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Observation of the Pairing Gap in a Strongly Interacting Fermi Gas

C. Chin,[1] M. Bartenstein,[1] A. Altmeyer,[1] S. Riedl,[1] S. Jochim,[1] J. Hecker Denschlag,[1] R. Grimm[1,2]*

We studied fermionic pairing in an ultracold two-component gas of ^{6}Li atoms by observing an energy gap in the radio-frequency excitation spectra. With control of the two-body interactions through a Feshbach resonance, we demonstrated the dependence of the pairing gap on coupling strength, temperature, and Fermi energy. The appearance of an energy gap with moderate evaporative cooling suggests that our full evaporation brought the strongly interacting system deep into a superfluid state.

1950s, pairing gap 的谱学观察，是超导的重大实验进展。

gap测量，为 investigating 配对粒子的性质提供了key。这些配对粒子是金属在低温下无摩擦电流的原因

开创性的BCS理论，在同时期也发展了起来。它表明，在简并费米海中的两个电子，可以通过有效的吸引相互作用，couple在一起，形成 delocalized 的，具有 bosonic 性质的复合粒子。

BCS Theory 预测低温下的 gap ∝ Tc 这与实验一致

In general，超流、超导物理，早已超出了 weak coupling 极限的BCS理论。

strong Coupling 极限下 Fermion 形成 localized Boson，可以发生BEC

BCS 极限和 BEC 极限由一个 Smooth 的 Crossover 连接，三十多年来，吸引了无数理论研究。

The spectroscopic observation of a pairing gap in the 1950s marked an important experimental breakthrough in research on superconductivity (*1*). The gap measurements provided a key to investigating the paired nature of the particles responsible for the frictionless current in metals at very low temperatures. The ground-breaking Bardeen-Cooper-Schrieffer (BCS) theory, developed at about the same time, showed that two electrons in the degenerate Fermi sea can be coupled by an effectively attractive interaction and will form a delocalized, composite particle with bosonic character. BCS theory predicted that the gap in the low-temperature limit is proportional to the critical temperature T_c for the phase transition, in agreement with the experimental measurements. In general, the physics of superconductivity and superfluidity go far beyond the weak-coupling limit of BCS theory. In the limit of strong coupling, paired fermions form localized bosons, and the system can undergo Bose-Einstein condensation (BEC). The BCS limit and the BEC limit are connected by a smooth BCS-BEC crossover, which has been a subject of great theoretical interest for more than three decades (*2*–*5*). The formation of pairs generally represents a key ingredient of superfluidity in fermionic systems, and the gap energy is a central quantity to characterize the pairing regime.

成分

配对的形成在 fermi系统超流中，起了关键作用。
而 gap energy 又是表征 pairing regime 的 central quantity
characterize

超冷 degenerate fermi Gases 实验的迅速进展
开启了独特的 testing ground.

The rapid progress in experiments with ultracold degenerate Fermi gases (*6*) has opened up a unique testing ground to study phenomena related to pairing and superfluidity at densities typically a billion times below the ones in usual condensed-matter systems. In cold-atom experiments, magnetically tuned scattering resonances (Feshbach resonances) serve as a powerful tool to control the two-body coupling strength in the gas (*7*). On the basis of such a resonance, a strongly interacting degenerate Fermi gas was recently realized (*8*). A major breakthrough then followed, with the creation of Bose-Einstein condensates of molecular dimers composed of fermionic atoms (*9*–*13*), which corresponds to the realization of a BEC-type superfluid in the strong coupling limit. By variation of the coupling strength, subsequent experiments (*12*, *14*–*18*) began to explore the crossover to a BCS-type system. This BEC-BCS crossover is closely linked to the predicted "resonance superfluidity" (*19*–*22*) and a "universal" behavior of a Fermi gas with resonant interactions (*23*, *24*). The observation of the condensation of atom pairs (*15*, *16*) and measurements of collective oscillations (*17*, *18*) support the expected superfluidity at presently attainable temperatures in Fermi gases with resonant interactions.

试验场

研究低密度下的配对、超流现象

比 condense matter 系统中低很多

cold atom 实验中，Feshbach resonances 为 control 2-body coupling strength (in gas) 提供了一个 powerfu tool

有了这样的 resonance，强相互作用 degenerate fermi gas 最近实现了

We prepared our ultracold gas of fermionic ^{6}Li atoms in a balanced spin-mixture of the two lowest sub-states $|1\rangle$ and $|2\rangle$ of the electronic $1s^2\,2s$ ground state, employing methods of laser cooling and trapping and subsequent evaporative cooling (*9*). A magnetic field B in the range between 650 to 950 G was applied for Feshbach tuning through a broad resonance centered at the field $B_0 \approx 830$ G. In this high-field range, the three lowest atomic levels form a triplet of states $|1\rangle$, $|2\rangle$, and $|3\rangle$, essentially differing by the orientation of the nuclear spin (m_I = 1, 0, -1, where m_I is the nuclear magnetic quantum number). In the resonance region with $B < B_0$, the s-wave scattering length a for collisions between atoms in states $|1\rangle$ and $|2\rangle$ is positive. Here, two-body physics supports a weakly bound molecular state with a binding energy $E_b = \hbar^2/(ma^2)$, where $\hbar$ is Planck's constant h divided by 2π and m is the atomic mass. Molecules formed in this state can undergo BEC (*9*–*13*). At $B = B_0$, the two-body interaction is resonant ($a \to \pm\infty$), corresponding to a vanishing binding energy of the molecular state. Beyond the resonance ($B > B_0$), the scattering length is negative ($a < 0$), which leads to an effective attraction. Here, two-body physics does not support a weakly bound molecular level, and pairing can only occur because of many-body effects.

Our experimental approach (*9*, *14*) facilitated preparation of the quantum gas in various regimes with controlled temperature, Fermi energy, and interaction strength. We performed evaporative cooling under conditions (*25*) in which an essentially pure molecular Bose-Einstein condensate containing $N = 4 \times 10^5$ paired atoms could be created as a starting point for the experiments. The final laser power of the evaporation ramp allowed us to vary the temperature T. The Fermi energy E_F (Fermi temperature $T_F = E_F/k_B$, with Boltzmann's constant k_B) was controlled by a recompression of the gas, which we performed by increasing the trap laser power after the cooling process (*25*). We then varied the interaction strength by slowly changing the magnetic field to the desired final value. The adiabatic changes applied to the gas after evaporative cooling proceeded with conserved entropy (*14*). Lacking a reliable method to determine the temperature T of a deeply degenerate, strongly interacting Fermi gas in a direct way, we characterized the system by the temperature T' measured after an isentropic conversion into the BEC limit (*25*). For

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a deeply degenerate Fermi gas, the true temperature T is substantially below our observable T' (*25*, *26*), but a general theory for this relation is not yet available.

Radio-frequency (RF) spectroscopy has been introduced as a powerful tool to study interaction effects in ultracold Fermi gases (*27–29*). Molecular binding energies have been measured for ^{40}K atoms (*29*), for which the potential of the method to observe fermionic pairing gap energies has also been pointed out. RF spectroscopy has been applied to ^{6}Li atoms to study interaction effects up to magnetic fields of 750 G (*28*). One important observation was the absence of mean-field shifts in the strongly interacting regime. This effect can be attributed to the fact that, in the relevant magnetic-field range, all s-wave scattering processes between ^{6}Li atoms in the states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are simultaneously unitarity-limited. This property of ^{6}Li is very favorable for RF spectroscopy because it suppresses shifts and broadening by mean-field effects.

We drove RF transitions from state $|2\rangle$ to the empty state $|3\rangle$ at ~80 MHz and monitored the loss of atoms in state $|2\rangle$ after weak excitation by a 1-s RF pulse, using state-selective absorption imaging (*14*). Our experiment was optimized to obtain a resolution of ~100 Hz, corresponding to an intrinsic sensitivity to interaction effects on the scale of ~5 nK, which is more than two orders of magnitude below the typical Fermi temperatures.

We recorded RF spectra for different degrees of cooling and in various coupling regimes (Fig. 1). We realized the molecular regime at $B = 720$ G ($a = +120$ nm). For the resonance region, we examined two different magnetic fields, because the precise resonance location B_0 is not exactly known. Our two values $B = 822$ G (*16*) and 837 G (*13*, *18*) may be considered as lower and upper bounds for B_0. We also studied the regime beyond the resonance with a large negative scattering length at $B = 875$ G ($a \approx -600$ nm). Spectra taken in a "hot" thermal sample at $T \approx 6T_F$ (where $T_F = 15$ μK) show the narrow atomic $|2\rangle \rightarrow |3\rangle$ transition line (Fig. 1, top) and serve as a frequency reference. We present our spectra as a function of the RF offset with respect to the bare atomic transition frequency.

Spectral signatures of pairing have been theoretically considered (*30–34*). A clear signature of the pairing process is the emergence of a double-peak structure in the spectral response as a result of the coexistence of unpaired and paired atoms. The pair-related peak is located at a higher frequency than the unpaired-atoms signal, because energy is required for pair breaking. For understanding of the spectra, both the homogeneous line shape of the pair signal (*31*, *33*) and the inhomogeneous line broadening due to the density distribution in the harmonic trap need to be taken into account (*34*). As an effect of inhomogeneity, fermionic pairing due to many-body effects takes place predominantly in the central high-density region of the trap, and unpaired atoms mostly populate the outer region of the trap where the density is low (*34–36*). The spectral component corresponding to the pairs thus shows a large inhomogeneous broadening in addition to the homogeneous width of the pair-breaking signal. For the unpaired atoms, the homogeneous line is narrow and the effects of inhomogeneity and mean-field shifts are negligible. These arguments explain why the RF spectra in general show a relatively sharp peak for the unpaired atoms together with a broader peak attributed to the pairs.

We observed clear double-peak structures already at $T'/T_F = 0.5$, which we obtained with moderate evaporative cooling down to a laser power of $P = 200$ mW (Fig. 1, middle, $T_F = 3.4$ μK). In the molecular regime $B = 720$ G, the sharp atomic peak was well separated from the broad dissociation signal (*29*), which showed a molecular binding energy of $E_b = h \times 130$ kHz $= k_B \times 6.2$ μK. For $B \rightarrow B_0$, the peaks began to overlap. In the resonance region [822 G and 837 G (Fig. 1)], we still observed a relatively narrow atomic peak at the original position together with a pair signal. For magnetic fields beyond the resonance, we could resolve the double-peak structure for fields up to ~900 G.

For $T'/T_F < 0.2$, realized with a deep evaporative cooling ramp down to an optical trap power of $P = 3.8$ mW, we observed the disappearance of the narrow atomic peak in the RF spectra (Fig. 1, bottom, $T_F = 1.2$ μK). This shows that essentially all atoms were paired. In the BEC limit (720 G), the dissociation line shape is identical to the one observed in the trap at higher temperature and Fermi energy. Here the localized pairs are molecules with a size much smaller than the mean interparticle spacing, and the dissociation signal is independent of the density. In the resonance region [822 G and 837 G (Fig. 1)], the pairing signal shows a clear dependence on density (Fermi energy), which becomes even more pronounced beyond the resonance (875 G). We attribute this to the fact that the size of the pairs becomes comparable to or larger than the interparticle spacing. In addition, the narrow width of the pair signal in this regime (Fig. 1, bottom, $B = 875$ G) indicates a pair localization in momentum space to well below the Fermi momentum $\hbar k_F = \sqrt{2mE_F}$ and thus a pair size exceeding the interparticle spacing.

To quantitatively investigate the crossover from the two-body molecular regime to the fermionic many-body regime, we measured the pairing energy in a range between 720 and 905 G. The measurements were performed after deep evaporative cooling ($T'/T_F < 0.2$) for two different Fermi temperatures, $T_F = 1.2$ μK and $T_F = 3.6$ μK (Fig. 2). As an effective pairing gap, we defined $\Delta\nu$ as the frequency difference between the pair-signal maximum and the bare atomic resonance. In the BEC limit, the effec-

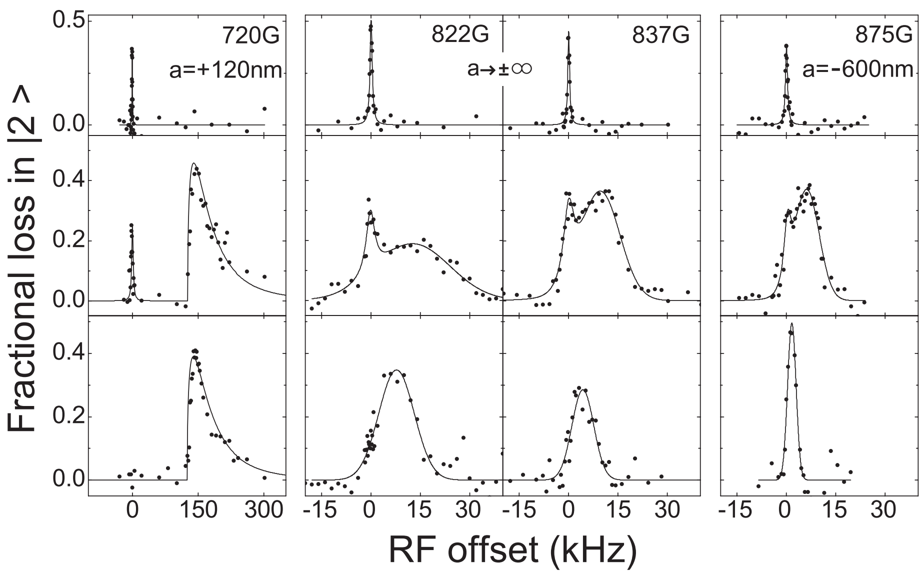

Fig. 1. RF spectra for various magnetic fields and different degrees of evaporative cooling. The RF offset ($k_B \times 1$ μK $\cong h \times 20.8$ kHz) is given relative to the atomic transition $|2\rangle \rightarrow |3\rangle$. The molecular limit is realized for $B = 720$ G (first column). The resonance regime is studied for $B = 822$ G and $B = 837$ G (second and third columns). The data at 875 G (fourth column) explore the crossover on the BCS side. Top row, signals of unpaired atoms at $T' \approx 6T_F$ ($T_F = 15$ μK); middle row, signals for a mixture of unpaired and paired atoms at $T' = 0.5T_F$ ($T_F = 3.4$ μK); bottom row, signals for paired atoms at $T' < 0.2T_F$ ($T_F = 1.2$ μK). The true temperature T of the atomic Fermi gas is below the temperature T', which we measured in the BEC limit. The solid lines are introduced to guide the eye.

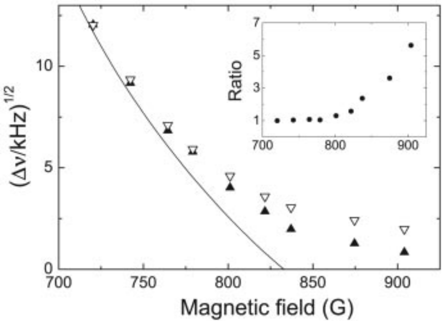

Fig. 2. Measurements of the effective pairing gap $\Delta\nu$ as a function of the magnetic field B for deep evaporative cooling and two different Fermi temperatures, T_F = 1.2 μK (solid symbols) and 3.6 μK (open symbols). The solid line shows $\Delta\nu$ for the low-density limit, where it is essentially given by the molecular binding energy (*25*). Inset: The ratio of the effective pairing gaps measured at the two different Fermi energies.

tive pairing gap $\Delta\nu$ simply reflects the molecular binding energy E_b (Fig. 2, solid line) (*25*). With an increasing magnetic field, in the BEC-BCS crossover, $\Delta\nu$ shows an increasing deviation from this low-density molecular limit and smoothly evolves into a density-dependent many-body regime where $h\Delta\nu < E_F$.

A comparison of the pairing energies at the two different Fermi energies (Fig. 2, inset) provides further insight into the nature of the pairs. In the BEC limit, $\Delta\nu$ is solely determined by E_b and thus does not depend on E_F. In the universal regime on resonance, E_F is the only energy scale, and we indeed observed the effective pairing gap $\Delta\nu$ to increase linearly with the Fermi energy. We found a corresponding relation $h\Delta\nu \approx 0.2\,E_F$. Beyond the resonance, where the system is expected to change from a resonant to a BCS-type behavior, $\Delta\nu$ was found to depend more strongly on the Fermi energy and the observed gap ratio further increased. We interpret this in terms of the increasing BCS character of pairing, for which an exponential dependence $h\Delta\nu/E_F \propto \exp(-\pi/2k_F|a|)$ is expected.

In a further series of measurements (Fig. 3), we applied a controlled heating method to study the temperature dependence of the gap in a way that allowed us to keep all other parameters constant. After production of a pure molecular Bose-Einstein condensate ($T' < 0.2T_F$) in the usual way, we adiabatically changed the conditions to B = 837 G and T_F = 1.2 μK. We then increased the trap laser power by a factor of nine (T_F increased to 2.5 μK), using exponential ramps of different durations. For fast ramps, this recompression was nonadiabatic and increased the entropy. By variation of the ramp time, we explored a range from our lowest temperatures up to $T'/T_F = 0.8$. The emergence of the gap with decreasing temperature is clearly visible in the RF spectra (Fig. 3). The marked increase of $\Delta\nu$ for decreasing temperature is

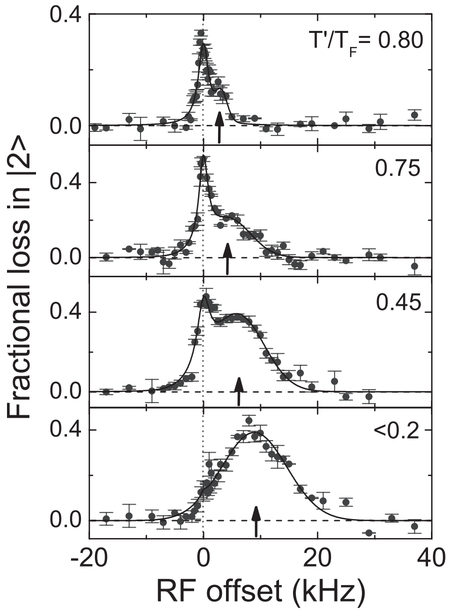

Fig. 3. RF spectra measured at B = 837 G and T_F = 2.5 μK for different temperatures T' adjusted by controlled heating. The solid lines are fits to guide the eye, using a Lorentzian curve for the atom peak and a Gaussian curve for the pair signal. The vertical dotted line marks the atomic transition, and the arrows indicate the effective pairing gap $\Delta\nu$.

in good agreement with theoretical expectations for the pairing gap energy (*5*).

The conditions of our experiment were theoretically analyzed for the case of resonant two-body interaction (*34*). The calculated RF spectra are in agreement with our experimental results and demonstrate how a double-peak structure emerges as the gas is cooled below $T/T_F \approx 0.5$ and how the atomic peak disappears with further decreasing temperature. In particular, the work clarifies the role of the "pseudo-gap" regime (*5*, *22*), in which pairs are formed before superfluidity is reached. According to the calculated spectra, the atomic peak disappears at temperatures well below the critical temperature for the phase-transition to a superfluid. A recent theoretical study of the BCS-BEC crossover at finite temperature (*36*) predicted the phase-transition to a superfluid to occur at a temperature that on resonance is only ~30% below the point where pair formation sets in.

We have observed fermionic pairing already after moderate evaporative cooling. With much deeper cooling applied, the unpaired atom signal disappeared from our spectra. This observation shows that pairing takes place even in the outer region of the trapped gas where the density and the local Fermi energy are low. Our results thus strongly suggest that a resonance superfluid is formed in the central region of the trap (*34*). Together with the observations of resonance condensation of fermionic pairs (*15*, *16*) and weak damping of collective excitations (*17*, *18*), our observation of the pairing gap provides a strong case for superfluidity in experiments on resonantly interacting Fermi gases.

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Supporting Online Material

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DOI: 10.1126/science.1100818originally published online July 22, 2004

degenerate fermi gas

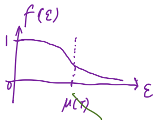
1 温度低，以致全部或大部分粒子都在 fermi level 以下

N个粒子
V. 体积
$n = \frac{N}{V}$. density

若是电子，$s = \pm 1/2$

$\frac{N}{2} = \frac{\frac{4}{3}\pi k_F^3}{(2\pi)^3/V} \Rightarrow \boxed{3\pi^2 n = k_F^3}$

巨正则 $f(\varepsilon) = \frac{1}{e^{\beta(\varepsilon - \mu(T))} + 1}$



$\beta \to \infty$ $\mu(\infty) = \text{const}. \Rightarrow$ $\varepsilon > \mu(\infty)$ $f = 0$

$\varepsilon < \mu(\infty)$ $f = 1$

$\mu(T)$? 是 T 的函数?

微正则 N. V. E

正则 N. V. T.

巨正则 μ. V. T.

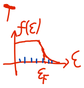
巨正则：给定 μ. V. T

给定 μ，分布的突变的地方给定
给定 T，分布突变的程度给定 } ⇒ 确定分布的形状
⇒ 确定总粒子数 N

给定 V. 同时 N 也给定了，确定 density！

实际体系 正则系综 N V T

$f(\varepsilon) = \frac{1}{e^{\beta\varepsilon} + 1}$



N: 限定态的个数
β: 形状参数 } ⇒ 确定 ε_F

V: N 确定 density

正则系综 density 本身就已经确定.

$N(\beta) = \sum_i \frac{1}{e^{\beta\varepsilon_i} + 1}$ 必须是分立的，
不然有无穷多能级，是非量子的，
体现不出 Pauli 不相容原理