

## Exercise 4

This is true due to the reduction in every step of the algorithm.

In every step of the algorithm each element whose counter is equal to 1 is deleted

therefore in one step, only a maximum of  $\frac{1}{2}w$  elements can be created, that are still alive in the current bucket. These elements would have a counter of 2 which is reduced to 1.

Let's say at  $B-j+1$   $\frac{1}{2}w$  elements are created that are still alive in the current bucket. This means in step  $B-j+2$  these elements have to be counted up by one, otherwise they would be deleted.

This also means in step  $B-j+2$  only  $\frac{1}{4}w$  elements can be newly created that are still alive.

Each next step only half of the previously newly created elements can be created to live in the current bucket.

This is bounded by  $\sum_{i=1}^j i * (\frac{1}{2})^{j+1-i}w$  which is smaller than  $jw$  for  $1 < j$

it can only be equal for the case  $j = 1$  when in the current Bucket  $w$  elements with counter 1 are created