

Exercise 1

We want to test if the probability the have a brain tumor $P(BRAIN_TUMOR)$ is 0.2 in the whole population. For that we have measured the presence of brain tumors in a subgroup of 100 people.
So we can formulate our null and anti hypothesis.

$$H_0: p = 0.2$$

$$H_1: p \neq 0.2$$

In the subgroup of 100 patients we observed the value $\hat{p} = \frac{|peoplewithbraintumor|}{100} = 0.1$. Under the null hypothesis where the probability to have a brain tumor is 0.2 the number of people suffering from brain tumor in a random selection of n people is $Bn(0.2, n)$ binomial distributed. By applying the central limit theorem we can define the test statistic:

$$V = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

, where $n = 100$. Moreover we know that V is $\mathcal{N}(0, 1)$ distributed. We now have that

$$V = \frac{0.1 - 0.2}{\sqrt{0.2 * 0.8/100}} = \frac{-0.1}{\sqrt{0.0016}} = \frac{-0.1}{0.04} = -2.5$$

Because the normal distrubution is symmetrical we can use $|V| = 2.5$. And for $\alpha = 0.01$ the z-value Z_N equals 2.58. So we conclude that we cannot reject the null hypothesis with a significance of 0.01. But we could reject the null hypothesis with a significance of 0.02 what is still pretty significant.