculator may be programmable but you are not allowed to use the programming tools.
3. There are 3 parts:

Part 1: Mandatory. You must answer all questions in this part, i.e. question 1 to question 16 (39 sub-questions) from page 2 to page 18. The total score of this part is 53.

Part 2: Optional: question 17 to question 26. Several questions have sub-questions, eg. 17.1, 17.2, etc. There are 33 sub-questions in this part from page 19 to page 29. Each sub-question is worth 1 point. **You can choose to answer how many sub-questions (N) and which sub-questions.** For example, you can choose to answer only 17.1 and 18.2. In the end, **your total score of this part is the correct credits within the N sub-questions you've chosen.** When you're about to submit your exam, please write down explicitly here which sub-questions you chose to be counted into the final score (**or clearly state "counted" next to those sub-questions**):

Your N = ..... (a number from 0 to 33), they are .....

Part 3: Extra credits: question 27 to question 33 from page 20 to page 33. There are 9 sub-questions in this part. Each is worth 1 point.

**You final score of the exam = (mandatory score + optional score + extra credits) / (53 + N)**

Good luck!

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## PART I. MANDATORY PART. YOU MUST ANSWER ALL QUESTIONS IN THIS PART.

**Question 1.** (3 points) You are given 6 formulas/descriptions/notations on the left and 9 statistical concepts on the right column. Please match each formulas/description to its corresponding statistical concepts. Notice there are **5 redundant concepts**, i.e. 5 concepts that do not have the corresponding descriptions.

Hence, you need to **match 6 formulas/descriptions to the most correct 6 concepts**. Each correct match is worth 0.5 point.

1. $\frac{s_{xy}}{s_x s_y}$	A. Sample mean
2. $s_{xy} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{N-1}$	B. Mean
3. $A \cup B$	C. Sample variance
4. $\frac{\sum X_i}{N}$	D. Variance
5. $P(A^c) = 1 - P(A)$	E. Standard deviation
6. $S_X^2 = \frac{\sum (X_i - \bar{X})^2}{N-1}$	F. Sample covariance
	G. Correlation coefficient
	H. Union event
	I. Intersection event
	J. Conditional event
	K. Complement event

Please write down your answer as specific as 1A, 2B, 3C... below:

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

**Question 2. (4 points)** You are given 8 formulas/descriptions/notations on the left and 11 statistical concepts on the right column. Please match each formulas/description to its corresponding statistical concepts. Notice there are **3 redundant concepts**, i.e. 3 concepts that do not have the corresponding descriptions.

Hence, you need to **match 8 formulas/descriptions to the correct 8 concepts**. Each correct match is worth 0.5 point.

1.  $f(x) = \frac{1}{b-a}$  for  $a \leq x \leq b$

2. *bell curve/bell shape*

3.  $\sum xf(x)$

4.  $\sigma$

5.  $\sum x^2 f(x) - \mu^2$

6.  $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

7.  $\sum x^2 f(x)$

8. Number of occurrences in a time period

A. Bernoulli distribution

B. Binomial distribution

C. Poisson distribution

D. Normal distribution

E. Uniform distribution

F. Standard deviation

G. Sample variance

H. Mean

I. Sample mean

J. Variance

K. Second moment,  $E(X^2)$

Please write down your answer as specific as 1A, 2B, 3C... below:

1.

2.

3.

4.

5.

6.

7.

8.

**Question 3.** (1 pt) The measure of dispersion which is not measured in the same units as the original data is the... (Choose one correct answer among the belows)

- a. correlation coefficient
- b. standard deviation
- c. coefficient of determination
- d. variance

**Question 4.** (3 pts) The following data is used for questions 4.1, 4.2, 4.3

The closing stock price of 7 corporations on Friday is shown below.

Corporation	Stock Price
A	83
B	87
C	83
D	89
E	85
F	90
G	91

4.1 (1 point) What is the type of this data set? (Choose a or b or c or d)

- a. Cross-sectional data
- b. Time-series data
- c. Panel data
- d. Longitudinal data

4.2 (1 point) What is the **mean** value of the stock price?

4.3 (1 point) What is the **median** value of the stock price?

**Question 5. (6 pts) The following data set is used for questions 5.1, 5.2, 5.3, 5.4, 5.5, 5.6**

The ABC Company has reported the number of commercial advertisings and the sale revenues as follows:

Month	Advertisings (campaigns)	Sale Revenues(\$)
1	33	40
2	27	30
3	7	10
4	15	20
5	20	25

You may find the below mysterious table useful to answer questions.

33	40	12.6	15	189	158.76	225
27	30	6.6	5	33	43.56	25
7	10	-13.4	-15	201	179.56	225
15	20	-5.4	-5	27	29.16	25
20	25	-0.4	0	0	0.16	0

5.1. What is the **sample variance** of the **number of advertisings**? (1 point)

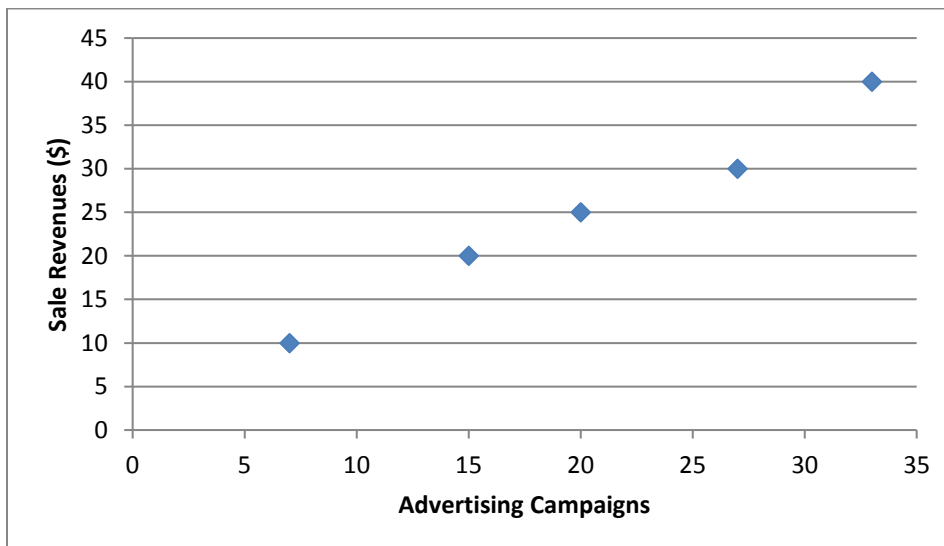
5.2. What is the **sample variance** of the **sales revenue**? (1 point)

Name:

Student ID:

5.3. Calculate the **sample covariance** of the Advertising Campaigns and the Sales Revenues? (1 point).

5.4. What is the name of the plot below? (1 point)



5.5. (1 pt) What do you think about the **sign and the magnitude** of the **sample correlation coefficient** of the Price Per Gallon and the Number of Gallons? Specifically, is the sample correlation coefficient negative, positive? Is it near to - 1, 0, or 1? (You can either calculate the correlation coefficient to answer this question, or you can look at the plot in the above question to answer this question.)

5.6. (1 pt) Which of the following statement is the **most complete correct**?

- a. The sample correlation coefficient of the two variables (number of advertisings and sales revenues) is positive. This means that an increase in the number of advertising is associated with an increase in the sales revenue.
- b. The sample covariance of the two variables (number of advertisings and sales revenues) is positive. This means that the two variables have a positive relationship.
- c. The sample correlation coefficient of the two variables (number of advertisings and sales revenues) is positive and close to 1. This means that an increase in the number of advertising leads to an increase in the sales revenue (causal relationship).
- d. The sample correlation coefficient of the two variables (number of advertisings and sales revenues) is negative and close to - 1. This means that an increase in the number of advertising leads to a decrease in the sales revenue (causal relationship).

**Question 6. (3 points)** Suppose  $Z$  is standard normal random variable, and let  $\sigma$  be the standard deviation of  $Z$ .

6.1. (1 pt) What is the probability that  $P(Z < 1.12)$ ?

6.2.(1 pt) What is the probability that  $P(Z > 2.11)$ ?

6.3.(1pt) What is the probability that  $Z$  is between  $-2\sigma$  and  $2\sigma$  ?



**Question 7. (3 points)** Suppose that a random variable  $X$  has the following probability density function:

$$f(x) = \frac{1}{40\sqrt{\pi}} e^{\frac{-(x-90)^2}{1600}}$$

7.1. (1 pt) What is the name of the distribution above? What is the mean value of  $X$ ?

7.2. (1 pt) What is the variance of  $X$ ? What is the standard deviation of  $X$ ?

7.3. (1 pt) Draw the pdf  $f(x)$  above. Put  $x$  on the horizontal axis and  $f(x)$  on the vertical axis. Indicate clearly the **location of the mean**. (You don't need to draw a very precise graph. You only need to show how it looks like and the location of the mean on the graph)

**Question 8.** (1pt) Is the following a valid probability mass function? Explain why.

f(x)	x	
0.2	0.2	
-0.2	0	
0.5	0.4	
0.5	0.4	
		Total

**Question 9.** (1pt) A machine is designed to fill toothpaste tubes with 5.8 ounces of toothpaste. The manufacturer does not want any underfilling or overfilling. The correct hypotheses to be tested are

- a.  $H_0: \mu \neq 5.8$     $H_a: \mu = 5.8$
- b.  $H_0: \mu = 5.8$     $H_a: \mu \neq 5.8$
- c.  $H_0: \mu > 5.8$     $H_a: \mu \leq 5.8$
- d.  $H_0: \mu \geq 5.8$     $H_a: \mu < 5.8$

**Question 10.** (1 pt) Is this probability mass function valid? Explain why?

f(x)	x	
0.2	-1	
0.1	0	
0.6	2	
		Total

**Question 11. (3pts)** An experimental diet to induce weight loss was followed for three months by a randomly selected group of 6 students with the following results.

Student	Loss in Pounds
1	15
2	16
3	18
4	15
5	14
6	12

**11.1.(1pt)** Find a point estimate for the average amount lost after three months on this diet.

**11.2. (1pt)** Find a point estimate for the variance of the amount lost on this diet. (Hint: you are given the mysterious table below. Please check carefully before using the table.)

Student	Loss in Pounds				
1	15	0	0	0	0
2	16	1	1	1	1
3	18	9	3	81	27
4	15	0	0	0	0
5	14	1	-1	1	-1
6	12	9	-3	81	-27

Name:

Student ID:

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**11.3.** (1pt) Find a point estimate for the standard deviation of the amount lost on this diet.

**Question 12.** (2pts) Assume that the population variance is 40, the population mean is 30, and the sample size is 150.

**12.1.** (1pt) What is the sampling distribution of the sample mean? Choose one answer:

- a. Normal distribution with mean 30 and standard deviation  $\frac{40}{150}$
- b. Normal distribution with mean 30 and standard deviation  $\sqrt{\frac{40}{150}}$
- c. Normal distribution with mean 30 and variance  $\frac{40}{\sqrt{150}}$
- d. Normal distribution with mean 30 and variance  $\frac{\sqrt{40}}{150}$
- e. Normal distribution with mean 30 and variance  $\sqrt{\frac{40}{150}}$

**12.2.** (1pt) What is the probability that the sample mean will be **less than 50**? Let variable Z follow standard normal distribution. Choose one answer:

- a.  $P(Z < \frac{50-30}{\frac{40}{150}})$
- b.  $P(Z < \frac{50-30}{\sqrt{\frac{40}{150}}})$
- c.  $P(Z < \frac{50-30}{\frac{40}{\sqrt{150}}})$
- d.  $1 - P(Z < \frac{50-30}{\frac{40}{\sqrt{150}}})$
- e.  $P(Z < \frac{50-30}{\frac{\sqrt{40}}{150}})$

**Question 13. (2 pts)** It is stated that 60% of primary care doctors think their patients receive unnecessary medical care (Reader's Digest, December 2011/January 2012). Suppose a sample of 200 primary care doctors were taken.

**13.1.** (1pt) Show the sampling distribution of the sample proportion of the doctors who think their patients receive unnecessary medical care. **Clearly state the name of the distribution, the mean and the variance of the sampling distribution.**

**13.2.** (1pt) What is the probability that the sample proportion will be **greater than 0.3**? Let Z be the standard normal distribution. Choose one answer among the followings:

a.  $P\left(Z < \frac{0.3-0.6}{\frac{\sqrt{0.6 \times 0.4}}{200}}\right)$  since  $N = 200$ ,  $p = 0.6$

b.  $P\left(Z < \frac{0.3-0.6}{0.0346}\right)$

c.  $1 - P\left(Z < \frac{0.3-0.6}{0.0346}\right)$

d.  $1 - P\left(Z < \frac{0.3-0.6}{\sqrt{0.6 \times 0.4}}\right)$

e.  $P\left(Z < \frac{0.3-0.6}{\frac{\sqrt{0.6 \times 0.4}}{200}}\right)$  since  $N = 200$ ,  $p = 0.6$

**Question 14. (2 pts)** In order to determine the average weight of carry-on luggage by passengers in airplanes, a sample of 15 pieces of carry-on luggage was weighed. The average sample weight was 20 pounds.

**14.1. (1pt)** Determine a 90% confidence interval estimate for the mean weight of the carry-on luggage. Assume that we know the **standard deviation of the population** to be 8 pounds. (The final result must be an interval [a,b]; you can leave your final result like the form in question 14.2, i.e. without giving a specific number in the end.)

**14.2. (1pt)** Determine a 90% confidence interval estimate for the mean weight of the carry-on luggage. Assume that the **sample standard deviation** is 8 pounds.

- a.  $\left[20 - 1.960 \times \frac{4}{\sqrt{15}}; 20 + 1.960 \times \frac{4}{\sqrt{15}}\right]$
- b.  $\left[20 - 1.645 \times \frac{4}{\sqrt{15}}; 20 + 1.645 \times \frac{4}{\sqrt{15}}\right]$
- c.  $\left[20 - 1.761 \times \frac{4}{\sqrt{14}}; 20 + 1.761 \times \frac{4}{\sqrt{14}}\right]$
- d.  $\left[20 - 1.761 \times \frac{4}{\sqrt{15}}; 20 + 1.761 \times \frac{4}{\sqrt{15}}\right]$
- e.  $\left[20 - 1.753 \times \frac{4}{\sqrt{14}}; 20 + 1.753 \times \frac{4}{\sqrt{14}}\right]$
- f.  $\left[20 - 1.753 \times \frac{4}{\sqrt{15}}; 20 + 1.753 \times \frac{4}{\sqrt{15}}\right]$

**Question 15. (4 pts)** A student believes that the average grade on the statistics final examination was 85. A sample of 36 final examinations was taken. The average grade in the sample was 80. Assume the population standard deviation is 12.

15.1.(1pt) State the null and alternative hypotheses testing the student's belief.

15.2 (1pt) Compute the test statistics. Clearly state whether it is the test statistics t or test statistics z.

15.3. (2pts) Test the hypotheses at the level of significance of 5%. Do you reject or accept  $H_0$ ? You can use whatever approach.



**WARNING: The multiple choice sub-questions in the question 16 have more than one correct answer. Read the sub-questions carefully.**

**Question 16.** (14 pts) Suppose  $X$  follows the normal distribution with the mean  $\mu = 50$  and the standard deviation  $\sigma = 40$

**16.1.**(1pt) Calculate  $P(X < 62.8)$ . Show your work, not just the final number.

**16.2.**(4pts) What is equal to  $P(-30.4 < X < 58.4)$ ? **Choose 4 correct answers** among the followings: (Hint: illustrating the probability by the graph of area under the density function is helpful.)

- a.  $P(X < 58.4) + P(X < -30.4)$
- b.  $P(Z < 0.21) - P(Z < -2.01)$
- c.  $P(X < 58.4) - P(X > -30.4)$
- d.  $0.5832 + 0.0222$
- e.  $0.5832 - 0.0222$
- f.  $P(X > 58.4) - P(X < -30.4)$
- g.  $P(X < 58.4) - P(X < -30.4)$
- h.  $P(X > -30.4) - P(X > 58.4)$
- i.  $P(X < 58.4) + P(X > -30.4)$

**16.3.**(3pts) What is  $P(X > 58.4 \text{ or } X < -26)$ ? **Choose 3 correct answers** among the followings:

- a.  $P(X > 58.4) + P(X < -26)$
- b.  $P(Z < 0.21) + P(Z < -1.90)$
- c.  $P(Z > 0.21) + P(Z < -1.90)$
- d.  $0.5832 + 0.0287$
- e.  $0.5832 + 0.9713$
- f.  $0.4168 + 0.0287$
- g.  $0.5832 - 0.0287$

**16.4.**(3pts) We want to find the value  $V$  such that the probability that  $X$  is greater than  $V$  is 0.025. **Which three** of the following statements are correct?

- a.  $P(X < V) = 0.025$
- b.  $P(X > V) = 0.025$
- c.  $P\left(Z < \frac{V-\mu}{\sigma}\right) = 0.975$
- d.  $P\left(Z > \frac{V-\mu}{\sigma}\right) = 0.975$
- e.  $P\left(Z < \frac{X-\mu}{\sigma}\right) = 0.975$
- f.  $\frac{X-\mu}{\sigma} = 1.96$
- g.  $\frac{V-\mu}{\sigma} = 1.96$

**16.5.**(3pts) What is the probability  $P(X > -10.4 \text{ or } X < -2.4)$ ? **Choose 3 correct answers** among the followings:

- a.  $P(X > -10.4) + P(X < -2.4)$
- b.  $P(X < -10.4) + P(X < -2.4)$
- c.  $P(Z > -1.51 \text{ or } Z < -1.31)$
- d.  $0.0655 + 0.0951$
- e.  $1 - 0.0655 + 0.0951$
- f.  $P(Z < -1.51) + P(Z < -1.31)$
- g.  $P(X < 10.4 \text{ or } X > 2.4)$
- h. 1

**PART II: OPTIONAL PART. YOU CAN CHOOSE TO ANSWER HOW MANY AND WHICH SUB-QUESTIONS. FOR EXAMPLE, YOU MAY ONLY CHOOSE SUB-QUESTION 17.1 AND 18.2**

**Question 17. (3 pts)** Bank of America has conducted a survey on how frequently consumers use credit cards in making a purchase. The results of the study provide the following information.

37% of consumers use a credit card when making a purchase.

**Among** those consumers who use a credit card, 60% are female.

55% of people interviewed for the survey are male.

Let  $F$  = event that a consumer is female.

Let  $U$  (use a credit card) = event that a consumer uses a credit card in making a purchase.

17.1. Determine the value of  $P(F)$ ? (1 pt)

17.2. Determine the value of  $P(F|U)$ ? (1 pt)

17.3. What is the probability that you see your friend, Susan, uses a credit card? You, of course, know that Susan is female. (1 pt)

**Question 18.** (4 pts)

A survey of 300 college seniors resulted in the following crosstabulation regarding their undergraduate major and their gender.

<b>Gender</b>	<b>Undergraduate Major</b>		<b>Total</b>
	<b>Business</b>	<b>Engineering</b>	
Female	70	30	100
Male	110	90	200
<b>Total</b>	180	120	300

18.1. (1pt) Show the joint probability distribution table, including marginal probabilities

18.2. (1pt) What is the probability that we will see a student who majors in business and is a female?

18.3. (1pt) What is the probability that we will see a student majoring in **engineering**?

18.4. (1pt) What is the probability that we will see a student majoring in engineering given that we know the student is a male?

**Question 19. (6 pts).** We have a discrete random variable  $X$ .

The possible values that  $X$  can take are  $-1, 0, 1, 2$ . The probability function of  $X$  is given as follows:

$x$	$f(x)$
$-1$	$0.1$
$0$	$0.3$
$1$	$0.3$
$2$	$0.3$

19.1. (1 pt) What is the probability that  $X$  is **greater than or equal to 1**?

19.2. (1 pt) What is the expected value of  $X$ ?

19.3. (1 pt) What is the variance of  $X$ ?

Suppose that we construct a new random variable  $Y = 2 + 3X$  based on the above random variable  $X$ . ( $X$  follows the distribution on the previous page. We are still in question 18.)

19.4. (1 pt) What is the **probability** that  $Y$  is less than or equal to 5?

19.5. (1 pt) What is the mean of  $Y$ ?

19.6. (1 pt) What is the variance of  $Y$ ?

**Question 20. (4 pts)**

Suppose  $X$  follows the binomial distribution with  $p = 0.3$  and the number of trials  $n = 10$ .

20.1. (1 pt) What is the probability that  $X = 2$ ? Choose one answer among the followings:

- a.  $\binom{10}{2} 0.3^2(1 - 0.3)^8$
- b.  $\binom{10}{2} 0.3^8(1 - 0.3)^2$
- c.  $0.3^2(1 - 0.3)^8$
- d.  $0.3^8(1 - 0.3)^2$

20.2. (1 pt) What is the probability that  $X \geq 2$ ? Choose one answer among the followings:

- a.  $\binom{10}{1} 0.3^1(1 - 0.3)^9 + \binom{10}{2} 0.3^2(1 - 0.3)^8$
- b.  $1 - \binom{10}{0} 0.3^0(1 - 0.3)^{10} - \binom{10}{1} 0.3^1(1 - 0.3)^9$
- c.  $1 - \binom{10}{1} 0.3^1(1 - 0.3)^9 - \binom{10}{2} 0.3^2(1 - 0.3)^8$
- d.  $\binom{10}{0} 0.3^0(1 - 0.3)^{10} + \binom{10}{1} 0.3^1(1 - 0.3)^9$

20.3. (1 pt) What is the expected value of  $X$ ?

20.4. (1 pt) What is the variance of  $X$ ?

**Question 21. (3 pts)** Suppose  $X$  follows a Poisson distribution with  $\mu = 4$

**21.1.** (1pt) What is the probability that  $X = 2$ ? Choose one correct answer.

a.  $\frac{4^2 e^{-4}}{2!}$

b.  $\frac{2^4 e^{-2}}{4!}$

c.  $\frac{2^4 e^{-4}}{2!}$

d.  $\frac{4^2 e^{-2}}{2!}$

**21.2.** (1pt) What is the probability that  $X \geq 2$ ? Choose one correct answer.

a.  $\frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!}$

b.  $\frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!}$

c.  $1 - \frac{4^0 e^{-4}}{0!} - \frac{4^1 e^{-4}}{1!}$

d.  $1 - \frac{4^1 e^{-4}}{1!} - \frac{4^2 e^{-4}}{2!}$

**21.3.** (1 pt) What is the probability that  $X \leq 1$ ? Choose one correct answer.

a.  $1 - 4e^{-4}$

b.  $4e^{-4}$

c.  $1 - 5e^{-4}$

d.  $5e^{-4}$



**Question 22. (5 pts)**

Consider the continuous random variable  $X$ , which has the uniform distribution over the interval from 20 to 28.

22.1. (1pt) Write down the probability density function  $f(x)$

22.2. (1pt) Draw the probability density function  $f(x)$

22.3. (1pt) What is the probability that  $X$  will take on a value between 21 and 25?

22.4. (1pt) What is the expected value/mean of  $X$ ?

22.5. (1pt) What is the variance of  $X$ ?

**Question 23. (2 pts)** On average, 7 cars arrive at the drive up window of a bank every hour. Define the random variable  $X$  to be the number of cars arriving in any hour.

23.1. (1pt) What is the probability that exactly 3 cars arrive in the next hour?

23.2. (1pt) Let  $Y$  be the number of cars arriving at the window from 11am to 11.30am. What is the distribution of  $Y$ ? (Give the **name of the distribution, and the probability density function**  $f(y)$  with clearly stating **the expected value**  $E(Y) = ?$  )

**Question 24.** (3pts) A major west coast city provides one of the most comprehensive emergency medical services in the world. The service goal is to respond to medical emergencies with a mean time of 12 minutes or less. An investigation is conducted at 40 medical emergencies. The sample mean is 13.25 minutes. The population standard deviation is believed to be 3.2 minutes.

The EMS director wants to perform a hypothesis test, with a 5% level of significance, to determine whether the service goal of 12 minutes or less is being achieved.

**24.1.** (1pt) Write down the null hypothesis and the alternative hypothesis to be tested.

**24.2.** (1pt) Compute the test statistics. Clearly state that whether this is the t-value or the z-value.

**24.3.** (1pt) Test the hypothesis at 5% significant level. Do you reject or accept  $H_0$ ? You can use whatever approach.

**Question 25. (1 pt)** Suppose that a random variable  $X$  has the uniform distribution with the support from 25 to  $H$ , where  $25 < H$

$$f(x) = \begin{cases} 0.25 & \text{for } 25 \leq x \leq H \\ 0 & , \text{ otherwise} \end{cases}$$

What is the appropriate value for  $H$ ? Explain how you found it.

**Question 26. (2pts)**

26.1. (1pt) Toss a coin, roll a die, and toss a coin again. We are interested in the number that appears in an outcome (for the coin, assume Head = 1, Tail = 0). Is this experiment a binomial experiment? Explain.

26.2. (1pt) Toss a coin 5 times identically and independently. This coin has a special property: whenever it has a Head, then the next time it is more likely to appear a Head again. Is this experiment a binomial experiment? Explain.

**PART III: EXTRA CREDITS. YOU GET EXTRA CREDITS IF YOUR ANSWER IS CORRECT.**

**Question 27.** (3 pts) The Bureau of Labor Statistics reported that the average yearly income of dentists in the year 2009 was \$110,000. A sample of 100 dentists, which was taken in 2010, showed an average yearly income of \$120,000. Assume the standard deviation of the sample of dentists in 2010 is \$36,000. We want to test to determine if there has been an (weakly) increase in the average yearly income of dentists between the year 2009 and 2010. (Note: “weakly” implies that equality can happen.)

27.1.(1pt) Provide the null and the alternative hypotheses.

27.2.(1pt) Compute the test statistic. State clearly whether this is a z-value or t-value.

27.3. (1pt) Test the hypothesis at 10% level of significance. Do you reject or accept  $H_0$ ? You can use whatever approach.

**Question 28.** (1pt) The age of students at a university are normally distributed with a mean of 21.

What percentage of the students in the university is at least 21 years old? Choose one answer among the followings:

- a. It could be any value, depending on the magnitude of the standard deviation
- b. 50%
- c. 21%
- d. 42%
- e.  $\sigma$  %

**Question 29.** (1pt) 40% of the parts produced by a machine are defective. We select 6 parts randomly. Let  $X$  be the number of parts that are not defective. Write down the probability function  $f(x)$  of  $X$ .

**Question 30. (1 pt)** There is a mathematical operation called Integration. To denote this operation, the symbol  $\int$  is used and it is called the Integral. What this operation does is that it gives the numerical measure of the area under a function. For example, if we calculate  $\int_a^b f(x)dx$ , it merely gives us the area under  $f(x)$  and between  $a$  and  $b$ . Knowing this, calculate the following expression (with reasoning):

$$\int_{-\infty}^{50} \frac{1}{\sqrt{50\pi}} e^{-\frac{(x-40)^2}{50}} dx$$

**Question 31. (1pt)** Roll a fair die 6 times. We are interested in whether the number that appears in an outcome of a rolling time is an odd number. Bob says the interested random variable is binomial? Charlie says it is not. Who is correct? Explain.



**Question 32.** (1pt) A continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} 0.01 & \text{for } 10 \leq x \leq 30 \\ 0.02 & \text{for } 30 < x < 70 \\ 0 & , \quad \text{otherwise} \end{cases}$$

What is the probability that X is between 20 and 50?

**Question 33.** (1pt) Calculate the following sum. You have to explain your answer to get a credit.

$$\sum_{x=0}^{\infty} x \cdot \frac{5^x e^{-5}}{x!} = 0 \times \frac{5^0 e^{-5}}{0!} + 1 \times \frac{5^1 e^{-5}}{1!} + 2 \times \frac{5^2 e^{-5}}{2!} + \dots$$