

## 1. Exercise

We have the following property:

$X$  is closed if and only if  $|D[Y]| < |D[X]|$  for every  $Y \supsetneq X$

and this means that  $D[Y] \subsetneq D[X]$ .

If  $|D[X \cup i]| \geq |D[X]|$  then  $X$  is not closed because it contradicts with the property.

So all proper supersets of  $X$  must have support count less than the support count of  $X$ .  $D[X \cup i] \subsetneq D[X]$  must hold for all  $i \in I \setminus X$  in order for  $X$  to be closed.

And if  $X$  is closed then  $D[X \cup i] \subsetneq D[X]$  for all  $i \in I \setminus X$  and the statement holds.

## 2. Exercise

Let  $X$  be an itemset and  $D[X]$  be the support count of  $X$

If we perform a closure operation  $c(X)$  then we can have either  $X$  ( $X$  is closed) or a superset containing  $X$ .

Now we know that :  $C(x) = ti(it(X))$

By definition  $it(X)$  returns all the lines containing all elements of  $X$ .

In other words :  $it(X) = it(X_1) \cap it(X_2) \dots \cap (X_n); X_1, X_2, \dots, X_n \in X$ .

So the output of  $it(X)$  will be all transactions that have the subset  $X$  in them, and from definition :  $ti(Y) = ti(Y_1) \cap ti(Y_2) \dots \cap (Y_n); Y_1, Y_2, \dots, Y_n \in Y$

So the output of  $ti(Y)$  will be all common items in the itemsets presented in  $Y_1, Y_2, \dots, Y_n$ , but we know that the output of  $it(X)$  contains all transactions having  $X$  in them, and so by performing  $ti$  to result of  $it(X)$  we will have an itemset containing  $X$ , and perhaps some other items, and this itemset must appear in all lines of the database (because of the definition), which means that result will always have a support count equal to  $D[X]$ .

In other words  $D[X] = D[c(X)]$