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1  #include <fstream>
2  #include <array>
3
4  //Constant expressions appearing in the problem
5  constexpr double PI = 3.14159265359;    //value of PI
6  constexpr size_t STEPS = 1000;    //number of steps
7
8  //Definition of data types in the problem
9  typedef double state_type;    //data type definition for dependant variables
    - array of x_0, x_1, ... x_n
10 typedef std::array<state_type, STEPS> solution;
11
12 //This is the differential Equation, with the higher order derivatives
13 state_type Pendulum(const state_type& x){
14     return - PI * PI * sin(x);
15 }
16
17 //The relaxation step
18 void relaxation_step(state_type (*Diff_Equation)(const state_type& x),
    solution& x, const double& dt){
19     solution temporary = x;
20     for (size_t i = 1; i < STEPS - 1; i++) {
21         x[i] = 0.5 * (temporary[i-1] + temporary[i+1] - dt * dt *
            Diff_Equation(temporary[i])); //Euler forward difference formula
22     }
23 }
24
25 int main(){
26     solution x_t{};    //variable to store the calculations
27
28     double t_0 = 0.0;    //initial time
29     double t_1 = 1.0;    //final time
30     double dt = (t_1 - t_0) / (STEPS - 1); //step size
31     state_type x_0 = 0.0, x_1 = PI / 4;    //boundary values for dependant
        variables
32
33     //repeat the relaxation step
34     for (size_t i = 0; i < STEPS; i++) {
35         x_t[i] = x_0 + i * (x_1 - x_0) / (STEPS - 1); //initial linear
            guess
36     }
37     for (size_t i = 0; i < 10000; i++) {
38         relaxation_step(Pendulum, x_t, dt);    //step forward
39     }
40
41     std::ofstream outfile;    //file handle to save the results in a file
42     outfile.open("./output/relaxation.txt", std::ios::out |
        std::ios::trunc );
43     for (size_t i = 0; i < STEPS; i++) {

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44     outfile << t_0 + i * dt << "\t" << x_t[i] << std::endl;
45     }
46     outfile.close();
47 }
```