RELAXATION-TIME MODEL

The relaxation time is

$$\tau(T,E,\mu) = 1/P_{\text{imp}} + 1/P_{\text{ac}} + 1/P_{\text{polar}}.$$

The scattering rates P are as follows.

⇒ **polar-optical phonon scattering rate**:

$$P_{\text{polar}}(T,E) = \sum_i \frac{C(T,E,e_i^{\text{LO}}) - A(T,E,e_i^{\text{LO}}) - B(T,E,e_i^{\text{LO}})}{Z(T,E,e_i^{\text{LO}})\,E^{3/2}} \tag{1}$$

where the sum is over longitudinal-optical phonons with energy e_i^{LO}. The functions A, B, C, and Z are given below. We define $x{=}E+\hbar\omega_{LO}$, $y{=}E-\hbar\omega_{LO}$, and $w{=}E/(\hbar\omega_{LO})$ with ω_{LO} the longitudinal optical frequency and E the electron energy; next, f is the Fermi-Dirac function, n the phonon occupation number (Planck distribution), θ the Heaviside step function, e the electron charge, $\overline{m}$ the average effective mass, ε_∞ and ε_s the high-frequency and static dielectric constants. We then have

$$A(E) = (n(\omega_{LO})+1)\frac{f(x)}{f(E)}\left\{(E+x)\sinh^{-1}(\sqrt{w}) - (Ex)^{1/2}\right\}$$

$$B(E) = \theta(y)\,n(\omega_{LO})\frac{f(y)}{f(E)}\left\{(E+y)\cosh^{-1}(\sqrt{w}) - (Ey)^{1/2}\right\}$$

$$C(E){=}2E\left[(n(\omega_{LO})+1)\frac{f(x)}{f(E)}\sinh^{-1}(\sqrt{w}) + \theta(y)\,n(\omega_{LO})\frac{f(y)}{f(E)}\cosh^{-1}(\sqrt{w})\right]$$

$$Z(E) = \frac{2}{W_0\sqrt{\hbar\omega_{LO}}};\quad W_0 = \frac{e^2}{4\pi\hbar}\left(\frac{2\overline{m}\omega_{LO}}{\hbar}\right)^{1/2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_s}\right).$$

⇒**acoustic phonon scattering rate**:

$$P_{\text{ac}}(T,E) = \frac{(2\overline{m})^{\frac{3}{2}}\,k_B T D^2\sqrt{E}}{2\pi\hbar^4\rho v^2} \tag{2}$$

where E is the electron energy, D the deformation potential of the band energies calculated at the band extrema, ρ the mass density, v the average sound velocity.

⇒**impurity scattering rate** (Brooks-Herring formula):

$$P_{\text{imp}}(T,E) = \frac{\pi n_I Z_I^2 e^4 E^{-3/2}}{\sqrt{2\overline{m}}\,(4\pi\epsilon_0\epsilon_s)^2}\left[\log\left(1+\frac{1}{x}\right) - \frac{1}{1+x}\right];\qquad x = \frac{\hbar^2 q_0^2}{8\overline{m}E}, \tag{3}$$

with n_I the ionized impurity concentration, Z_I the impurity charge (we assume $Z_I{=}1$), ε_0 the vacuum permittivity, and $q_0{=}\sqrt{e^2 n_I/(\varepsilon_0\varepsilon k_B T)}$ the Debye screening wavevector.