Quantum signatures of black hole mass superpositions

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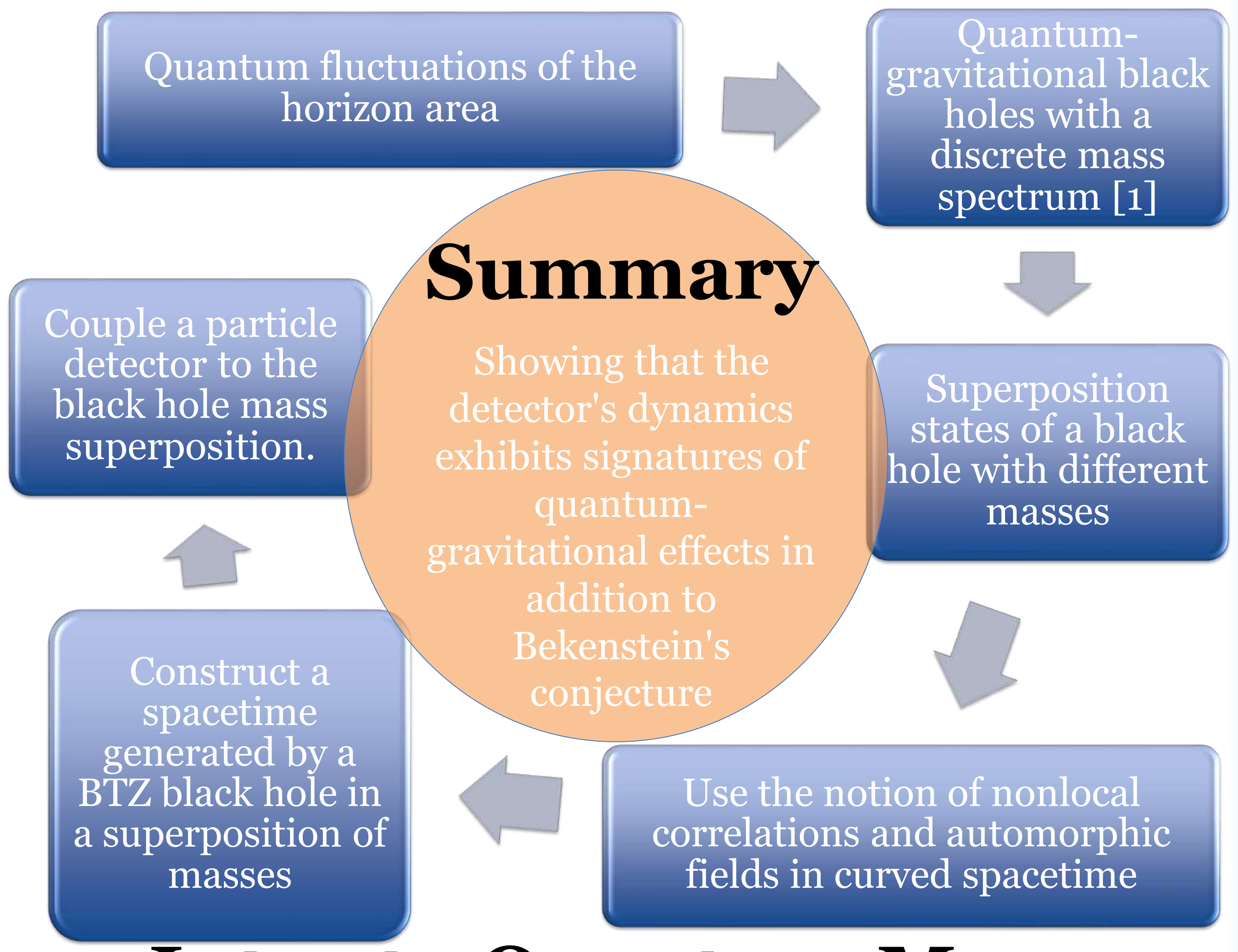

Intro to Quantum Mass Superposition

- The quantization of the mass was argued to modify the character of Hawking radiation to possess a discrete emission spectrum with even spacing between energy levels [1].
- The existence of mass-quantized black holes implies that they may also exist in superpositions of mass eigenstates.
- A mass-superposed black hole => e.g. of quantum superposition of spacetimes as such
  - ➢ the different masses individually define a unique classical solution to Einstein's field equations
  - ➢ the resulting amplitudes of the mass-superposition state correspond to associated 'spacetime states'.

Understanding the effects that arise in such a spacetime superposition is an important stepping stone towards understanding how spacetime itself may be quantized.

AdS-Rindler and the BTZ black hole

The BTZ black hole is a (2+1)-dimensional solution to Einstein's field equations with a negative cosmological constant $\Lambda = -1/l^2$ where l is the anti-de Sitter (AdS) length scale (the BTZ spacetime is asymptotically AdS) [2]. We begin with the AdS-Rindler metric, which is given by

$$ds^2 = -f(r)dt^2 + f(r)^{-1}\, dr^2 + r^2 d\varphi^2 \tag{1}$$

where $f(r) = (r^2/l^2 - 1)$ and φ takes values on the full real line.

A quotient of AdS-Rindler spacetime [44–46] under the identification $\Gamma: \phi \to \phi + 2\pi\sqrt{M}$ (making it a true angular coordinate) and appropriately rescaling the temporal and radial coordinates with a mass parameter $\sqrt{M}$. This yields the BTZ metric,

$$ds^2 = -g(r)\, dt^2 + g(r)^{-1}dr^2 + r^2 d\varphi^2 \tag{2}$$

where $g(r) = (r^2/l^2 - M)$ and $\sqrt{M}l < r < \infty$, $-\infty < t < \infty$, $\phi \in [0, 2\pi)$. The spacetime has a local Tolman temperature $T_L = r_H/(2\pi l^2 p\sqrt{g(R)})$, where R denotes the radial coordinate and $r_H = \sqrt{M}l$ is the radial coordinate of the event horizon.

Automorphic field theory on the BTZ black hole spacetime

- The theory of automorphic fields for the BTZ spacetime, are constructed from the ordinary (massless scalar) fields $\hat{\psi}$ in (2+1)-dimensional AdS spacetime (AdS_3) via the identification Γ, yielding

$$\hat{\phi}(\mathsf{x}) := \frac{1}{\sqrt{\mathcal{N}}}\sum_n \eta^n \hat{\psi}(\Gamma^n \mathsf{x}) \tag{3}$$

Where, $x = (t, r, \phi)$, $\mathcal{N} = \sum_n \eta^{2n}$ and $\eta = \pm 1$ (corresponding to untwisted and twisted fields respectively). The Wightman functions (two-point correlation functions) between the spacetime points x, x' is

$$\begin{aligned} W_{\text{BTZ}}^{(D)}(\mathsf{x},\mathsf{x}') &= \frac{1}{\mathcal{N}}\sum_{n,m}\eta^n\eta^m\langle 0|\hat{\psi}(\Gamma_D^n \mathsf{x})\hat{\psi}(\Gamma_D^m \mathsf{x}')|0\rangle \\ &= \frac{1}{\mathcal{N}}\sum_{n,m}\eta^n\eta^m W_{\text{AdS}}(\Gamma_D^n\mathsf{x}, \Gamma_D^m\mathsf{x}') \end{aligned} \tag{4}$$

Where $\Gamma_D^n : (t, r, \phi) \to (t, r, \phi + 2\pi n\sqrt{M_D})$ in a BTZ spacetime where the black hole mass M_D is fixed.

- To perform our analysis, we consider the field quantized on a background arising from superposing BTZ spacetimes with different black hole masses.
- The black hole–quantum field system can be described in the tensor product Hilbert space $\mathcal{H} = \mathcal{H}_{BH} \otimes \mathcal{H}_F$, where we consider the black hole to be (without loss of generality) in a symmetric superposition1 of two mass states |MA⟩, |MB⟩ while the field is in the AdS vacuum |0⟩

$$W_{\text{BTZ}}^{(AB)}(\mathsf{x},\mathsf{x}') = \frac{1}{\mathcal{N}}\sum_{n,m}\eta^n\eta^m W_{\text{AdS}}(\Gamma_A^n\mathsf{x}, \Gamma_B^m\mathsf{x}') \tag{5}$$

a Wightman function characterising amplitudes between fields associated with two BTZ black holes with different masses.

Particle Detectors

Coupling a particle detector to the black hole–quantum field system

Understand the physical effects induced by a spacetime associated with a black hole in a superposition of two masses, MA and MB.

The Unruh-deWitt particle detector model, a pointlike two-level system coupled to the field [3] considered here in its ground state |0⟩ at a fixed radial coordinate RD, which here means it is in a superposition of proper distances from the black hole horizon.

The detector couples to the field and black hole via the interaction Hamiltonian.

$$\hat{U} = I + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3) \tag{6}$$

The evolution of the system relative to the coordinate time t is described as

$$\hat{H}_{\text{int.}} = \lambda\eta(\tau)\hat{\sigma}(\tau)\sum_{D=A,B}\hat{\phi}(\mathsf{x}_D)\otimes|M_D\rangle\langle M_D| \tag{7}$$

The time-evolution operator expanded in the Dyson series to leading order in the coupling strength,

$$|\psi(t_f)\rangle = e^{-i\hat{H}_{0,S}t_f}\hat{U}(t_i,t_f)e^{i\hat{H}_{0,S}t_i}|\psi(t_i)\rangle \tag{8}$$

where the first- and second-order terms are

$$\begin{aligned} \hat{U}^{(1)} &= -i\lambda\int_{t_i}^{t_f}\mathrm{d}\tau\,\hat{H}_{\text{int.}}(\tau), \\ \hat{U}^{(2)} &= -\lambda^2\int_{t_i}^{t_f}\mathrm{d}\tau\int_{t_i}^{\tau}\mathrm{d}\tau'\hat{H}_{\text{int.}}(\tau)\hat{H}_{\text{int.}}(\tau'). \end{aligned} \tag{9}$$

a measurement in the symmetric-antisymmetric superposition basis, |±i ⟩, then (upon tracing out the final states of the field) one finds that the ground and excited state probabilities of the detector are given by

$$\begin{aligned} P_G^{(\pm)} &= \frac{1}{2}\Big(1 \pm \cos(\Delta E\Delta t)\Big)\Big[1 - \frac{\lambda^2}{2}\Big(P_A + P_B\Big)\Big], \\ P_E^{(\pm)} &= \frac{\lambda^2}{4}\Big(P_A + P_B \pm 2\cos(\Delta E\Delta t)L_{AB}\Big), \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{P_D}{\sigma} &= \frac{\sqrt{\pi}H_0(0)}{8} - \frac{i}{8\sqrt{\pi}}\text{PV}\int_{-t_f/2l}^{t_f/2l}\frac{\mathrm{d}z\,X_0(2lz)H_0(2lz)}{\sinh(z)} \\ &\quad + \frac{1}{4\sqrt{2\pi}\sum_n\eta^{2n}}\sum_{n\neq m}\text{Re}\int_0^{t_f/l}\frac{\mathrm{d}z\,X_0(lz)H_0(lz)}{\sqrt{\beta_{nm} - \cosh(z)}} \\ \beta_{nm} &= \frac{1}{\gamma_D^2}\left[\frac{R_D^2\cosh(2\pi(n-m)\sqrt{M_D})}{M_D l^2} - 1\right] \end{aligned}$$

$$\begin{aligned} \frac{L_{AB}}{\sigma} &= \frac{Y_0}{\sum_n\eta^{2n}}\sum_{n,m}\text{Re}\int_0^{t_f/l}\frac{\mathrm{d}z\,Z_0(lz)Q_0(lz)}{\sqrt{\alpha_{nm} - \cosh(z)}}. \\ \alpha_{nm} &= \frac{1}{\gamma_A\gamma_B}\left[\frac{R_D^2\cosh(2\pi(m\sqrt{M_A} - n\sqrt{M_B}))}{\sqrt{M_A M_B}l^2} - 1\right] \end{aligned}$$

Results

The response of the detector is sensitive to the mass ratio of the superposed black hole, exhibiting signatory peaks at rational values of $\sqrt{MB/MA}$.

This prescription for the detection of a quantum-gravitational effect:

- The first of its kind
- Constitutes a new method for investigating effects implied by Bekenstein's conjecture about the quantization of a black hole's mass.

Specifically, the allowed mass values of the BTZ black hole, assuming the Bohr-Sommerfeld quantization scheme for the horizon radius, are given by [4]:

$$r_H = \sqrt{M}l = n, \qquad n = 1, 2, \ldots$$

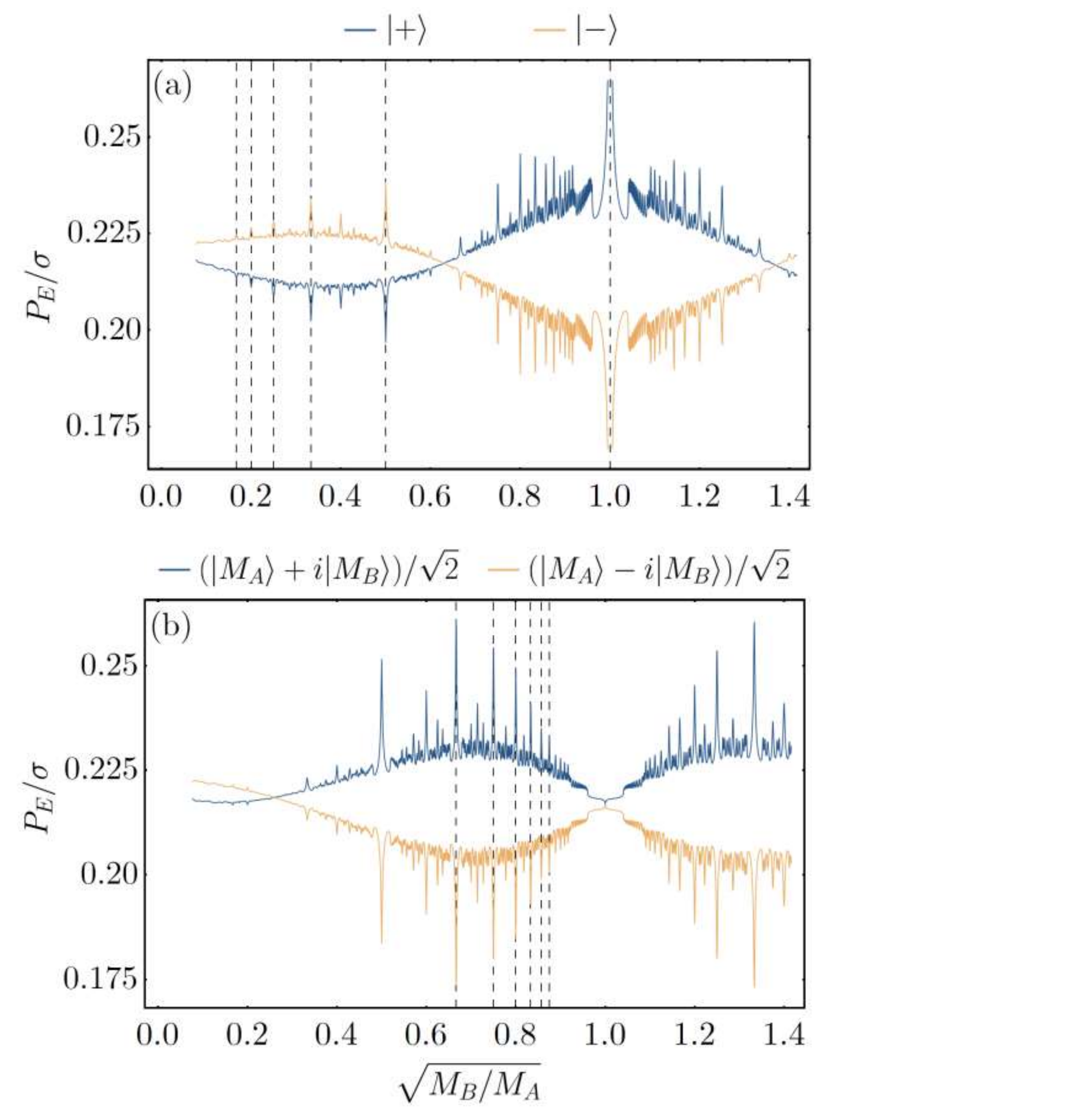

FIG. 1. Transition probability of the detector as a function of the square root of the mass ratio of the superposed amplitudes. The measurement basis corresponding to the relevant plot is indicated by the figure legends. In (a), the dashed lines correspond to $\sqrt{M_B/M_A} = 1/n$ where $n = \{1, \ldots 6\}$. In (b), the dashed lines correspond to $\sqrt{M_B/M_A} = (n-1)/n$ where $n = \{3, \ldots 8\}$. Moreover, the oscillating cross term in (b) is $\pi/2$ out-of-phase with that for the black hole measured in the (anti)symmetric basis. In all plots we have also used $l/\sigma = 5$, $R_D/\sigma = 25$, $t_f = 5\sigma$ and $M_A l^2 = 2$.

References

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