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1  #include <iostream>
2  #include <fstream>
3  #include <cmath>
4  #include <array>
5  #include <map>
6
7  //Constant expressions appearing in the problem
8  constexpr size_t dimension = 2;    //dimension of the reduced 1st-order    ↗
    problem
9  constexpr double PI = 3.14159265359;    //value of PI
10 constexpr double rollnum = 0.226121014;    //my roll number
11
12 //Definition of data types in the problem
13 typedef std::array<double, dimension> state_type;    //data type definition ↗
    for dependant variables - array of x_0, x_1, ... x_n
14 typedef std::map<double, state_type> solution;    //data type definition for ↗
    storing the list of calculated values ((hash)map of time -> state)
15
16 //Class template for the Runge Kutta solver using Butcher tableau
17 template <class State_Type, size_t order> class explicit_rk {
18     //data type definitions for storing the Butcher tableau
19     typedef std::array<double, order> butcher_coefficients;
20     typedef std::array<std::array<double, order>, order> butcher_matrix;
21 private:
22     //information about the Butcher tableau
23     butcher_matrix a;
24     butcher_coefficients b, c;
25     //temporary variables for intermediate steps
26     std::array<State_Type, order> k;
27 public:
28     //Constructor - just copy the Butcher tableau
29     explicit_rk(butcher_matrix A, std::array<double, order> B,    ↗
        std::array<double, order> C) : a(A), b(B), c(C) {
30         k = {};    //zero-initialize k
31     }
32
33     //Destructor - nothing to do
34     ~explicit_rk() {}
35
36 }
37
38 //The stepper function, calculates x_{n+1} given the differential    ↗
    equation, x_{n}, t and step size
39 void do_step(void (*Diff_Equation)(const State_Type& x, const double& ↗
    t, State_Type& dxdt), State_Type& x, const double& t, const double& ↗
    dt) {
40     State_Type result = x;    //temporary variable for storing the    ↗
        result
41

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42 //loops for evaluating k1, k2 ... k_n
43 for (size_t i = 0; i < order; i++) {
44     State_Type sum{}, dxdt; //temporary variables for k's and the
45                               derivatives
46     for (size_t j = 0; j < i; j++) {
47         sum = sum + dt * a[i][j] * k[j]; //compute a_{ij} * k_j
48     }
49     sum = x + sum; //compute x_{n} + a_{ij} * k_j
50     Diff_Equation(sum, t + c[i] * dt, dxdt); //evaluate dx/dt
51     at (x_{n} + a_{ij} * k_j, t_{n} + c_{i} * dt) according to
52     Runge Kutta
53     k[i] = dxdt; //store the dx/dt as k_i
54 }
55
56 //loop for calculating x_{n+1} using the k's
57 for (size_t i = 0; i < order; i++) {
58     result = result + dt * b[i] * k[i]; //weighted average of k's
59     with b's as weights
60 }
61
62 //return the result
63 x = result;
64 }
65 };
66
67 //Overload the + operator to be able to add two vectors
68 state_type operator + (state_type const& x, state_type const& y) {
69     state_type z;
70     for (size_t i = 0; i < dimension; i++) {
71         z[i] = x[i] + y[i]; //add the individual components and store in z
72     }
73     return z; //return the resulting vector z
74 }
75
76 //Overload the * operator to be able to multiply numbers and vectors
77 state_type operator * (double const& a, state_type const& x) {
78     state_type z;
79     for (size_t i = 0; i < dimension; i++) {
80         z[i] = a * x[i]; //multiply the individual components and store
81         in z
82     }
83     return z; //return the resulting vector z
84 }
85
86 //This is the differential Equation, reduced to first-order
87 void Pendulum(const state_type& x, const double& t, state_type& dxdt) {
88     dxdt[0] = x[1];
89     dxdt[1] = -4.0 * PI * PI * sin(x[0]);
90 }

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86
87 int main() {
88     //Using the class template, creates a class object for the Runge Kutta ↗
89     solver with a given butcher tableau
90     explicit_rk <state_type, 4> rk4_stepper(
91         { 0,0,0,0,    //Butcher a matrix
92         .5,0,0,0,
93         0,.5,0,0,
94         0,0,1,0 },
95         { 1.0 / 6.0 , 1.0 / 3.0 , 1.0 / 3.0 , 1.0 / 6.0 },    //Butcher b ↗
96         coefficientants
97         { 0.0 , 0.5 , 0.5 , 1.0 }); //Butcher c coefficients
98
99     solution x_t;    //variable to store the calculations
100
101     size_t STEPS = 1000; //number of steps
102     double t_0 = 0.0;    //initial time
103     double t_1 = 1.0;    //final time
104     double dt = (t_1 - t_0) / (STEPS - 1); //step size
105     state_type x = { 0.0, rollnum };    //initial values for dependant ↗
106     variables
107
108     //Step through the domain of the problem and store the solutions
109     x_t[t_0] = x;    //store initial values
110     for (size_t i = 0; i < STEPS; i++) {
111         rk4_stepper.do_step(Pendulum, x, NULL, dt);    //step forward
112         x_t[t_0 + i * dt] = x;    //store the calculation
113     }
114
115     std::ofstream outfile;    //file handle to save the results in a file
116     outfile.open("tableau.txt", std::ios::out | std::ios::trunc);
117     for (auto const& temp : x_t) {
118         outfile << temp.first << "\t" << temp.second[0] << "\t" << ↗
119         temp.second[1] << std::endl;
120     }
121     outfile.close();
122 }
```