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1 #include <iostream>
2 #include <fstream>
3 #include <cmath>
4 #include <array>
5 #include <map>
6
7 //Constant expressions appearing in the problem
8 constexpr size_t dimension = 2; //dimension of the reduced 1st-order problem
9 constexpr double PI = 3.14159265359; //value of PI
10 constexpr double rollnum = 0.226121014; //my roll number
11
12 //Definition of data types in the problem
13 typedef std::array<double, dimension> state_type; //data type definition for
    dependant variables - array of x_0, x_1, ... x_n
14 typedef std::map<double, state_type> solution; //data type definition for
    storing the list of calculated values ((hash)map of time -> state)
15
16 //Overload the + operator to be able to add two vectors
17 state_type operator + (state_type const& x, state_type const& y) {
18     state_type z;
19     for (size_t i = 0; i < dimension; i++) {
20         z[i] = x[i] + y[i]; //add the individual components and store in z
21     }
22     return z; //return the resulting vector z
23 }
24
25 //Overload the * operator to be able to multiply numbers and vectors
26 state_type operator * (double const& a, state_type const& x) {
27     state_type z;
28     for (size_t i = 0; i < dimension; i++) {
29         z[i] = a * x[i]; //multiply the individual components and store
    in z
30     }
31     return z; //return the resulting vector z
32 }
33
34 //Class template for the Runge Kutta solver using Butcher tableau
35 template <class State_Type, size_t order> class explicit_rk {
36     //data type definitions for storing the Butcher tableau
37     typedef std::array<double, order> butcher_coefficients;
38     typedef std::array<std::array<double, order>, order> butcher_matrix;
39 private:
40     //information about the Butcher tableau
41     butcher_matrix a;
42     butcher_coefficients b, c;
43     //temporary variables for intermediate steps
44     std::array<State_Type, order> k;
45 public:
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46 //Constructor - just copy the Butcher tableau
47 explicit_rk(butcher_matrix A, butcher_coefficients B,
48             butcher_coefficients C) : a(A), b(B), c(C) {
49     k = {}; //zero-initialize k
50 }
51 //Destructor - nothing to do
52 ~explicit_rk() {
53 }
54 }
55
56 //The stepper function, calculates  $x_{n+1}$  given the differential
57 //equation,  $x_n$ ,  $t$  and step size
58 void do_step(void (*Diff_Equation)(const State_Type& x, const double&
59 t, State_Type& dxdt), State_Type& x, const double& t, const double&
60 dt) {
61     State_Type result = x; //temporary variable for storing the
62     result
63
64     //loops for evaluating  $k_1, k_2 \dots k_n$ 
65     for (size_t i = 0; i < order; i++) {
66         State_Type sum{}, dxdt; //temporary variables for k's and the
67         derivatives
68         for (size_t j = 0; j < i; j++) {
69             sum = sum + dt * a[i][j] * k[j]; //compute  $a_{ij} * k_j$ 
70         }
71         sum = x + sum; //compute  $x_n + a_{ij} * k_j$ 
72         Diff_Equation(sum, t + c[i] * dt, dxdt); //evaluate  $dx/dt$ 
73         at ( $x_n + a_{ij} * k_j$ ,  $t_n + c_{ij} * dt$ ) according to
74         Runge Kutta
75         k[i] = dxdt; //store the  $dx/dt$  as  $k_i$ 
76     }
77
78     //loop for calculating  $x_{n+1}$  using the k's
79     for (size_t i = 0; i < order; i++) {
80         result = result + dt * b[i] * k[i]; //weighted average of k's
81         with b's as weights
82     }
83
84     //return the result
85     x = result;
86 }
87 };
88
89 //This is the differential Equation, reduced to first-order
90 void Pendulum(const state_type& x, const double& t, state_type& dxdt) {
91     dxdt[0] = x[1];
92     dxdt[1] = -4.0 * PI * PI * sin(x[0]);
93 }

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86
87 int main() {
88     //Using the class template, creates a class object for the Runge Kutta ➤
89     solver with a given butcher tableau
90     explicit_rk <state_type, 4> rk4_stepper(
91         { 0,0,0,0,    //Butcher a matrix
92         .5,0,0,0,
93         0,.5,0,0,
94         0,0,1,0 },
95         { 1.0 / 6.0 , 1.0 / 3.0 , 1.0 / 3.0 , 1.0 / 6.0 },    //Butcher b ➤
96         coefficientants
97         { 0.0 , 0.5 , 0.5 , 1.0 }); //Butcher c coefficients
98
99     solution x_t;    //variable to store the calculations
100
101     size_t STEPS = 1000; //number of steps
102     double t_0 = 0.0;    //initial time
103     double t_1 = 1.0;    //final time
104     double dt = (t_1 - t_0) / (STEPS - 1); //step size
105     state_type x = { 0.0, rollnum };    //initial values for dependant ➤
106     variables
107
108     //Step through the domain of the problem and store the solutions
109     x_t[t_0] = x;    //store initial values
110     for (size_t i = 0; i < STEPS; i++) {
111         rk4_stepper.do_step(Pendulum, x, NULL, dt);    //step forward
112         x_t[t_0 + i * dt] = x;    //store the calculation
113     }
114
115     std::ofstream outfile;    //file handle to save the results in a file
116     outfile.open("tableau.txt", std::ios::out | std::ios::trunc);
117     for (auto const& temp : x_t) {
118         outfile << temp.first << "\t" << temp.second[0] << "\t" << ➤
119         temp.second[1] << std::endl;
120     }
121     outfile.close();
122 }

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