

2) Xác định mqh phức tạp thuật toán theo ki' pháp trên đây.

B<sub>1</sub>:

a) worst-case

$$f(x) = 50x + 7 \leq 51x \Leftrightarrow x \geq 7$$

$$\text{vs } x \geq 7, c = 51, f(x) \leq 51x.$$

$$\Rightarrow O(n).$$

b) Best-case.

$$f(x) = 50x + 7 \geq 7 \quad \forall x \geq 0.$$

$$\Rightarrow \forall x \geq 0, \forall c, f(x) \geq 7 \Rightarrow \Omega(7).$$

c) Average case.

$$50x \leq f(x) \leq 51(x)$$

$$\Rightarrow \text{ki' pháp } \theta.$$



Bài 2:

a)  $2n(n-1)/2 \in O(n^3)$  True

Thật vậy:

$$f(n) = \frac{2n(n-1)}{2} = n^2 - n \leq n^2 \leq n^3.$$

$$\forall n > 0$$

Vậy  $\forall n > 0, c=1, f(n) \leq n^3$   
 $\Rightarrow O(n^3)$ .

b)  $2n(n-1)/2 \in O(n^2)$  True.

$$\text{Do } f(n) = n^2 - n \leq n^2 \forall n > 0$$

Vậy  $\forall n > 0, c=1, f(n) \leq n^2$ .

$$\Rightarrow f(n) \in O(n^2)$$

c)  $2n(n-1)/2 \in \Theta(n^3)$  False.



$$f(n) = n^2 - n \leq n^2 \quad \forall n > 0$$

$$f(n) = n^2 - n \geq n^2 - n \cdot \frac{n}{2} = n^2 - \frac{n^2}{2} = \frac{1}{2}n^2 \quad \forall n > 0$$

$$\frac{1}{2}n^2 \leq f(n) \leq n^2$$

$$\Rightarrow f(n) \in \Theta(n^2)$$

d)  $2n(n-1)/2 \in \Theta(\Omega(n))$  True.

$$f(n) = 2n(n-1)/2 = n^2 - n \geq \frac{n^2}{2}$$

$$\forall n > 0$$

$$f(n) \in \Omega\left(\frac{n^2}{2}\right)$$

$$\Rightarrow f(n) \in \Omega(n)$$

Bài 5:

Sắp xếp theo tăng dần:

$$\sqrt{n}, \ln^3 n, 2 \lg(n+50)^5, 0.05n^{10} + 3n^3 + 1,$$

$$3^{2n}, 3^{3n}, (n^2+3)!$$



Bài 7:

a) If  $f(n) \in O(g(n))$  then  $g(n) \in \Omega(f(n))$ .  
True.

Thật vậy:

Nếu  $f(n) \in O(g(n))$

$\Leftrightarrow \exists n > n_0, c = c_0$  sao cho

$$f(n) \leq c_0 \cdot g(n).$$

$$\Leftrightarrow g(n) \geq \frac{1}{c_0} \cdot f(n)$$

$$(g(n), f(n) > 0)$$

b)  $\Theta(\alpha f(n)) = \Theta(f(n))$  where  $\alpha > 0$

False, điều này xảy ra khi  $\alpha = 0$

c)  $\Theta(f(n)) = O(f(n)) \cap \Omega(f(n))$ .

True, Bởi.

⊕  $f(n) \in O(f(n))$  thì  
 $\exists n > n_0, c = c_0$  sao cho  
 $f(n) \leq c_0 \cdot f(n).$

⊕  $f(n) \in \Omega(f(n))$

$\Leftrightarrow \exists n > n_0, c = c_0$  sao cho



$$g(n) \geq c_1 \cdot g(n).$$

Hay

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n)).$$

d) For any two non negative function  $f(n)$  and  $g(n)$  defined of the set of non negative integer, either

$$f(n) \in O(g(n)), \text{ or } f(n) \in \Omega(g(n)), \text{ or both}$$

CM.

$$\textcircled{+} TH_1: \text{gds. } f(n) \geq g(n),$$

$$\Rightarrow \exists k \in \mathbb{N}, \text{ sao cho}$$

$$f(n) \geq k \cdot g(n)$$

$$\Rightarrow f(n) \in \Omega(g(n)). \Rightarrow n^4 \in \Omega(2n^3)$$

VD:

$$f(n) = n^4$$

$$g(n) = 2n^3$$

$$\cancel{f(n) = 4}$$

$$n^4 \geq 2n^3$$

$$\textcircled{+} TH_2: f(n) \leq g(n) \text{ klu'at}$$

$$\exists k \in \mathbb{N}, \text{ sao cho}$$

$$f(n) \leq k \cdot g(n)$$

$$\Rightarrow f(n) \in O(g(n))$$

VD:

$$f(n) = n+1$$

$$g(n) = n^2$$

$$n+1 \leq n^2$$

$$\Rightarrow n+1 \in O(n^2)$$

$$\textcircled{+} TH_3: f(n) = g(n) \text{ klu'at}$$

$$\exists k, q \in \mathbb{N}^+, \text{ sao cho}$$

$$q \cdot g(n) \leq f(n) \leq k \cdot g(n)$$

$$f(n) \in O(g(n)), f(n) \in \Omega(g(n))$$

$$\Rightarrow \text{dpcau}$$

HAPLUS

VD:

$$f(n) = 0.5 \cdot g(n) = n^3$$

$$\Rightarrow 0.5n^3 \leq n^3 \leq 5n^3$$