3) Lemma 4

Sonntag, 27. Juni 2021 12:50

- Let entry $t = (e, f, \Delta) \in D$
- We assume that t was created when processing $a_n = e$
- We also assume that t is not deleted when processing the next i items after a_n with $i \in \mathbb{N}_0$ and we say $m = n + i$
- We now show that for entry t we always have $f \leq f_e \leq f + \varepsilon m$
- Proof of $f \leq f_e$:
  This property must hold because increments of f by 1 only happen if e occurs ✓
- Proof of $f \leq f_e + \varepsilon m$:

- For each new occurrence of e f_e grows by 1
- For each new occurrence of e f is incremented by 1
- But f is also decremented by 1 at each new bucket boundary

- After creation of t there are at most $\frac{i+1}{w}$ bucket bounderies that appear while processing $a_n, \dots, a_{n+i}$
- Let dec be the number of decrements of f after creation of t that are done while processing the next i elements after a_n

$$\Rightarrow \text{dec} \leq \frac{i+1}{w} \leq \frac{i+1}{\lceil \frac{1}{\varepsilon} \rceil} \leq \varepsilon \cdot (i+1)$$

- Now we introduce an index for fe and denote as $fe_{,j}$ the value of fe after processing a_j
- We introduce the same indexing for f
- So to finish the proof we want to show $fe_{,m} \leq \varepsilon m + f_m$
- Since t was created when processing a_n, we have $fe_{,n-1} \leq \varepsilon(n-1)$ according to Lemma 3
- Let inc be the number of increments of f that happen while processing the next i elements after a_n

$$\Rightarrow fe_{,m} = fe_{,n-1} + 1 + \text{inc}$$
$$\leq \varepsilon(n-1) + 1 + \text{inc}$$
$$\leq \varepsilon(n-1) + 1 + \text{inc} \overbrace{- \text{dec} + \varepsilon(i+1)}^{\text{dec} \leq \varepsilon(i+1) \text{ from above}}$$
$$= \varepsilon(n-1+i+1) + 1 + \text{inc} - \text{dec}$$
$$= \varepsilon m + f_m \checkmark$$