$$\Delta\phi = \int d r \cdot (\nabla\phi - A) \tag{6.89}$$

Therefore the current cross the junction is

$$J = \frac{\delta L_{\text{junction}}}{\delta A} = \mathcal{A}\, F'(\Delta\phi)\, \frac{\delta\Delta\phi}{\delta A} = \mathcal{A}\, F'(\Delta\phi)\, \frac{\delta A}{\delta A} = \mathcal{A}\, F'(\Delta\phi) \tag{6.90}$$

Now we apply a voltage V across the junction. Because we know that

$$\partial_0\, \phi(r) = -V(r), \tag{6.91}$$

it is easy to notice that

$$\Delta\phi = -V\, t + \text{constant} \tag{6.92}$$

Therefore:

$$J = \mathcal{A}\, F'(-V\, t + \text{constant}) \tag{6.93}$$

By applying a fixed V, we found that the current is changing with time. Because F is a periodic function, F' is also a periodic function. So J is aperiodic function of t, and the periodicity is $\pi\, \hbar/\text{e}\, \Delta V$

6.5. High Temperature superconductivity

High Tc compounds is a family of materials: La2-xSrxCuO4, Bi2Sr2Can-1CunO2n+4+x (BSCCO), YBa2Cu3O7-x, etc

6.5.1. Lattice structure

- Common feature: CuO2 planes separated by some barrier
- a undoped compound:
  - 1 hole per unit cell.
  - Cu site has lower potential energy, so electrons want to say on Copper
  - Strong Coulomb repulsion on the Cu site (it cost a lot of energy to put two holes on the same site). Basically, we can only have one hole per Cu site,
- The system is an insulator (Every Cu has one hole, then the hole cannot move any more).
- Each Cu site has one hole, and the hole carrier spin 1/2. So we have a lattice of spin. Anti-Ferromagnetic ordering.

6.5.2. Dope the system

Q: How to change electron density?

Option #1: Dope the system (replace some of the atom by another type of atom with different number of electrons, or remove some atoms from the systems).

- Dope electron or hole? Depending on the elements, we may be adding electron or removing electron from the system. Adding electron is know as "electron doping". Add hole is known as "hole doping".

Option #2, Gating

- too weak for high Tc compounds

Q: Which technique is good?

A: If one can choose from these two options, gating is in principle better, because it doesn't induce disorders. But in many cases gating only change the electron density by a small amount, so we have no choice by using the option #1.

Some high Tc compounds are hole doped and some are electron doped. But in most cases, "high Tc" means hole doped high Tc compounds, because hole doped case have a higher Tc.

6.5.3. Phase diagram

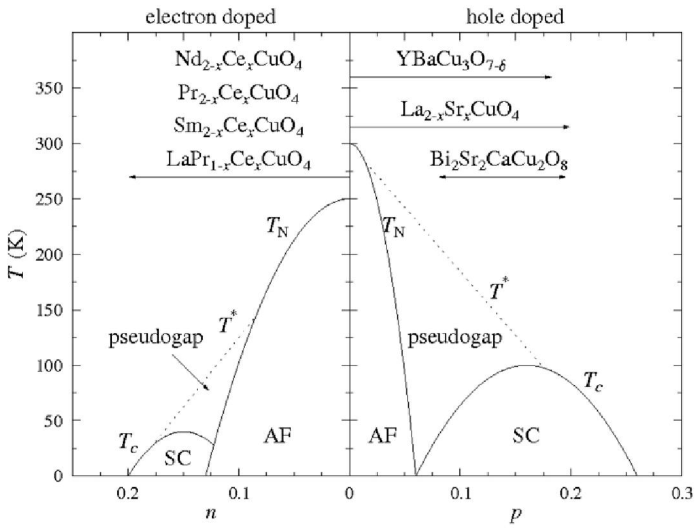

Fig. 6. Schematic Phase diagrams of hole-doped (right side) and electron doped (left side) high Tc superconductors (from wikipedia)

6.5.4. Tc

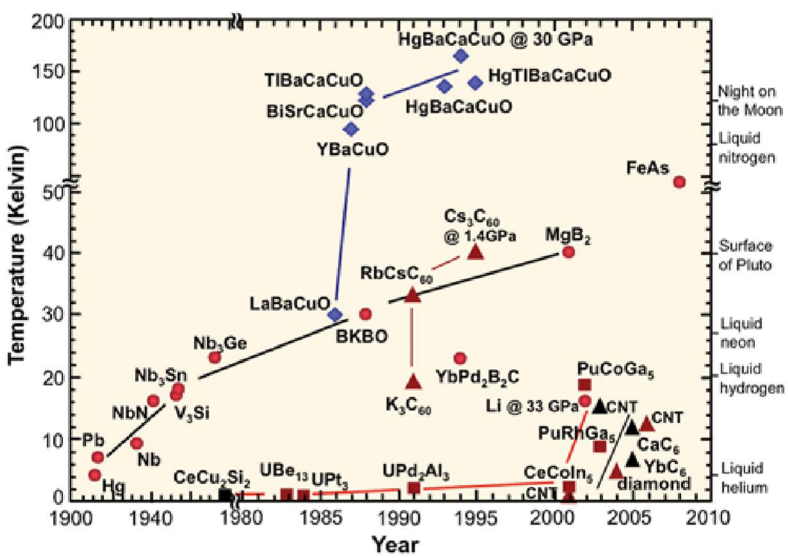

Fig. 7. Tc vs year

Iron based superconductor shows some similarity to electron doped high-Tc compounds (but they also have major differences). They have similar Tc. Both of them have strong e-e interaction, but the e-e interaction is less strong compare to hole-doped high Tc.

6.5.5. The order parameter (d-wave)

Order parameter is a function of momentum

$$\Delta_k = \langle c_{k\uparrow}\, c_{-k\downarrow}\rangle \tag{6.94}$$

In conventional superconductors (BCS theory), we assume Δ is isotropic, $\Delta_k = \langle c_{k\uparrow}\, c_{-k\downarrow}\rangle$ = constant. But in general it is a function of $\vec{k}$

$$\Delta_k = \Delta\left(\vec{k}\right) = \Delta(\,|\,k\,|,\,\theta_k) \tag{6.95}$$

The dependence on |k| is not important, because we know only electrons near the Fermi surface matters.

$$\Delta_k = \Delta\left(\vec{k}\right) = \Delta(\,|\,k\,|,\,\theta_k) \approx \Delta(k_F,\,\theta_k) = \Delta(\theta_k) \tag{6.96}$$

Let's consider the case $\Delta(\theta_k)$ is a real function. For conventional SCs and high Tc SCs, we can always make Δ real by choosing a proper gauge, but there are cases where Δ is NOT real and cannot be made real.

For conventional superconductors, $\Delta(\theta_k)$ varies as a function of θ, but the sign never change (s-wave)

For high T_c superconductors, $\Delta(\theta_k)$ changes sign for four times as we go around from $\theta_k = 0$ to 2π. It has a d-wave symmetry

The superconducting phase of high Tc materials are described very well by a d-wave pairing state, if one don't ask why the electrons pair up at such a high temperature. The mystery of high Tc is in fact not about the superconducting phase, but about the normal phase. The normal phase is a bad metal and we don't really have a theoretical understand about it. Since we don't know the normal phase, we don't know why electron turns into a superconductor.

6.5.6. Normal phase

- Like a Fermi liquid at high doping.
- Not a Fermi liquid at low doping (known as nonFermi liquid or strange metal, or pseudo-gap phase)

Pseudo gap,

For the under-doped side, there is an temperature scale, T^*, at which the behavior of many experiential measurable quantity changes.

Different experiments give different T^*.

Below T^*, the density of state shows that there seems to be a insulating gap, but not a full insulating gap (a pseudo-gap).

Q: Whether this is related with the superconducting gap or this is due to some other ordering tendency?

A: Unclear.

Q: Whether T^* is a phase transition or a crossover?

A: Unclear. Because different experiments give different T^*, it is probably a crossover (phase transition should have a well-define transition temperature T_c and all experiment should give the same T_c). However, in the presence of disorder, the transition temperature T_c many have some distribution. So it is NOT impossible to think T^* as a phase transition.

6.5.7. Mechanics for superconductivity: under debating

What we know:

- Not phonon driven
  - too weak to get such a high Tc.
  - Isotope effect is weak.
  - Phonon prefer s-wave pairing
- Very likely, it is due to e-e interaction and probably involves spin
  - e-e interaction is very strong
  - spin-spin coupling is very strong
  - Tc is too high to be explained by other interactions, which are all too weak.

Q: How can repulsion lead to pairing?

A: Unclear, but we have some hint.

6.5.8. Attraction: superconductor vs. inhomogeneity