EXERCISES FOR THE OPTICS COURSE AT THE CAS 2024

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At the CERN Accelerator School 2024 (Spa, Belgium, 10-22 November) a course on optics design is organized. This course is intended as an introduction for accelerator physicists who want to start the design of accelerator optics, as well as for people from other fields who would like to get a basic knowledge of the principles of accelerator design. The aim is that the participants design a realistic accelerator optics, after having followed the lectures on lattice cells, insertions and imperfections...

This should be done following

- Part 1 (to be covered during November 12$^{\text{th}}$, 13$^{\text{th}}$, 15$^{\text{th}}$ and 16$^{\text{th}}$ sessions): a series of steps in form of exercises the participants have to solve and implement in an accelerator design program (`Xsuite`),
- Part 2 (to be covered during November 18$^{\text{th}}$ and 19$^{\text{th}}$ sessions): by developing in four groups a theme of study. On 19$^{\text{th}}$ November (afternoon session) all groups will present their work (15 min presentation + 10 min discussion).

In this document, we present the exercises (Part 1) and the themes of study (Part 2) for this course.

Part 1. Exercises

1. A 20 GeV/c synchrotron

Design a machine for protons at a momentum of 20 GeV/c with the following basic parameters:

- circumference = 1000 m,
- quadrupole length L_q = 3.0 m,
- 8 FODO cells,
- dipole length is 5 m, maximum field is 3 T.

Apply the knowledge from previous lectures at this school and define a lattice cell according to the boundary conditions (position of dipole magnets and quadrupoles) and find the optics (strength of dipoles and quadrupoles) so that β_{max} is around 300 m (use first the thin approximation approach). Implement it in `Xsuite` format using thin lenses for all elements and verify the calculations (check that the β_{max} value obtained from the `Xsuite` TWISS table is around 300 m).

Prepare a simple code to make the following basic checks: (I) the machine is circular, and its reference orbit is closed (use the `survey` method), (II) that the orbit is closed, and (III) that the lattice is stable.

2. Beam envelope and machine aperture

Assume the aperture requires a beam size 10 $\sigma \leq 31.4$ mm. Start with the lattice from Exercise 1 and modify it so that the β_{max} satisfies this requirement (please use rounded numbers for convenience). The normalized beam emittance is $\epsilon_n = 2.0\ \mu$m. The circumference and the energy must not be changed, all other parameters may be modified. HINT: you can achieve the results in at least two ways (analytical or via `Xsuite` matching), choose the one you prefer.

3. Exercise 3: chromaticity correction

Start with the lattice from Exercise 2 and modify it, so you can correct the chromaticity for both planes to zero.

Try first to calculate approximately the required strengths. Implement your correction scheme in your previous `Xsuite` lattice description and verify your calculation.

Use `Xsuite` to compute the exact strengths required by matching the global parameters *dqx* and *dqy*. Compare the results with your calculations.

4. Dispersion matching

The purpose of this exercise is to insert dispersion suppressors into the existing regular lattice, in order to create low-dispersion straight sections. Such straight sections with very small dispersion are very useful for the installation of RF equipment, wigglers, undulators, beam instrumentation, collimation systems etc., or to house an experiment.

Start with the lattice from the previous exercise and first double the circumference to 2000 m. Change the phase advance to $\phi = 60°$ per cell. Insert two straight sections (each of 2 cells): i.e., cells without bending magnets, but keep the same focusing of the quadrupoles. Insert the two straight sections opposite in azimuth in the ring. Modify now the lattice to keep the horizontal dispersion function small ($<$ 1-2 m) along this straight section, i.e. set up a dispersion suppressor.

You can do this by adding or removing bending magnets or changing the bending radius of some or all the bending magnets. At this stage, do not change the focusing properties in any of the cells.

It is a good practice to check with the `survey` method the geometry of the machine as done in Exercise 1.

5. Transfer lines

Build a transfer line of 10 m with 4 quads of L=0.4 m (centred at 2, 4, 6, and 8 m). With K1 respectively of 0.1, 0.1, 0.1, 0.1 m^{-2}. Can you find a periodic solution of this lattice? Compute the final optical condition starting from $(\beta_x, \alpha_x, \beta_y, \alpha_y) = (1\ \text{m}, 0, 2\ \text{m}, 0)$.

Match the optics of the line (K-values) to the downstream synchrotron, assume that the injection point of the synchrotron has $(\beta_x, \alpha_x, \beta_y, \alpha_y) = (2\ \text{m}, 0, 1\ \text{m}, 0)$.

Add 2 horizontal correctors in the transfer line, and use `Xsuite` to compute the 4 terms of the transfer matrix between the corrector kicks and the x-x' position of the beam at the end of the line. Using this matrix, compute the correctors strength needed to have ($\Delta x = 1$ mm, $\Delta x' = 0$)

Part 2. Themes of study

1. Low-β insertion

Start from the lattice of Exercise 4 and design a symmetric insertion with a low-β section in a dispersion free region. The β should be small at least in one plane and should have a waist at an "interaction point". Choose the low-β (usually called β^*) at your own discretion. Think and develop different options $\beta_x^* = \beta_y^*$ (round beams) or $\beta_x^* \neq \beta_y^*$ (flat beams).

Evaluate the effect of the low-β insertion on the chromaticity and rematch it to zero.

2. Closed orbit correction

Start from the lattice of Exercise 3 and assume random misalignments of the quadrupoles of r.m.s. 0.1 mm in the horizontal and 0.2 mm in the vertical plane. Calculate the expected r.m.s. orbit and verify with `Xsuite` (see Closed orbit and trajectory correction from the `Xsuite` website). Add the necessary equipment to be able to correct the closed orbit in both planes. Estimate first the maximum necessary strength of the orbit correctors assuming a maximum quadrupole displacement of 1 mm. Use `Xsuite` to correct the orbit in both planes. What is the effect of the correction on the dispersion?

Optional: Now remove the correction and repeat the exercise by adding a skew quadrupole. Power the skew quadrupole until you start to see the coupling between the horizontal and vertical orbits. Perform again the `Xsuite` orbit correction and compare the results with the uncoupled case.

3. Tracking particles

Start from the lattice of Exercise 3 and set up a single particle tracking to study the stability of the beams. Use the thin lens version for tracking with `Xsuite`.

- Select appropriate particle amplitudes. Change the tune and sextupole strengths to observe the effect.
- Change the tunes to see the effect.
- Change the strengths of the chromaticity sextupoles to see the effect.

From the position of the particle for the different turns and using a mathematical software of your choice, compute the tune spectrum of the particle.

4. Beam injection oscillations

Start with Exercise 3 and create a matched distribution (e.g. start with 20k particles). Track the distribution assuming a horizontal injection error (e.g., $\Delta x = 1$ mm). Simulate the evolution of the x-distribution in the first 1k turns. Can you see the betatronic oscillation of the beam centroid? Change the beam distributions to have an oscillation of the beam envelope. Can you check the relation between the frequency of the centroid and of the beam envelope (e.g. do an FFT)?

5. Beam extraction

Start with Exercise 3 of the first week and define a point on your machine to be used as extraction point, $\bar{s}$. Assume that for extracting the beam, you need to move the horizontal closed orbit at ($x = 2$ mm, $x' = 0$ mrad) in $\bar{s}$.

Add 4 correctors and design a closed bump for that purpose, i.e., ($x = 2$ mm, $x' = 0$ mrad) in $\bar{s}$. Write a python function to set the values of the correctors as a function of the $x(\bar{s})$, $x'(\bar{s})$ and the machine tunes.

Position an extraction kicker in your lattice to extract the beam. A kicker is a fast corrector that can reach the full field in a fraction of the beam revolution period (it is important to observe that the beam sees the kicker field for only one passage). Plot (1) the closed orbit of the machine with the closed bump and (2) the trajectory of the beam considering also the kicker during the last turn (dimension the extraction kick to have $\Delta x(\bar{s}) = 2$ mm, that is, the total trajectory offset has to be x= 4 mm at $x(\bar{s})$).