# 3 (i)

Samstag, 19. Juni 2021 01:51

Proof by induction over $m$

→ Base case $m=1$:

- By line 1 of the algorithm we have $S = \{a_l\}$ for $l=1$

  $\Rightarrow P(S=\{a_1\}) = 1$ ✓

→ We assume that the theorem holds for $m>0$

→ Induction step:

- Let $S_i$ be the set $S$ after processing $a_i$ for $i \in [m+1]$
- Let $HEAD_i$ be the HEAD-event when processing $a_i$ for $i \in [m+1]$
- We now consider a stream of length $m+1$
- $P(S_{m+1} = \{a_{m+1}\}) = P(HEAD_{m+1}) = \frac{1}{m+1}$

  $\Rightarrow P(S=\{a_{m+1}\}) = \frac{1}{m+1}$ ✓

- Let $l \in [m]$
- By our assumption we know

  $P(S_m = \{a_l\}) = \frac{1}{m}$

  $\Rightarrow P(S_{m+1} = \{a_l\}) = P(S_m = \{a_l\} \text{ and } \neg S_{m+1} = \{a_{m+1}\})$

  indep. events because "$S_{m+1} = \{a_{m+1}\}$" determined by indep. coin toss → $= P(S_m=\{a_l\}) P(\neg S_{m+1} = \{a_{m+1}\})$

  $= \frac{1}{m} \cdot \left(1 - \frac{1}{m+1}\right) = \frac{1}{m} \cdot \frac{m}{m+1}$

  $= \frac{1}{m+1}$ ✓

- All in all, for $j \in [m+1]$: $P(S=\{a_j\}) = \frac{1}{m+1}$ □