# 3) Lemma 4

Dienstag, 29. Juni 2021 15:04

- Let $t = (e, f, \Delta) \in D$
- Proof of $f \leq f_e$:
  - $t$ is created when processing the element $e$ with $f = 1$
  - $f$ is only increased with further processings of $e$
  - We obtain that $f$ must be bounded by the number of occurences of $e$ in stream
    $\Rightarrow f \leq f_e$
- Proof of $f_e \leq f + \varepsilon m$
  - Let $f_e'$ be the number of times $e$ occurs in stream before creation of entry $t$ in $D$
  - Let $t$ be created when processing $a_{m'}$ and $m = m' + i$
  - Since we assume that $t \in D$ after processing $a_m$, there is at least one increase of $f$ for each decrease that happens at a bucket boundary between processing $a_{m'+1}$ and $a_m$

- Let $b$ be this number of bucket boundaries that accurs when processing $a_{m'+1}, \dots, a_m$ and let inc be the number of increases of $f$ since creation of $t$
- Now we have to prove

$$f_e = f_e' + 1 + inc \leq f + \varepsilon m$$

- We assume $inc = b$ because this results in $f$ being as small as possible. This is good since we want to use $f$ for bounding $f_e$.
  $\Rightarrow$ We obtain $f = 1 + inc - b = 1 + b - b = 1$

- According to Lemma 3 we have

$$f_e' \leq \varepsilon(m' - 1)$$

- We now do the following:

$$f_e' + 1 + inc \leq f + \varepsilon m$$

$$\Leftrightarrow$$

$$f_e' + 1 + b \leq 1 + \varepsilon(m' - 1 + 1 + i)$$

$$\Leftrightarrow$$

$$f_e' + 1 + b \leq \varepsilon(m' - 1) + 1 + \varepsilon(1 + i)$$

- Since we know $f_e' \leq \varepsilon(m' - 1)$ we just need to prove

$$\underbrace{b \leq \varepsilon(1 + i)}_{\text{This holds}}$$

because $b = \left\lceil \frac{i}{w} \right\rceil = \left\lceil \frac{i}{\lceil \frac{1}{\varepsilon} \rceil} \right\rceil$ ✓