Exercise 4

We define the set T of transaction identifiers with $T = V_1$ and the groundset I with $I = V_2$. Now we create a transaction database D in which the entry with tid v_1 contains all items v_2 so that $(v_1, v_2) \in E$. D must have size polynomial in $|V_1|$ and $|V_2|$ because the number of rows is bounded by $|V_1|$ and the number of items per row is bounded by $|V_2|$.
If there are k elements in V_1 that all connect with the same k elements from V_2, there must be k rows/tids containing the same itemset. And if there are k rows containing elements of the same k-subset of V_2, all of the vertices corresponding to the tids of these rows must be connected to each vertex in this k-subset. All in all, the existence of a balanced bibartite clique of size k is equivalent to the existence of a k-frequent itemset of size at least k.
In conclusion, it cannot be decided in polynomial time if there is a frequent itemset consisting of at least k items for a threshold t = k. Otherwise, one could solve the Balanced Bipartite Clique Problem in polyonomial time by building the transaction database D and checking for the existence of a k-frequent itemset of size at least k.