

Game Playing

Chapter 4

Outline

- Overview
- Minimax search
- Adding alpha-beta cutoffs
- Additional refinements

Overview

Old beliefs

Games provided a structured task in which it was very easy to measure success or failure.

Games did not obviously require large amounts of knowledge, thought to be solvable by straightforward search.

Overview

Chess

The average branching factor is around 35.

In an average game, each player might make 50 moves.

One would have to examine 35^{100} positions.

Overview

- Improve the **generate procedure** so that only good moves are generated.
- Improve the **test procedure** so that the best moves will be recognized and explored first.

Overview

- Improve the **generate procedure** so that only good moves are generated.

plausible-moves vs. legal-moves

- Improve the **test procedure** so that the best moves will be recognized and explored first.

less moves to be evaluated

Overview

- It is not usually possible to search until a goal state is found.
- It has to evaluate individual board positions by estimating how likely they are to lead to a win.

Static evaluation function

- Credit assignment problem (Minsky, 1963).

Overview

- Good plausible-move generator.
- Good static evaluation function.

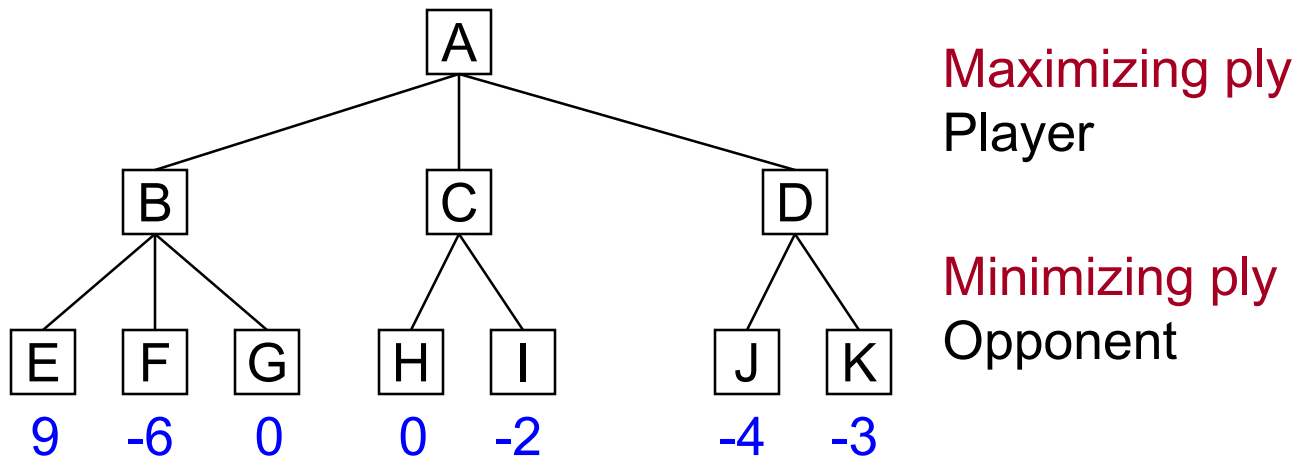
Overview

- What search strategy should we use then?
 - One-person game
 - Two-person game

Minimax Search

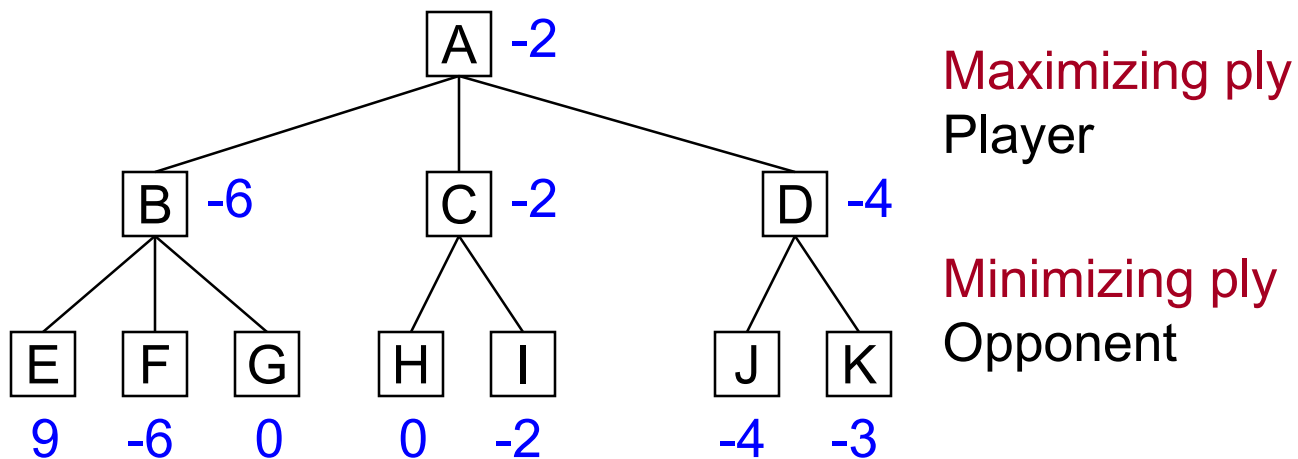
- Depth-first and depth-limited search.
- At the player choice, maximize the static evaluation of the next position.
- At the opponent choice, minimize the static evaluation of the next position.

Minimax Search



Two-ply search

Minimax Search



Two-ply search

Ví dụ: https://en.wikibooks.org/wiki/Artificial_Intelligence/Search/Adversarial_search/Minimax_Search

Minimax Search

Player(Position, Depth):

for each $S \in \text{SUCCESSORS}(\text{Position})$ do

$\text{RESULT} = \text{Opponent}(S, \text{Depth} + 1)$

$\text{NEW-VALUE} = \text{VALUE}(\text{RESULT})$

 if $\text{NEW-VALUE} > \text{MAX-SCORE}$, then

$\text{MAX-SCORE} = \text{NEW-VALUE}$

$\text{BEST-PATH} = \text{PATH}(\text{RESULT}) + S$

return

$\text{VALUE} = \text{MAX-SCORE}$

$\text{PATH} = \text{BEST-PATH}$

Minimax Search

Opponent(Position, Depth):

for each $S \in \text{SUCCESSORS}(\text{Position})$ do

$\text{RESULT} = \text{Player}(S, \text{Depth} + 1)$

$\text{NEW-VALUE} = \text{VALUE}(\text{RESULT})$

 if $\text{NEW-VALUE} < \text{MIN-SCORE}$, then

$\text{MIN-SCORE} = \text{NEW-VALUE}$

$\text{BEST-PATH} = \text{PATH}(\text{RESULT}) + S$

return

$\text{VALUE} = \text{MIN-SCORE}$

$\text{PATH} = \text{BEST-PATH}$

Minimax Search

Any-Player(Position, Depth):

for each $S \in \text{SUCCESSORS}(\text{Position})$ do

$\text{RESULT} = \text{Any-Player}(S, \text{Depth} + 1)$

$\text{NEW-VALUE} = - \text{VALUE}(\text{RESULT})$

 if $\text{NEW-VALUE} > \text{BEST-SCORE}$, then

$\text{BEST-SCORE} = \text{NEW-VALUE}$

$\text{BEST-PATH} = \text{PATH}(\text{RESULT}) + S$

return

$\text{VALUE} = \text{BEST-SCORE}$

$\text{PATH} = \text{BEST-PATH}$

Minimax Search

MINIMAX(Position, Depth, Player):

- MOVE-GEN(Position, Player).
- STATIC(Position, Player).
- DEEP-ENOUGH(Position, Depth)

Minimax Search

1. if DEEP-ENOUGH(Position, Depth), then return:

 VALUE = STATIC(Position, Player)
 PATH = nil
2. SUCCESSORS = MOVE-GEN(Position, Player)
3. if SUCCESSORS is empty, then do as in Step 1

Minimax Search

4. if SUCCESSORS is not empty:

$RESULT-SUCC = MINIMAX(SUCC, Depth+1, Opp(Player))$

$NEW-VALUE = - VALUE(RESULT-SUCC)$

 if $NEW-VALUE > BEST-SCORE$, then:

$BEST-SCORE = NEW-VALUE$

$BEST-PATH = PATH(RESULT-SUCC) + SUCC$

5. Return:

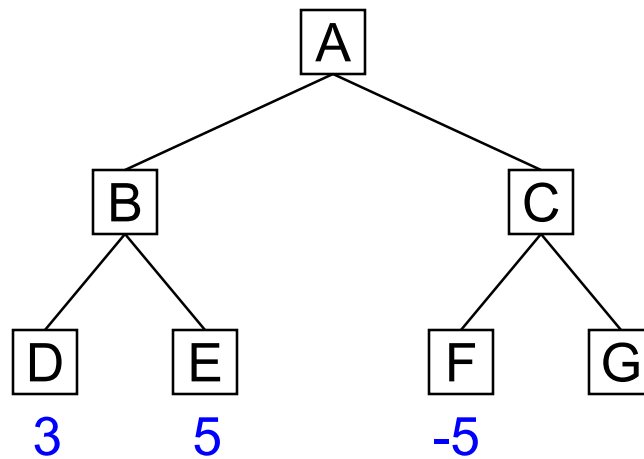
$VALUE = BEST-SCORE$

$PATH = BEST-PATH$

Adding Alpha-Beta Cutoffs

- Depth-first and depth-limited search.
branch-and-bound
- At the player choice, maximize the static evaluation of the next position.
> α threshold
- At the opponent choice, minimize the static evaluation of the next position.
< β threshold

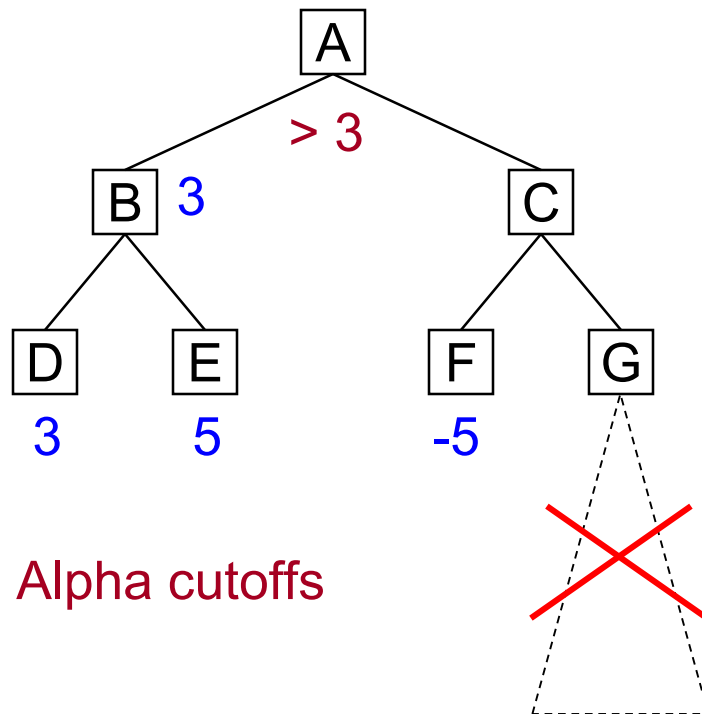
Adding Alpha-Beta Cutoffs



Maximizing ply
Player

Minimizing ply
Opponent

Adding Alpha-Beta Cutoffs

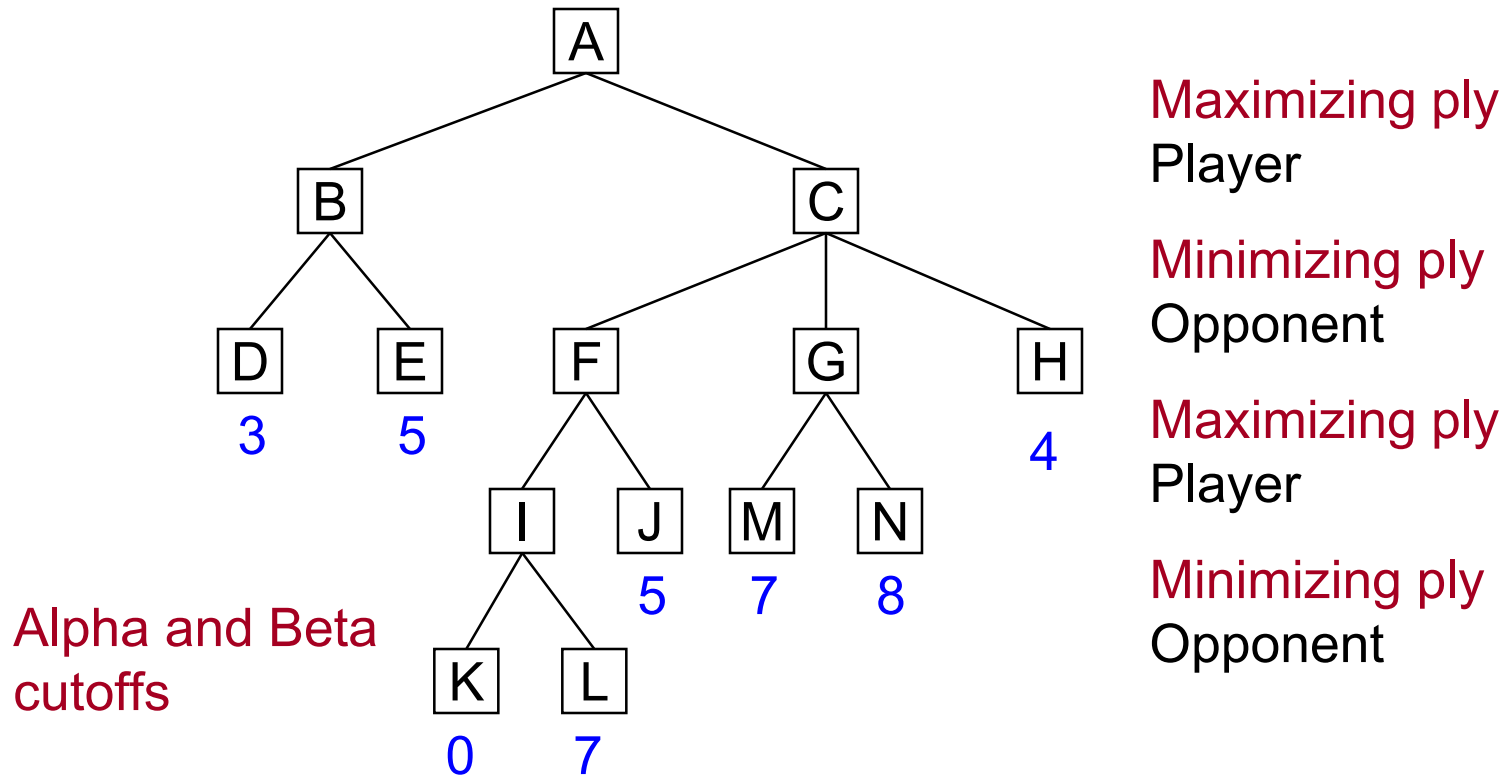


Maximizing ply
Player

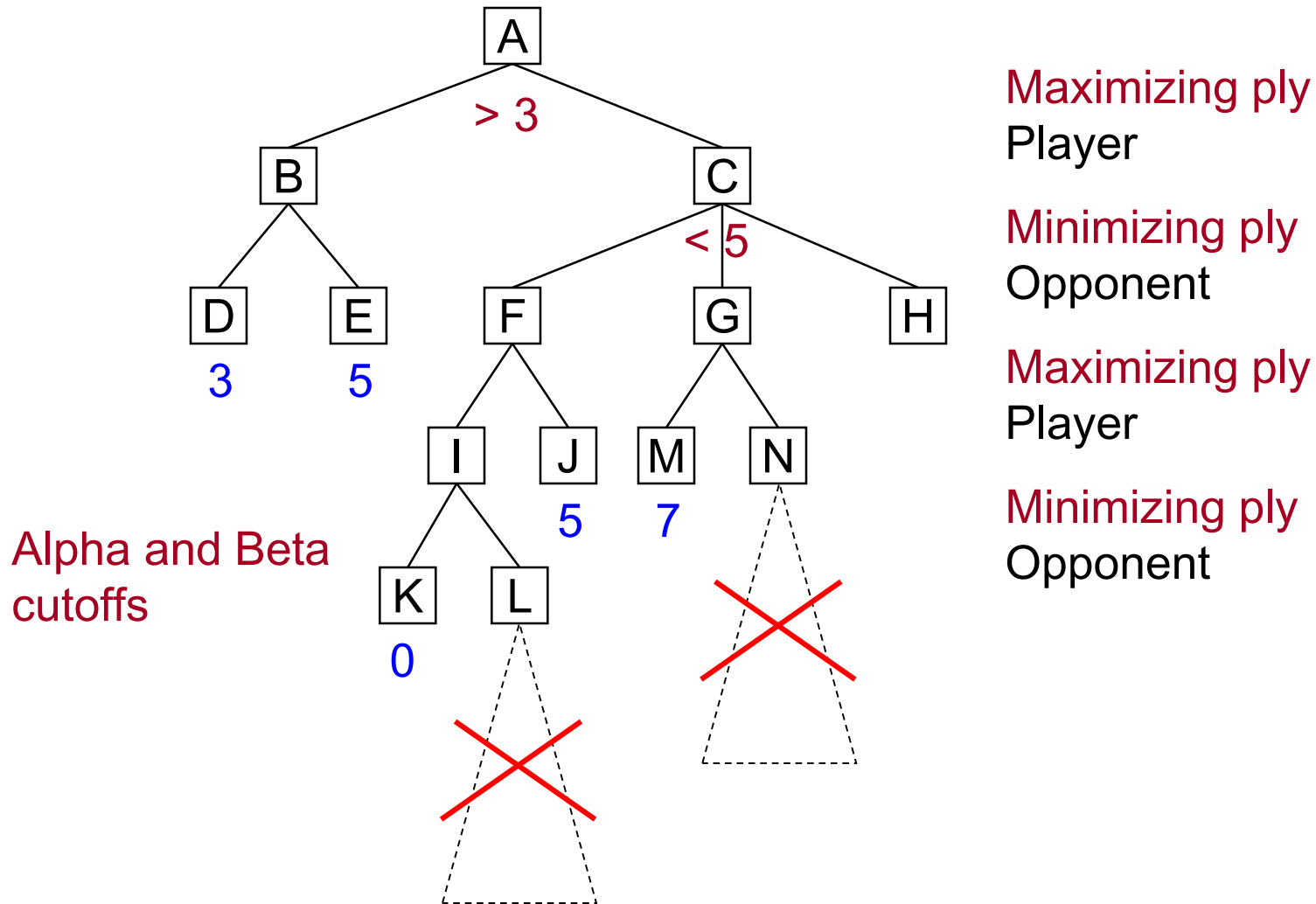
Minimizing ply
Opponent

Alpha cutoffs

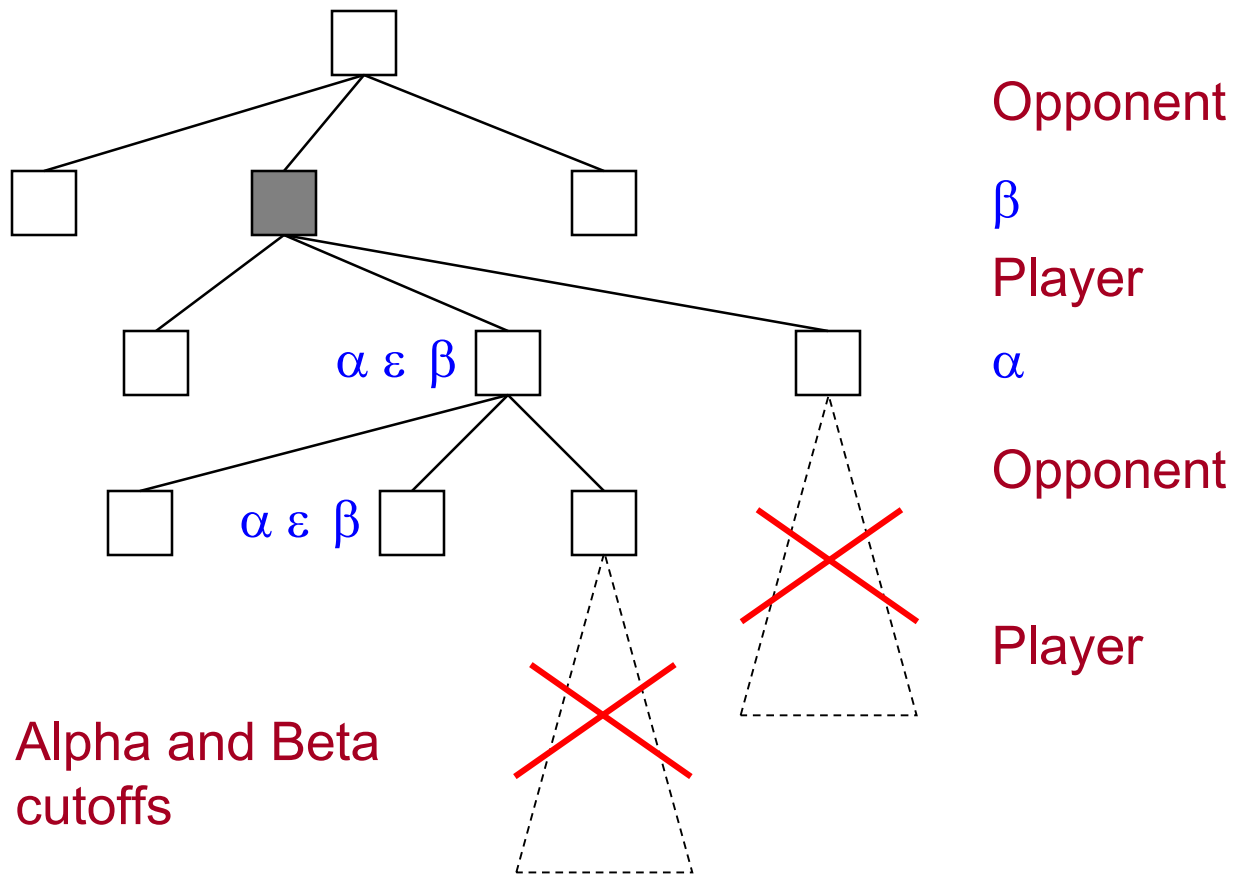
Adding Alpha-Beta Cutoffs



Adding Alpha-Beta Cutoffs



Adding Alpha-Beta Cutoffs



Player(Position, Depth, α , β):

for each $S \in \text{SUCCESSORS}(\text{Position})$ do

 RESULT = Opponent(S, Depth + 1, α , β)

 NEW-VALUE = VALUE(RESULT)

 if NEW-VALUE > α , then

α = NEW-VALUE

 BEST-PATH = PATH(RESULT) + S

 if $\alpha \geq \beta$ then return

 VALUE = α

 PATH = BEST-PATH

return

 VALUE = α

 PATH = BEST-PATH

Opponent(Position, Depth, α , β):

for each $S \in \text{SUCCESSORS}(\text{Position})$ do

$\text{RESULT} = \text{Player}(S, \text{Depth} + 1, \alpha, \beta)$

$\text{NEW-VALUE} = \text{VALUE}(\text{RESULT})$

 if $\text{NEW-VALUE} < \beta$, then

$\beta = \text{NEW-VALUE}$

$\text{BEST-PATH} = \text{PATH}(\text{RESULT}) + S$

 if $\beta \leq \alpha$ then return

$\text{VALUE} = \beta$

$\text{PATH} = \text{BEST-PATH}$

return

$\text{VALUE} = \beta$

$\text{PATH} = \text{BEST-PATH}$

Any-Player(Position, Depth, α , β):

for each $S \in \text{SUCCESSORS}(\text{Position})$ do

RESULT = **Any-Player**(S , Depth + 1, $-\beta$, $-\alpha$)

NEW-VALUE = $-\text{VALUE}(\text{RESULT})$

 if **NEW-VALUE** $> \alpha$, then

$\alpha = \text{NEW-VALUE}$

BEST-PATH = **PATH**(**RESULT**) + S

 if $\alpha \geq \beta$ then return

VALUE = α

PATH = **BEST-PATH**

return

VALUE = α

PATH = **BEST-PATH**

Adding Alpha-Beta Cutoffs

MINIMAX-A-B(Position, Depth, Player, UseTd, PassTd):

- **UseTd**: checked for cutoffs.
- **PassTd**: current best value

Adding Alpha-Beta Cutoffs

1. if DEEP-ENOUGH(Position, Depth), then return:

 VALUE = STATIC(Position, Player)
 PATH = nil
2. SUCCESSORS = MOVE-GEN(Position, Player)
3. if SUCCESSORS is empty, then do as in Step 1

Adding Alpha-Beta Cutoffs

4. if SUCCESSORS is not empty:
 $\text{RESULT-SUCC} = \text{MINIMAX-A-B}(\text{SUCC}, \text{Depth} + 1, \text{Opp}(\text{Player}), -\text{PassTd}, -\text{UseTd})$

 $\text{NEW-VALUE} = -\text{VALUE}(\text{RESULT-SUCC})$

 if $\text{NEW-VALUE} > \text{PassTd}$, then:
 $\text{PassTd} = \text{NEW-VALUE}$

 $\text{BEST-PATH} = \text{PATH}(\text{RESULT-SUCC}) + \text{SUCC}$

 if $\text{PassTd} \geq \text{UseTd}$, then return:
 $\text{VALUE} = \text{PassTd}$

 $\text{PATH} = \text{BEST-PATH}$
5. Return:
 $\text{VALUE} = \text{PassTd}$

 $\text{PATH} = \text{BEST-PATH}$

Additional Refinements

- Futility cutoffs
- Waiting for quiescence
- Secondary search
- Using book moves
- Not assuming opponent's optimal move

Homework

Exercises 1-7, 9 (Chapter 12)