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It is sufficient to show that for any $X \subseteq I$:

X is a transversal of $H(S) \iff X \notin cl(S)$.

According to this equivalence, $Tr(H(S))$ contains all minimal itemsets that are not in $cl(S)$. Since $cl(S)$ is the set of all frequent itemsets that can be derived from S , the set of minimal itemsets not in $cl(S)$ must be equal to the negative border. The proof of the above equivalence follows below.

" \Rightarrow ":

Let X be a transversal of $H(S)$. So we have $\forall B \in Bd^+(cl(S)) : X \cap (I - B) \neq \emptyset$. Lets do a proof by contradiction and assume $X \in cl(S)$. This means X is a frequent itemset that can be derived from a set B in $Bd^+(S)$ as a subset. So we obtain $X \subseteq B$ and $X \cap (I - B) \neq \emptyset$. But this is a contradiction because X cannot contain only elements from B while also containing elements from the complement from B at the same time.

" \Leftarrow ":

Let $X \notin cl(S)$ be true. Lets again do a proof by contradiction and assume X is not a transversal of $H(S)$. This means there has to be at least one $B \in Bd^+(cl(S))$ so that $X \cap (I - B) = \emptyset$. Since $X \subseteq I$, it follows that $X \subseteq B$. But this is obviously a contradiction to our first assumption $X \notin cl(S)$.