```
(*Definition of the exact integral and its RK4 approximation*)
K1[h_] := h f[tn, x[tn]]
K2[h_] := h f[tn + 1 / 2 h, x[tn] + 1 / 2 K1[h]]
K3[h_] := h f[tn + 1 / 2 h,  x[tn] + 1 / 2 K2[h]]
K4[h_] := h f[tn + h, x[tn] + K3[h]]
RK4Approx[h_] := 1 / 6 (K1[h] + 2 K2[h] + 2 K3[h] + K4[h])
Exact[h_] := Integrate[ f[t, x[t]], {t, tn, tn + h} ]

(*We define y[t] so that higher order total derivatives of x[k]
 in terms of derivatives of f[t,x[t]] are automatically calculated*)
y[t_] := f[t, x[t]]
x'[t_] := y[t]
x''[t_] := y'[t]
x'''[t_] := y''[t]
x''''[t_] := y'''[t]
(*Simplify the difference of exact integral and RK4 approximation up to 5th order*)
FullSimplify[Series[Exact[h] - RK4Approx[h], {h, 0, 5}]]
```

Out[ ]=

$$
\begin{aligned}
&\frac{1}{2880}\Big(-f[\mathrm{tn},x[\mathrm{tn}]]^4\, f^{(0,4)}[\mathrm{tn},x[\mathrm{tn}]] + 24\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]^3\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]] + f[\mathrm{tn},x[\mathrm{tn}]]^3 \\
&\quad\Big(6\, f^{(0,2)}[\mathrm{tn},x[\mathrm{tn}]]^2 - 2\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(0,3)}[\mathrm{tn},x[\mathrm{tn}]] - 4\, f^{(1,3)}[\mathrm{tn},x[\mathrm{tn}]]\Big) - \\
&\quad 6\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]^2\, f^{(2,0)}[\mathrm{tn},x[\mathrm{tn}]] + 6\, f^{(1,1)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(2,0)}[\mathrm{tn},x[\mathrm{tn}]] - \\
&\quad 6\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]]\Big(3\, f^{(0,2)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]] + f^{(2,1)}[\mathrm{tn},x[\mathrm{tn}]]\Big) - \\
&\quad 6\, f[\mathrm{tn},x[\mathrm{tn}]]^2\Big(f^{(0,3)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]] + \\
&\qquad 3\, f^{(0,2)}[\mathrm{tn},x[\mathrm{tn}]]\Big(2\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]^2 - f^{(1,1)}[\mathrm{tn},x[\mathrm{tn}]]\Big) + f^{(2,2)}[\mathrm{tn},x[\mathrm{tn}]]\Big) + \\
&\quad 4\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]\Big(-3\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(1,1)}[\mathrm{tn},x[\mathrm{tn}]] + f^{(3,0)}[\mathrm{tn},x[\mathrm{tn}]]\Big) + \\
&\quad 2\, f[\mathrm{tn},x[\mathrm{tn}]] \\
&\qquad\Big(3\Big(4\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]^4 - 4\, f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]^2\, f^{(1,1)}[\mathrm{tn},x[\mathrm{tn}]] + 2\, f^{(1,1)}[\mathrm{tn},x[\mathrm{tn}]]^2 - \\
&\qquad\quad 2\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(1,2)}[\mathrm{tn},x[\mathrm{tn}]] + f^{(0,2)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(2,0)}[\mathrm{tn},x[\mathrm{tn}]] + \\
&\qquad\quad f^{(0,1)}[\mathrm{tn},x[\mathrm{tn}]]\Big(-8\, f^{(0,2)}[\mathrm{tn},x[\mathrm{tn}]]\, f^{(1,0)}[\mathrm{tn},x[\mathrm{tn}]] + f^{(2,1)}[\mathrm{tn},x[\mathrm{tn}]]\Big)\Big) - \\
&\qquad 2\, f^{(3,1)}[\mathrm{tn},x[\mathrm{tn}]]\Big) - f^{(4,0)}[\mathrm{tn},x[\mathrm{tn}]]\Big)\, h^5 + O[h]^6
\end{aligned}
$$