Exercise 4

We have $I := \{A, C, D, E, K, M, N, O, Y\}$.

In order to mine **frequent** itemset we are going to use $\rho = \rho'$ as defined in the lecture.

START:
S = $\emptyset$
$max(\rho(inc(\emptyset, A))\backslash\emptyset) = max(\rho(A)) = max(I) = Y \nleq A$
$max(\rho(inc(\emptyset, C))\backslash\emptyset) = max(\rho(C)) = max(I) = Y \nleq C$
$max(\rho(inc(\emptyset, D))\backslash\emptyset) = max(\rho(D)) = max(I) = Y \nleq D$
$max(\rho(inc(\emptyset, E))\backslash\emptyset) = max(\rho(E)) = max(KE) = K \nleq E$
$max(\rho(inc(\emptyset, K))\backslash\emptyset) = max(\rho(K)) = max(K) \leq K \Rightarrow$ **and** $\{K\}$ **is a closed frequent itemset.**

S=$\{K\}$
$max(\rho(inc(K, A))\backslash K) = max(\rho(KA)\backslash K) = max(I\backslash K) = Y \nleq A$
$max(\rho(inc(K, C))\backslash K) = max(\rho(KC)\backslash K) = max(I\backslash K) = Y \nleq C$
$max(\rho(inc(K, D))\backslash K) = max(I\backslash K) = Y \nleq D$
$max(\rho(inc(K, E))\backslash K) = max(I\backslash K) = Y \nleq E$
$max(\rho(inc(K, M))\backslash K) = max(\rho(M)\backslash K) = max(MKY\backslash K) = max(MY) = Y \nleq M$
$max(\rho(inc(K, N))\backslash K) = max(\rho(N)\backslash K) = max(I\backslash K) = Y \nleq N$
$max(\rho(inc(K, O))\backslash K) = max(\rho(O)\backslash K) = max(OKE\backslash K) = max(OE) = O \leq O \Rightarrow \{OKE\}$ **is a frequent closed itemset**.

S = $\{OKE\}$
$max(\rho(inc(OKE, A))\backslash OKE) = max(\rho(OKEA)\backslash OKE) = max(I\backslash OKE) = Y \nleq A$
$max(\rho(inc(OKE, C))\backslash OKE) = max(\rho(OKEC)\backslash OKE) = max(I\backslash OKE) = Y \nleq C$
$max(\rho(inc(OKE, D))\backslash OKE) = max(\rho(OKED)\backslash OKE) = max(I\backslash OKE) = Y \nleq D$
$max(\rho(inc(OKE, M))\backslash OKE) = max(\rho(MO)\backslash OKE) = max(I\backslash OKE) = y \nleq M$
$max(\rho(inc(OKE, N))\backslash OKE) = max(\rho(ON)\backslash OKE) = max(I\backslash OKE) = Y \nleq N$
$max(\rho(inc(OKE, Y))\backslash OKE) = max(\rho(Y)\backslash OKE) = max(KY\backslash OKE) = Y \leq Y \Rightarrow \{KY\}$ **is a closed frequent itemset.**

S = $\{KY\}$
$max(\rho(inc(KY, A))\backslash KY) = max(\rho(KYA)\backslash KY) = max(I\backslash KY) = O \nleq A$
$max(\rho(inc(KY, C))\backslash KY) = max(\rho(KYC)\backslash KY) = max(I\backslash KY) = O \nleq C$
$max(\rho(inc(KY, D))\backslash KY) = max(\rho(KYD)\backslash KY) = max(I\backslash KY) = O \nleq D$
$max(\rho(inc(KY, E))\backslash KY) = max(I\backslash KY) = O \nleq O$

$max(\rho(inc(KY, M))\backslash KY) = max(\rho(MY)\backslash KY) = max(I\backslash KY) = O \nleq M$
$max(\rho(inc(KY, N))\backslash KY) = max(\rho(NY)\backslash KY) = max(I\backslash KY) = O \nleq N$
$max(\rho(inc(KY, O))\backslash KY) = max(\rho(YO)\backslash KY) = max(I\backslash KY) = O \leq O$. And thats the end of the algorithm since I is closed but not frequent.

Hence the closed frequent itemsets are: $\{K, KY, OKE\}$