

```
In[1]:= K1[h_] := h f[tn, x[tn]]
K2[h_] := h f[tn + 1 / 2 h, x[tn] + 1 / 2 K1[h]]
K3[h_] := h f[tn + 1 / 2 h, x[tn] + 1 / 2 K2[h]]
K4[h_] := h f[tn + h, x[tn] + K3[h]]
RK4Approx[h_] := 1 / 6 (K1[h] + 2 K2[h] + 2 K3[h] + K4[h])
Exact[h_] := Integrate[ f[t, x[t]], {t, tn, tn + h} ]
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In[7]:= y[t_] := f[t, x[t]]
x'[t_] := y[t]
x''[t_] := y'[t]
x'''[t_] := y''[t]
x''''[t_] := y'''[t]
FullSimplify[Series[Exact[h] - RK4Approx[h], {h, 0, 5}]]
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$$\begin{aligned} \text{Out[12]} = & \frac{1}{2880} \\ & \left(-f[tn, x[tn]]^4 f^{(0,4)}[tn, x[tn]] + 24 f^{(0,1)}[tn, x[tn]]^3 f^{(1,0)}[tn, x[tn]] + f[tn, x[tn]]^3 \right. \\ & \quad \left(6 f^{(0,2)}[tn, x[tn]]^2 - 2 f^{(0,1)}[tn, x[tn]] f^{(0,3)}[tn, x[tn]] - 4 f^{(1,3)}[tn, x[tn]] \right) - \\ & \quad 6 f^{(0,1)}[tn, x[tn]]^2 f^{(2,0)}[tn, x[tn]] + 6 f^{(1,1)}[tn, x[tn]] f^{(2,0)}[tn, x[tn]] - \\ & \quad 6 f^{(1,0)}[tn, x[tn]] \left(3 f^{(0,2)}[tn, x[tn]] f^{(1,0)}[tn, x[tn]] + f^{(2,1)}[tn, x[tn]] \right) - \\ & \quad 6 f[tn, x[tn]]^2 \left(f^{(0,3)}[tn, x[tn]] f^{(1,0)}[tn, x[tn]] + \right. \\ & \quad \left. 3 f^{(0,2)}[tn, x[tn]] \left(2 f^{(0,1)}[tn, x[tn]]^2 - f^{(1,1)}[tn, x[tn]] \right) + f^{(2,2)}[tn, x[tn]] \right) + \\ & \quad 4 f^{(0,1)}[tn, x[tn]] \left(-3 f^{(1,0)}[tn, x[tn]] f^{(1,1)}[tn, x[tn]] + f^{(3,0)}[tn, x[tn]] \right) + \\ & \quad 2 f[tn, x[tn]] \\ & \quad \left. \left(3 \left(4 f^{(0,1)}[tn, x[tn]]^4 - 4 f^{(0,1)}[tn, x[tn]]^2 f^{(1,1)}[tn, x[tn]] + 2 f^{(1,1)}[tn, x[tn]]^2 - \right. \right. \right. \\ & \quad \left. 2 f^{(1,0)}[tn, x[tn]] f^{(1,2)}[tn, x[tn]] + f^{(0,2)}[tn, x[tn]] f^{(2,0)}[tn, x[tn]] + \right. \\ & \quad \left. f^{(0,1)}[tn, x[tn]] \left(-8 f^{(0,2)}[tn, x[tn]] f^{(1,0)}[tn, x[tn]] + f^{(2,1)}[tn, x[tn]] \right) \right) - \\ & \quad \left. 2 f^{(3,1)}[tn, x[tn]] \right) - f^{(4,0)}[tn, x[tn]] \Big) h^5 + O[h]^6 \end{aligned}$$