Introduction to Algorithms
Lecture 16 Computational Complexity

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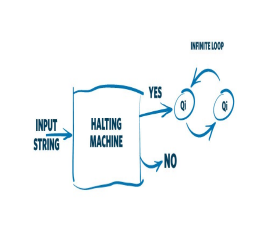
2024 spring in

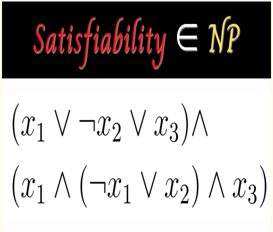
Outline

Overview

1. We have discussed various methods to design efficient algorithms
2. Many problems do not have efficient algorithms, e.g.



(a) HALTING PROBLEM



(b) SATISFIABILITY

(c) GO GAME PROBLEM

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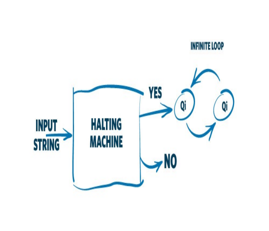

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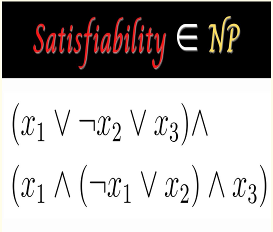

(e) SATISFIABILITY

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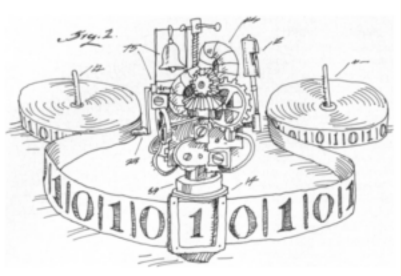
Computational Complexity

Study computational efficiency of problems:

1. Can computers solve it?
2. How long? Does it have efficient (poly-time) algorithms?

History

1. Turing defined the math model of modern computer around 1936

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   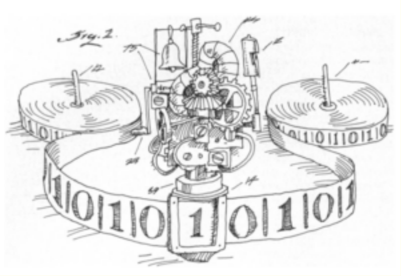

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   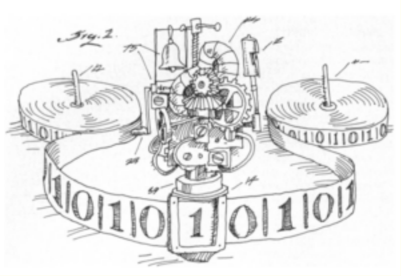

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3. Turing-Church Theorem/Thesis: Any physical computation can be simulated by a Turing machine (with poly-time overhead)
4. ☺Turing machine is simple enough to argue the limitation of modern computers

Turing Machines

An informal introduction:

1. Consider it as a low-level programming language — even lower than assembly language
2. A fixed number (say $\leqslant 100$) of instructions
3. Its description has a fixed number of lines/instructions
4. Its input is all $\{0, 1\}$-strings, i.e., $\{0, 1\}^*$
5. Its output is $\{0, 1\}$
6. Unbound memory space — different from automata

An Assembly Language Program

```
;
; Program to multiply a number by the constant 6
;
        .ORIG   x3050
        LD      R1, SIX
        LD      R2, NUMBER
        AND     R3, R3, #0      ; Clear R3.  It will
                                ; contain the product.
; The inner loop
;
AGAIN   ADD     R3, R3, R2
        ADD     R1, R1, #-1     ; R1 keeps track of
        BRp     AGAIN           ; the iteration.
;
        HALT
;
NUMBER  .BLKW   1
SIX     .FILL   x0006
;
        .END
```

Math Models

Binary Encoding of Problems — Languages

1. Only consider decision problems for now: For an instance/input I, the output is either 1 (means YES) or 0 (means NO)
2. Fix a decision problem Q, encode the instance I as a binary string $enc(\mathrm{I})$ in $\{0,1\}^*$
3. Define the language of Q as $L_Q = \{enc(\mathrm{I}) : \mathrm{Q}(\mathrm{I}) = 1\}$

- Example 1 PRIME: For a binary number $t \in \{0,1\}^*$, output 1 iff it is a prime. $L_{\mathrm{PRIME}} = \left\{ t_0 \cdots t_n \in \{0,1\}^* \,\middle|\, \sum_{i=0}^{n} t_i \cdot 2^i \text{ is a prime} \right\} = \{10, 11, 101, 111, 1011, 1101, \dots\}$

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- Example 2 CONNECTIVITY: For a graph G, output 1 iff G is a connected. $L_{\mathrm{CONN}} = \left\{ T \in \{0, 1\}^* \middle| T \text{ presents a connected graph} \right\}$

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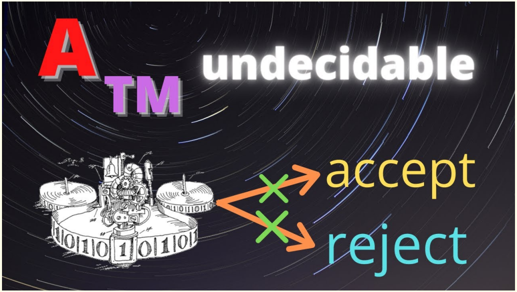

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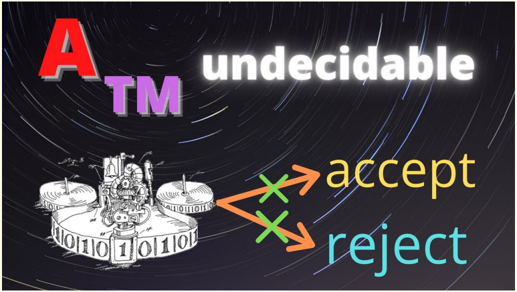
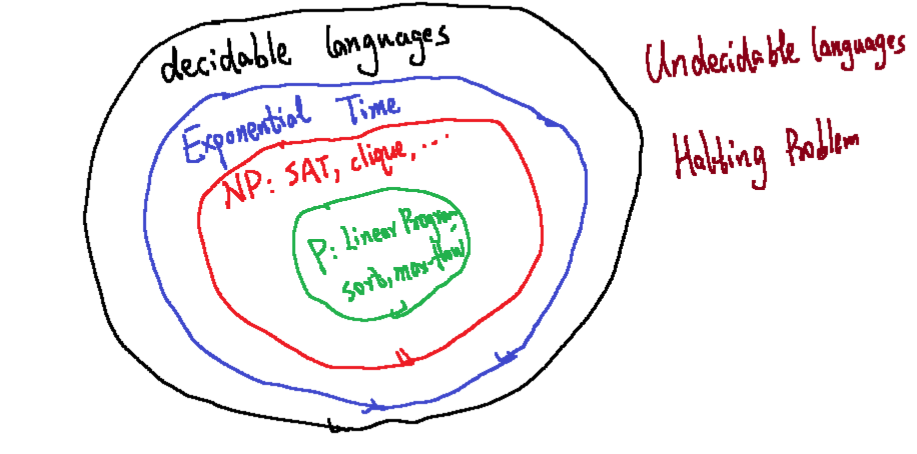


2. Use their binary encodings and executions on simple models to prove that some problem is the hardest in a class (called complete) like SAT problems in NP and LP in P

Computational Classes



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Main Question

Where is the limit of computation? — Are there problems/languages that can not be solved by computers (called undecidable)?

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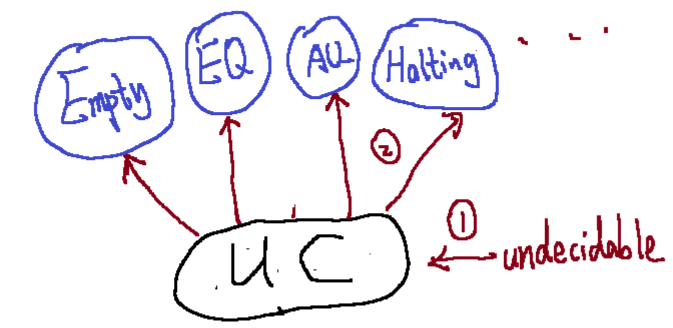
2. EMPTY: Given a program M, will it ever output YES?
3. ALL: Given a program M, will it accept any input?
4. EQ: Given two programs M_1 and M_2, are they equivalent?

Road Map

To show they are undecidable,

1. Show Language UC is undecidable — the hardness of UC is the cornerstone of undecidable theory
2. Reduce UC to Halting and other problems



— if a computer solves any other prob, then it solves UC, which is impossible from (1)

1st Undecidable Language

Definition

1. Sort all programs from 1 to ∞: $M_1, M_2, \ldots$
2. Sort all inputs in $\{0,1\}^*$ from 1 to ∞: $\mathrm{I}_1, \mathrm{I}_2, \ldots$
3. For each $\alpha \in \{0,1\}^*$, consider the natural number corresponding to it

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3. For each $\alpha \in \{0,1\}^*$, consider the natural number corresponding to it
4. Define $UC(\alpha) = 0$ only if $M_\alpha(\mathrm{I}_\alpha) = 1$; o.w. $UC(\alpha) = 1$ when $M_\alpha(\mathrm{I}_\alpha) = 0$ or never halts

Theorem 1

UC is undecidable

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1. Reduce UC to Halting — direction is very important!
2. Halting is decidable $\Rightarrow$ UC is decidable
3. Since THM1, this is impossible. So Halting is undecidable.

EMPTY Problem

Definition

Given a program M, output 1 if $M(x) = 1$ for any $x \in \{0, 1\}^*$.

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1. We leave the rest problems in homework
2. One more question: Is this language decidable or not?

   Given a program M, an input x, and t, output 1 if M accepts x in t steps.

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> Given a program M, an input x, and t, output 1 if M accepts x in t steps.

3. Next: What is the limit of **efficient computation**?

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Definition

P is the class of all problems with a (deterministic) polynomial time algorithm — same as P := class of poly-time algorithms

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2. Issue 1: How about randomized poly-time algorithms? Most researchers believe Class(randomized poly-time algorithms)=P ☺
3. Issue 2: Definition P is too strict — the algorithm runs in poly-time for all inputs. Average-case complexity ☺

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3. Issue 2: Definition P is too strict — the algorithm runs in poly-time for all inputs. Average-case complexity ☺
4. Issue 3: Lack of precision, is n^{10} efficient? ☹
5. Issue 4: How about quantum algorithms? Not sure whether general quantum computers are realizable ☹

Relation between Languages

Given two problems A and B, how to compare their hardness?

1. Prime is equivalent to composite
2. How to compare connectivity and set cover?

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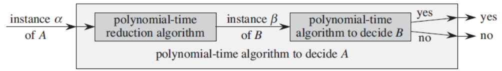
Poly Time Reduction (a.k.a. Cook/Karp Reduction)



$A \leq_p B$: poly-time reduction from A to B

Requirements:

1. Reduction time is polynomial
2. Map Yes instance to Yes instance and No instance to No — the most technical part

— for now, $A \leqslant_p B$ means that B is harder than A; essentially, $B \in \mathrm{P} \Rightarrow A \in \mathrm{P}$

Complete problems

Complete Problems of a class

A problem P is complete in the class $\mathcal{C}$ only if (1) $P \in \mathcal{C}$; (2) any problem $Q \in \mathcal{C}$ has a reduction $Q \leqslant_p P$

— informally, P is the “hardest" problem in $\mathcal{C}$

1. If there is a poly-time algorithm of P, then every problem in $\mathcal{C}$ has poly-time algorithms

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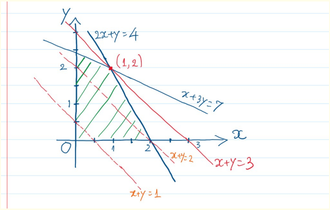
2. Example 1: Linear programming is complete in P



3. Another P-complete problem: Given a program M, input x, and string S, determine $M(x) = 1$ in $|S|$ steps or not

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Background

There are many problems not in P:

Example

Given a program M, input x, and number N, determine $M(x) = 1$ in N steps or not

1. This problem is exponential-time complete.
2. Question: What's difference between the P-complete problem — Given M, x, and string S, determine $M(x) = 1$ in $|S|$ steps or not?

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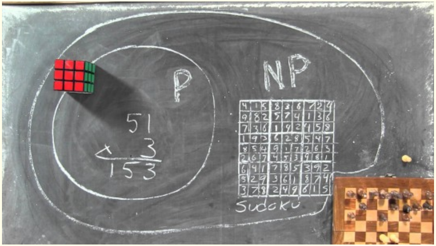
3. However, this problem is not very interesting
4. The most interesting class probably not in P is NP



What is NP?

The original definition is by non-deterministic Turing machine. We will consider the modern interpretation:

Definition of NP

A language/problem L is in NP only if $\exists$ an algorithm, called verifier, V such that

1. For any $x \in L$, $\exists$ a proof y s.t. $V(x, y) = 1$ in poly$(|x|)$ time — completeness
2. For any $x \notin L$, for any proof y, $V(x, y) = 0$ in poly$(|x|)$ time — soundness

1. $V(x, \cdot)$ is a non-deterministic algorithm but $V(x, y)$ is deterministic

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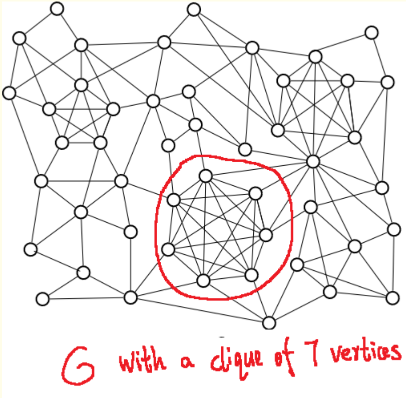
1. $V(x, \cdot)$ is a non-deterministic algorithm but $V(x, y)$ is deterministic
2. In Case 1, y depends on x
3. $|y| = \text{poly}(n)$ because V runs in poly-time
4. Example 1: $L_{\text{CLIQUE}} = \left\{(G, k) : \exists \text{ a clique of size } k \text{ in } G\right\}$

Example 1: Clique

1. y is a subset of k vertices in G
2. $V\Big((G,k),y\Big)$ checks all pairs in y are connected in G



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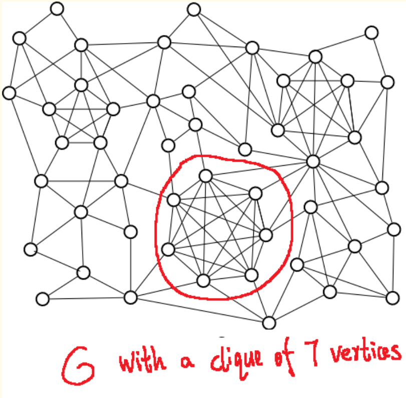

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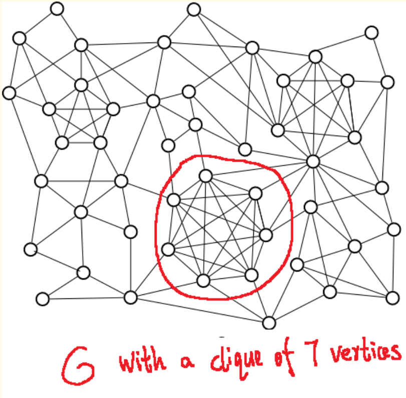
4. Soundness: If $(G,k) \notin L_{\text{CLIQUE}}$ — $\forall$ k-subset in G, V rejects it since it is not a clique



Example 2: SAT problem

1. Description: each instance Φ has n Boolean variables $x_1, \ldots, x_n$ and m clauses $C_1, \ldots, C_m$
2. $L_{\mathrm{SAT}} = \left\{ \Phi \text{ is satisfiable} : \exists \sigma \in \{T, F\}^n \text{ s.t. } C_1(\sigma) = C_2(\sigma) = \cdots = C_m(\sigma) = T \right\}$

$C_1: x_1 \vee x_2 \vee \overline{x_4} \vee \overline{x_n}$

$C_2: \overline{x_3} \vee \overline{x_{n-2}} \vee \overline{x_{n-1}} \vee x_n$

$C_3: x_5$

$C_4: x_6 \vee x_7$

$\vdots$

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3. y is a string $\in \{T, F\}^n$ as the assignment on x
4. V verifies $C_1(y) = C_2(y) = \cdots = C_m(y) = T$
5. Equivalent to define Φ as a CNF $C_1 \wedge C_2 \wedge C_3 \wedge \cdots \wedge C_m$

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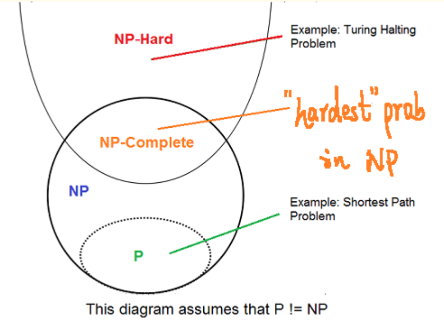

4. Which problem? — NP-complete

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NP-complete: “Hardest" problems in NP under poly-time reduction

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2. A concrete plan to show $\mathrm{P} \neq \mathrm{NP}$: (1) Find an NP-complete problem L; (2) Prove that L does not admit poly-time algorithms

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3. Good news: A long list of natural NP-complete problems: Clique, SAT, independent set, ...
4. Bad news: Extremely difficult to show: a problem *does not* have poly-time algorithms
5. Since $\mathrm{P} \neq \mathrm{NP}$ is very plausible, another way: problem Q is NP-complete means $Q \notin \mathrm{P}$s.t. Q does not have a poly-time algorithm

More about NP

1. If we can not design poly-time algorithms for Q, showing Q is NP-complete gives an excuse

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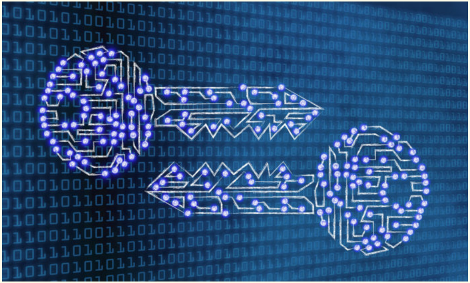

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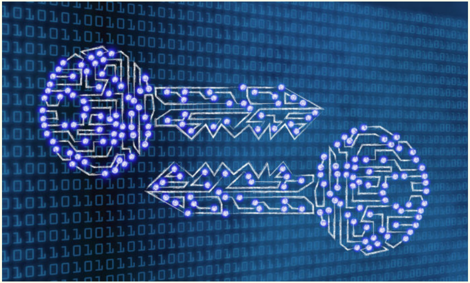


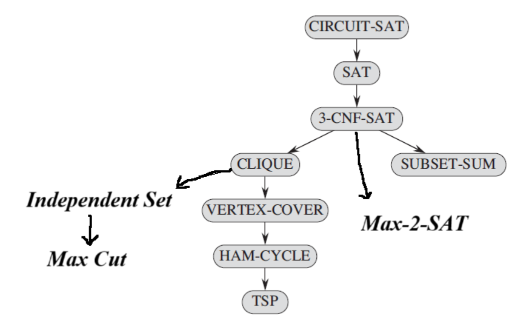
   In fact, cryptography assumptions are much stronger than P$\neq$NP
3. Surprisingly, very few hard problems are presumably not NP-complete: graph isomorphism, factoring, . . .;

NP-compelete

Next question: How to show a problem is NP-complete?

Recall definition

A problem L is NP-complete iff (1) $L \in \mathrm{NP}$; (2) for any $Q \in \mathrm{NP}$, $Q \leqslant_p L$.



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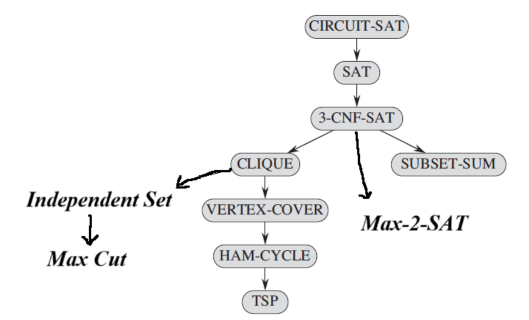

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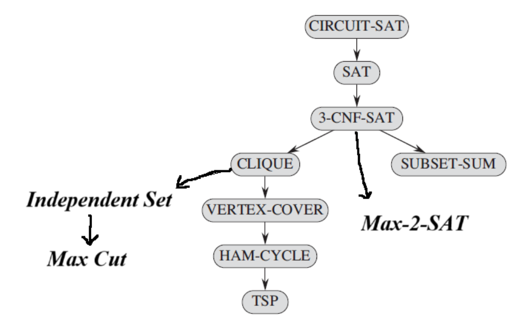
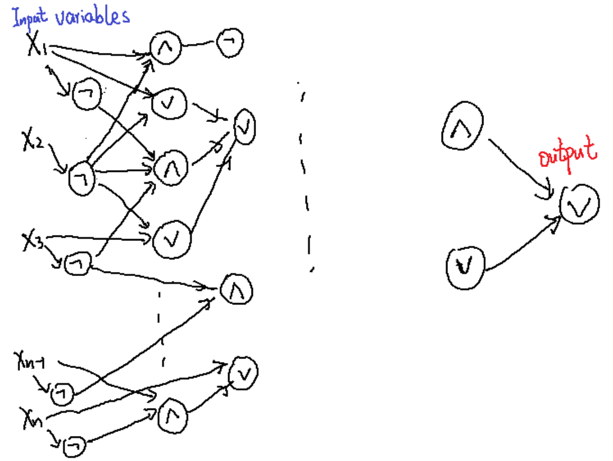
1. But which problem shall we begin with?
2. Roadmap of NPC problems



CIRCUIT-SAT

Description

$L_{\text{CIRCUIT-SAT}} = \{C : C \text{ is satisfiable}\}$ where circuit C is a DAG with AND $\wedge$, OR $\vee$, NOT $\neg$ gates and n input variables $x_1, \ldots, x_n$



CIRCUIT-SAT (II)

Theorem 34.7

$L_{\mathrm{CIRCUIT-SAT}}$ is NP-complete.

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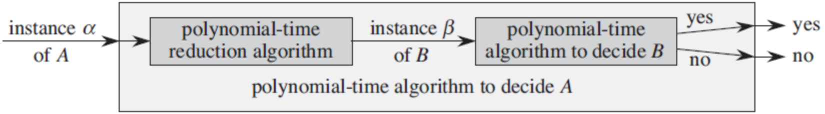
1. Lemma 34.5 in CLRS: $L_{\mathrm{CIRCUIT-SAT}}$ is in NP



$A \leq_p B$: poly-time reduction from A to B

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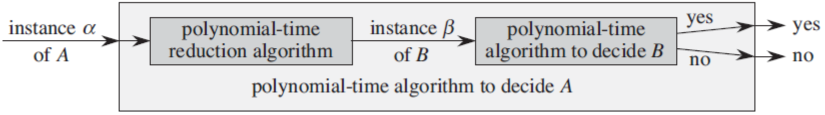

$A \leq_p B$: poly-time reduction from A to B

2. Lemma 34.6 in CLRS: $\forall Q \in \mathrm{NP}$, $Q \leqslant_p L_{\text{CIRCUIT-SAT}}$ — roughly, fix V and x then treat proof y as Boolean variables such that execution on the hardware level is a circuit

1. To show $Q \leqslant_p L_{\mathrm{CIRCUIT-SAT}}$, consider Q's verifier V such that $Q(x) = 1$ iff $\exists y$ s.t. $V(x, y) = 1$.
2. Our goal is to show a reduction that given V and x, outputs a circuit $C_{V,x}(y)$ s.t. $C_{V,x}(y) = V(x, y)$

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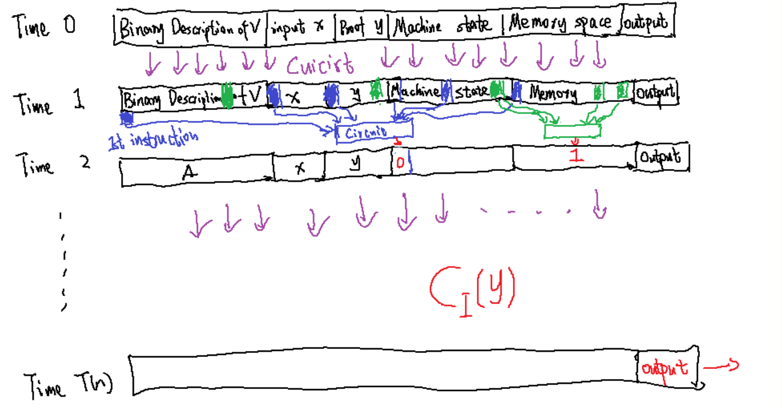
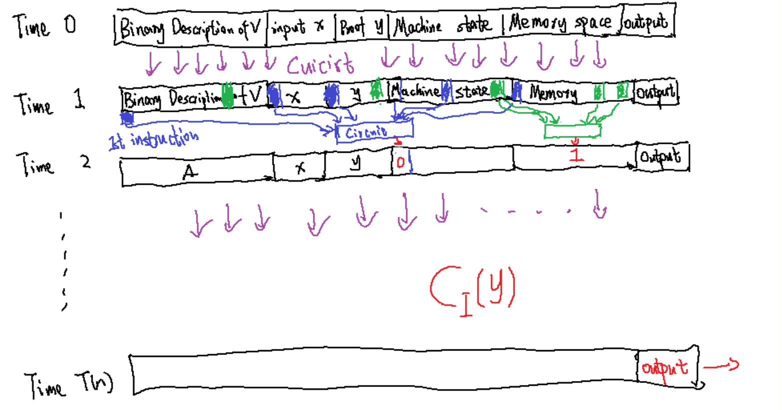
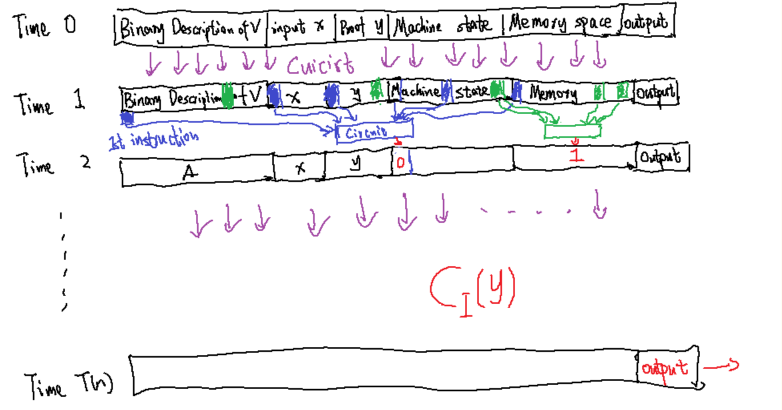
2. Our goal is to show a reduction that given V and x, outputs a circuit $C_{V,x}(y)$ s.t. $C_{V,x}(y) = V(x, y)$
3. In one sentence, $C_{V,x}(y)$ is the circuit computing $V(I, y)$ in hardware level





1. Because the time and memory space of V is $\text{poly}(n)$, its size is $\text{poly}(n)$



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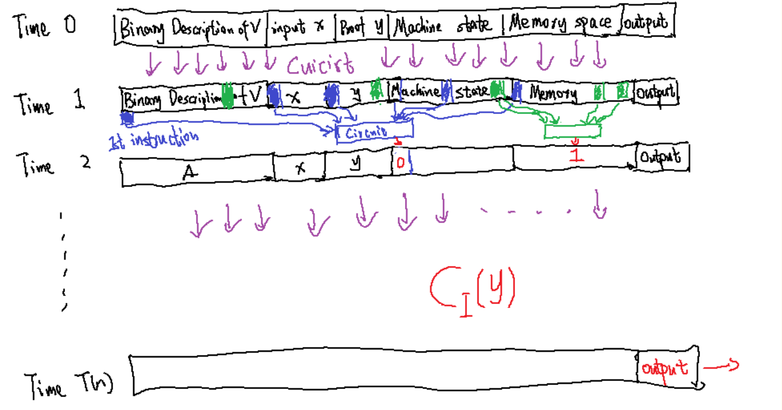

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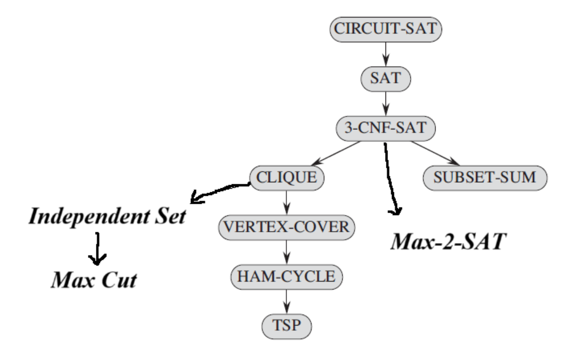
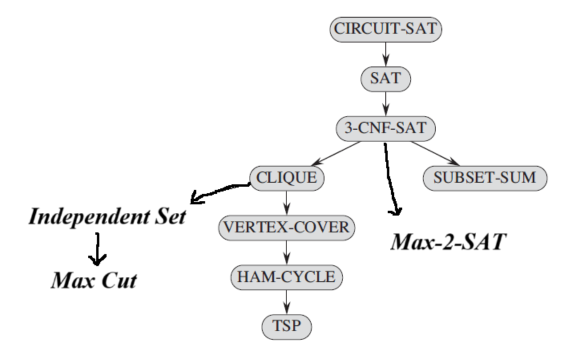
2. We can compute $C_{V,x}(y)$ in $\text{poly}(n)$ time
3. Because $C_{V,x}(y) = V(x,y)$ for any y, $x \in L_Q$ iff $C_{V,x}$ is satisfiable ☺

Road Map



1. CIRCUIT-SAT is the 1st non-trivial NP-complete problem

Road Map



1. CIRCUIT-SAT is the 1st non-trivial NP-complete problem
2. Next show SAT is NP-complete:

> Each instance Φ has n Boolean variables $x_1, \ldots, x_n$ and m clauses $C_1, \ldots, C_m$ such that $L_{\mathrm{SAT}} = \{\Phi : \exists \sigma \in \{T, F\}^n \text{ s.t. } C_1(\sigma) = C_2(\sigma) = \cdots = C_m(\sigma) = T\}$

SAT Problem

Theorem 34.9 in CLRS

SAT is NP-complete

SAT Problem

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1. SAT is in NP

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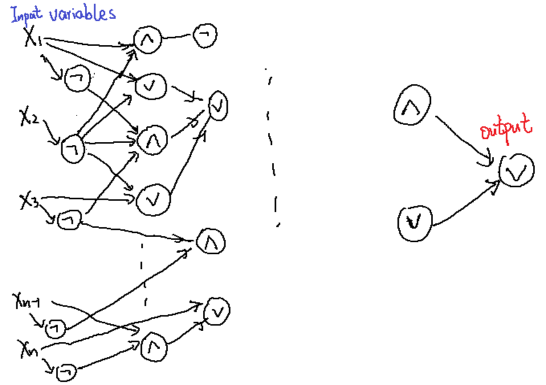
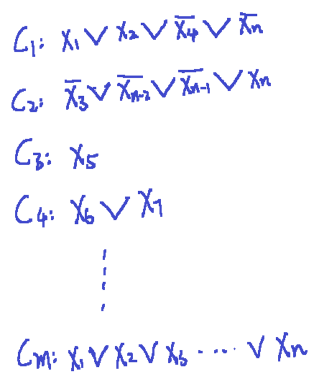
1. SAT is in NP
2. Reduce CIRCUIT-SAT $\rightarrow$ SAT

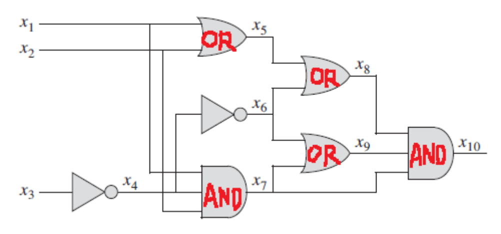


$\Rightarrow$



Basic Idea

1. Assign a Boolean variable to each gate to denote its evaluation:



2. Question: How to make sure $x_4 = \overline{x_3}$ and $x_5 = x_1 \vee x_2$?
3. Completeness & Soundness: $\exists x$ s.t. $C(x) = \mathit{True} \iff \exists \sigma$ s.t. all clauses in $\Phi(\sigma)$ are true

3-SAT

Definition: Each instance Φ has n Boolean variables $x_1, \ldots, x_n$ and m clauses $C_1, \ldots, C_m$ of width 3 such that
$L_{\mathrm{3SAT}} = \{\Phi : \exists \sigma \in \{T, F\}^n$ s.t. $C_1(\sigma) = C_2(\sigma) = \cdots = C_m(\sigma) = T\}$

Theorem 34.10 in CLRS

3SAT (CNF) is NP-complete.

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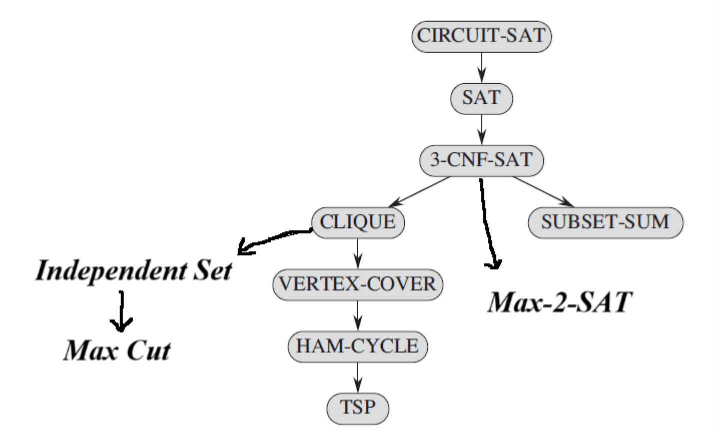
Proof Sketch:

1. 3SAT is in NP
2. Reduce SAT to 3SAT

Outline

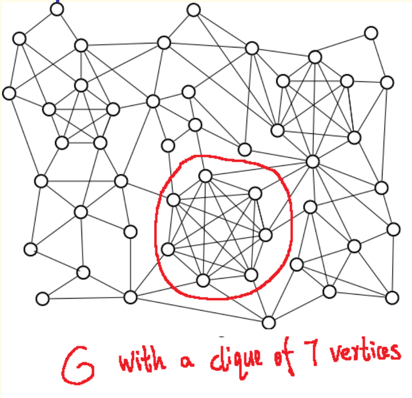
Overview



To show a problem Q is NP-complete

1. Easy part: $Q \in \mathrm{NP}$
2. Tricky part: Reduce a NP-complete problem (known ones like CIRCUIT-SAT, SAT, 3SAT) to Q

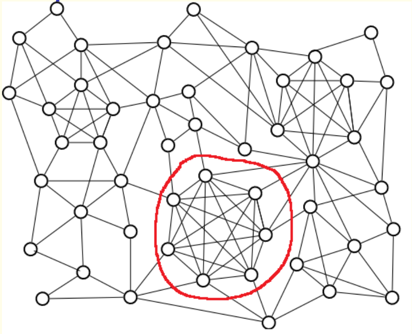
Clique



Theorem 34.11 in CLRS

$L_{\mathrm{CLIQUE}} := \{(G, k) : \exists$ a k-clique in $G\}$ is NP-complete.

Clique



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Proof Sketch:

1. It is in NP.
2. Reduce 3SAT to CLIQUE.

Discussion

1. Gadgets are the key in reductions — more gadgets next!
2. We only discussed decision problem so far: Given (G, k), output YES to indicate $\exists$ a k-clique in G.

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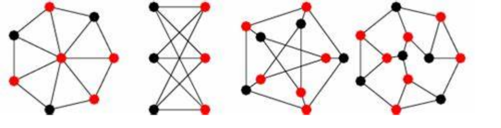
4. Question: Is the optimization problem NP-complete?
   — While it is harder than the decision problem, can not prove it is in NP ☹
5. Definition: For a problem P, if $Q \leqslant_p P$ for any $Q \in \mathrm{NP}$, call p NP-hard
6. Examples of NP-hard: Largest clique in G; find an assignment to maximize # clauses for 3SAT

Set Cover

A vertex cover of $G = (V, E)$ is a subset $S \subseteq V$ such that every edge has at least one endpoint in S



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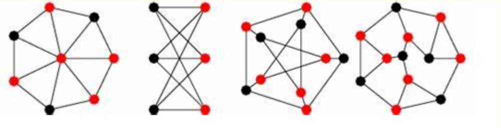

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$L_{\mathrm{Vertex-Cover}} = \{(G, k) : G$ has a vertex cover of size $k\}$ is NP-complete

1. $L_{\mathrm{Vertex-Cover}}$ is in NP
2. Reduce CLIQUE to Vertex-Cover

Subset Sum

Theorem 34.15 in CLRS

$L_{\mathrm{Subset-Sum}} = \left\{ \langle S = (s_1, \ldots, s_n), t \rangle : \exists S' \subseteq S \text{ with summation } = t \right\}$ is NP-complete

1. $L_{\mathrm{Subset-Sum}}$ is in NP
2. Reduce 3SAT to Subset-Sum

Reduction

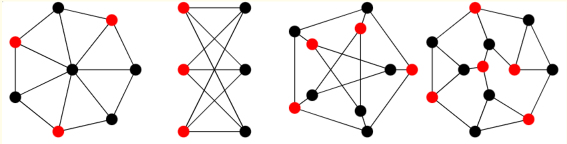
$C_1 = x_1 \vee \overline{x_2} \vee \overline{x_3}$, $C_2 = \overline{x_1} \vee \overline{x_2} \vee \overline{x_3}$, $C_3 = \overline{x_1} \vee \overline{x_2} \vee x_3$, $C_4 = x_1 \vee x_2 \vee x_3$

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
v_1	=	1	0	0	1	0	0	1
v_1'	=	1	0	0	0	1	1	0
v_2	=	0	1	0	0	0	0	1
v_2'	=	0	1	0	1	1	1	0
v_3	=	0	0	1	0	0	1	1
v_3'	=	0	0	1	1	1	0	0
s_1	=	0	0	0	1	0	0	0
s_1'	=	0	0	0	2	0	0	0
s_2	=	0	0	0	0	1	0	0
s_2'	=	0	0	0	0	2	0	0
s_3	=	0	0	0	0	0	1	0
s_3'	=	0	0	0	0	0	2	0
s_4	=	0	0	0	0	0	0	1
s_4'	=	0	0	0	0	0	0	2
t	=	1	1	1	4	4	4	4

More NP-hard Problems

Max Independent Set

Given a graph G, it is NP-hard to find the maximal independent set.



Which problem shall we reduce from?

Max Cut

Theorem

Given a graph G, it is NP-hard to find a max cut.

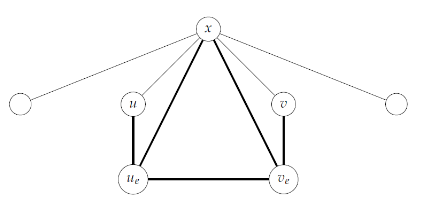
Reduction from Max Independent Set:

Max Cut

Theorem

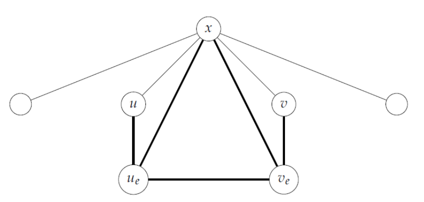
Given a graph G, it is NP-hard to find a max cut.

Reduction from Max Independent Set: Given $G = (V, E)$ from MIS, construct $G' = (V', E')$ for Max Cut:



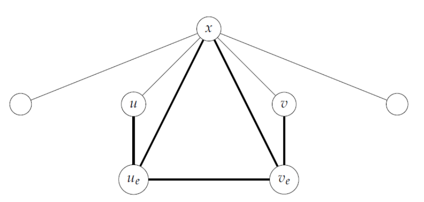
$V' = x \cup V \cup \{u_e, v_e : \forall e \in E\}$
$E' = \{(x, v) : \forall v \in V\} \cup \{e - \text{gadget} : \forall e \in E\}$
where (u, v)-gadget $:= (x, u_e), (x, v_e), (u, u_e), (v, v_e), (u_e, v_e)$

Analysis



1. $\Rightarrow$ If $I \subset V$ is independent of size k, define a cut $S := I$ then for each (u, v)-gadget, add u_e to S if $v \in S$ or add v_e to S if $u \in S$
2. OBS: cut S is $|I| + 4 \cdot |E|$

Analysis



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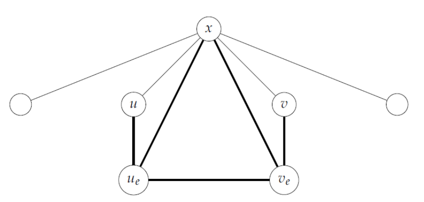
4. Consider $I := S \cap V$ — suppose $\exists m(I)$ edges inside I s.t. it is not independent

Analysis



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4. Consider $I := S \cap V$ — suppose $\exists m(I)$ edges inside I s.t. it is not independent
5. For a gadget of $e = (u, v)$, if both u and $v \in I$, S cuts at most 3 edges in this gadget. Otherwise, S cuts 4 edges.
6. $E(S, S') \leqslant |I| + 4 \cdot |E| - |m(I)| \Rightarrow$ deleting one point in each $m(I)$ provides an independent set of size $|I| - |m(I)| \geqslant k$

2-SAT

2-SAT

1. n Boolean variables $x_1, \ldots, x_n \in \{T, F\}$ and m clauses of width 2 like $x_1 \vee \overline{x_2}, \overline{x_3} \vee \overline{x_4}, x_2 \vee x_3, \ldots$
2. Find an assignment to satisfy all clauses.

Different from 3-SAT, 2-SAT is in P; however, MAX-2-SAT is NP-hard

1. Why 2-SAT $\in \mathrm{P}$?

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2. $x_1 \vee \overline{x_2}$ implies $x_1 = F \Rightarrow x_2 = F$ and $x_2 = T \Rightarrow x_1 = T$
3. Construct a directed graph on $2n$ vertices and draw the above $2m$ relations

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3. Construct a directed graph on $2n$ vertices and draw the above $2m$ relations
4. If $\exists$ a cycle contains $x_i = T$ and $x_i = F$, no solution
5. Otherwise, assign a variable to T and repeat it

Max-2-SAT

Max-2-SAT

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Reduction from 3-SAT:

Max-2-SAT

Max-2-SAT

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Reduction from 3-SAT:

1. For each clause $C = x \vee y \vee z$ in 3-SAT, consider 10 clauses in 2-SAT with an extra variable w_C
2. $x, y, z, w_C, \overline{x} \vee \overline{y}, \overline{x} \vee \overline{z}, \overline{y} \vee \overline{z}, x \vee \overline{w_C}, y \vee \overline{w_C}, z \vee \overline{w_C}$

Max-2-SAT

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3. $C(\sigma) = \mathit{True} \Rightarrow$ 7 of them are satisfied with some w_C; otherwise at most 6
4. Claim: 3-SAT is satisfiable $\Leftrightarrow$ value(2-SAT) $= 7m$

Discussion

1. There are poly-time algorithms for special cases of Vertex-Cover and Subset-Sum
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2. Example 1: Vertex-Cover of bipartite graphs is in P
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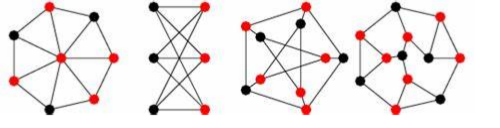
4. There are various ways to design the reductions!
5. Most important thing: The direction is from a known NP-complete problem to this problem!

Outline

Introduction

Many interesting problems, like finding the smallest Vertex-Cover in G, are NP-hard



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In both theory and practice, what can we do for NP-hard problems?

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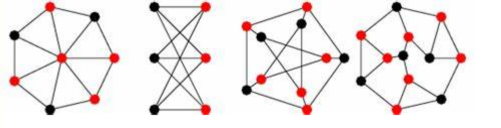

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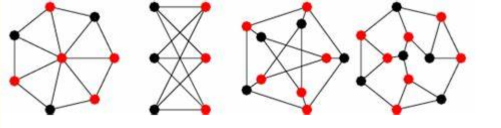

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2. Local search heuristics (like evolution)
3. Approximation Algorithms

Approximation Algorithms

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Key Insight

Design a trade-off between time and accuracy

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1. This is an important idea in CS: in big data algorithms, design trade-offs between space and accuracy
2. What do we mean accuracy?
3. Suppose our algorithm finds a solution with value ANS
4. Define ANS/OPT as the approximation ratio — >1 for minimizations and <1 for maximizations

Vertex-Cover

Given G, find a vertex-cover with a minimum size

Theorem 35.1 in CLRS

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Recall that for Set-Cover, the approximation algorithm is $\ln n$.

Traveling Salesman Problem — Hamilton Cycle

Description: Given a weighted graph $G = (V, E)$, find a sequence $\vec{u} = (u_0, \ldots, u_n)$ of V s.t. (1) $u_0 = u_n$ and each vertex appears once; (2) the total weight $\sum_{i=1}^{k} c(u_{i-1}, u_i)$ is minimum.

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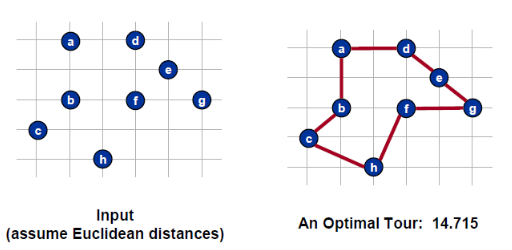

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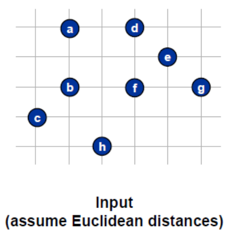

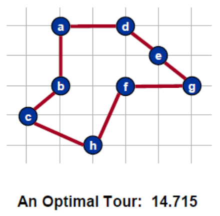

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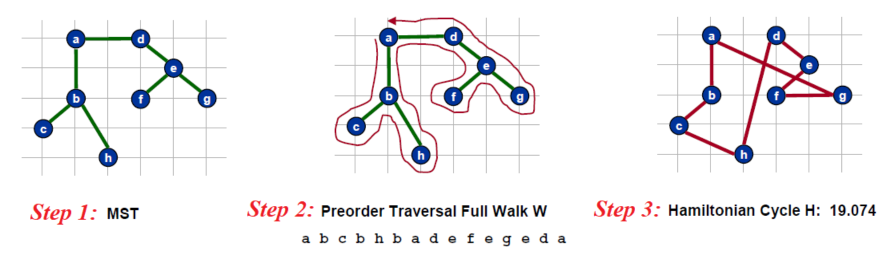
Remark: The triangle inequality is necessary, otherwise no constant approx unless $\mathrm{P}{=}\mathrm{NP}$— Section 35.2.2 in CLRS

2-Approx of TSP



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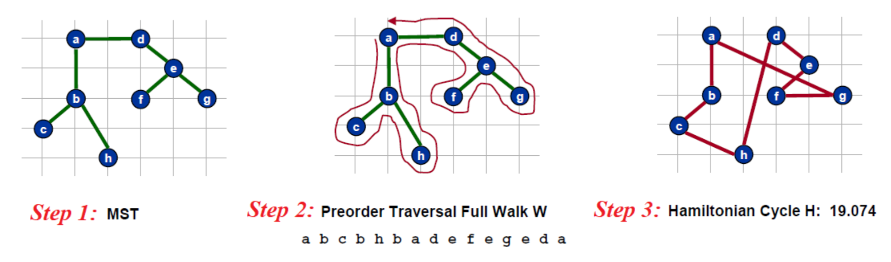

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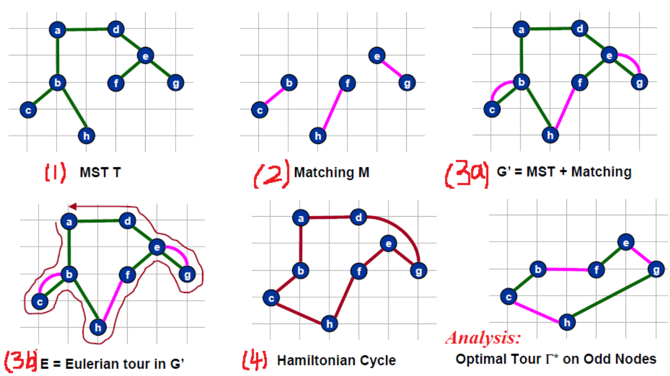
Christofides Algorithm — short-cut via the Eulerian Tour



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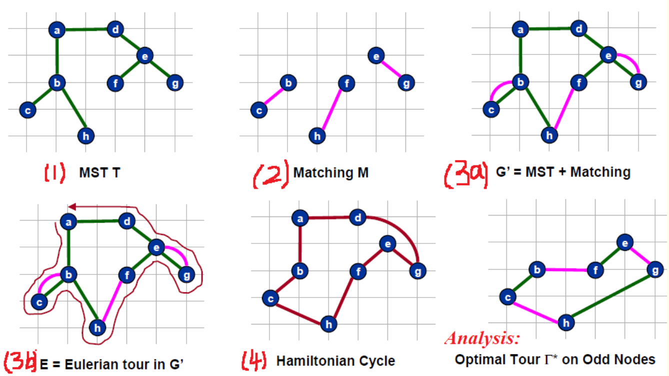

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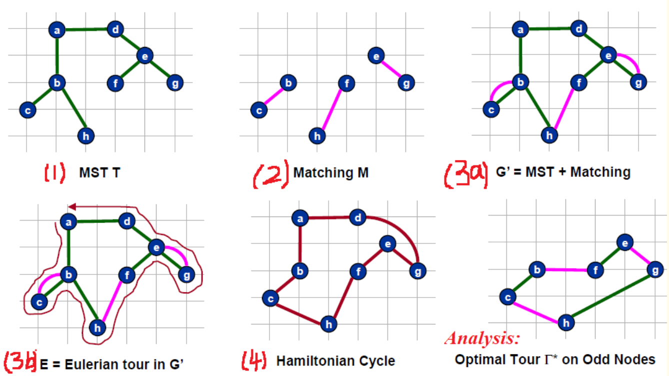

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Approximating MIS

Here is a strange claim about approximating max-independent-set

Claim

For some $\alpha < 1$, if $\exists$ an α-approximation algorithm in P for MIS, then $\sqrt{\alpha}$-approximating MIS is also in P.

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4. Next, how to finish the proof?

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In fact, we could consider k-fold OR-power for any k!

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For some $\alpha < 1$, if there is an α-approximation algorithm in P for MIS, then $\alpha^{1/k}$-approximating MIS is also in $\mathrm{P}$for any k.

1. If there is a 0.001-approximation, setting $k = 800$, we obtain a 0.99-approximation.
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5. 3 steps: NP-hard to distinguish $\exists$ an independent set of size $n/10$ or MIS $\leqslant 0.99 \cdot n/10$; then apply $O(\log n)$-fold OR-power; finally, sparsify the product to $n^{O(1)}$ size

Summary

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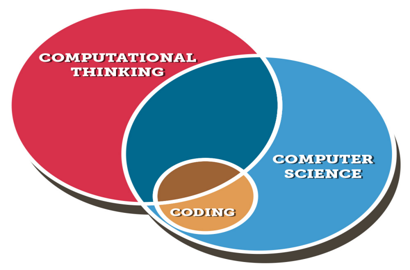
5. Many natural problems are NP-complete: SAT, CLIQUE, SUBSET-SUM, Vertex-Cover, . . .
6. There are many ways to cure NP-complete ☺

For this course



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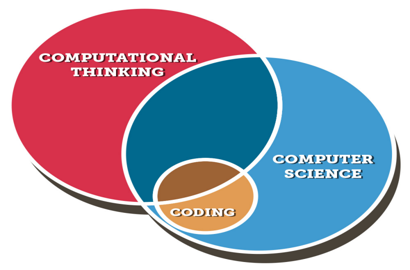

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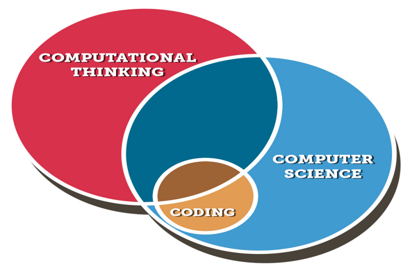

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3. Hope you enjoy this course and learn something (at least) ☺

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