Exercise 2.

Antisymetry

def: $A \preceq B$ and $A \preceq A$ then $A = A$
Let $A \neq A$ and $A \preceq A$:
$\Rightarrow$ there is a $max(x \in B)$ with $x \notin A$
this x is greater than all elements of $A \backslash B$
$B \preceq A$:
$\Rightarrow$ there is a $max(y \in A)$ with $x \notin B$
this y is greater than all elements of $B \backslash A$
both statements don't make sense together
$\Rightarrow$ A=B

Transitivity

def: $A \preceq B$ and $B \preceq C$ then $A \preceq C$
$A \preceq B \iff A = B\, or\, max((A \cup B) \backslash (A \cap B)) \in B$
$B \preceq C \iff B = C\, or\, max((B \cup C) \backslash (B \cap C)) \in C$
so as A can be either equal to B or contained in it (by definition of the order), and B can be either equal or contained in it, so we know that A is definitely contained in C.
so $max((A \cup C) \backslash (A \cap C)) \in C$
$\Rightarrow$ transitivity holds.
and $\preceq$ is total order on 2^I