Fluid Statics

Objective

By studying this section, you will learn:

- the meaning of the density of a material and the average density of a body
- what is meant by the pressure in a fluid, and how it is measured
- how to calculate the buoyant force that a fluid exerts on a body immersed in it

Background

> This shark must swim constantly to keep from sinking to the bottom of the ocean, yet the orange tropical fish can remain at the same level in the water with little effort. Why is there a difference.

Fluids play a vital role in many aspects of everyday life.

We begin our study with fluid statics, the study of fluids at rest in equilibrium situations. Like other equilibrium situations, it is based on Newton's first and third laws. We will explore the key concepts of density, pressure and buoyancy.

Density

Density of any material is defined as its mass per unit volume. A homogeneous material such as ice or iron has the same density throughout. We use ρ (Greek letter rho) for density. If a mass m of homogeneous material has volume V, the density ρ is

$$\rho = \frac{m}{V} \tag{1}$$

Two objects made of the same material have the same density even though they may have different masses and different volumes. That's because the ratio of mass to volume is the same for both objects.

Densities of Some Common Substances:

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

The SI unit of density is the kilogram per cubic meter $(1\ \mathrm{kg/m^3})$. The cgs unit, the gram per cubic centimeter $(1\ \mathrm{g/cm^3})$, is also widely used:

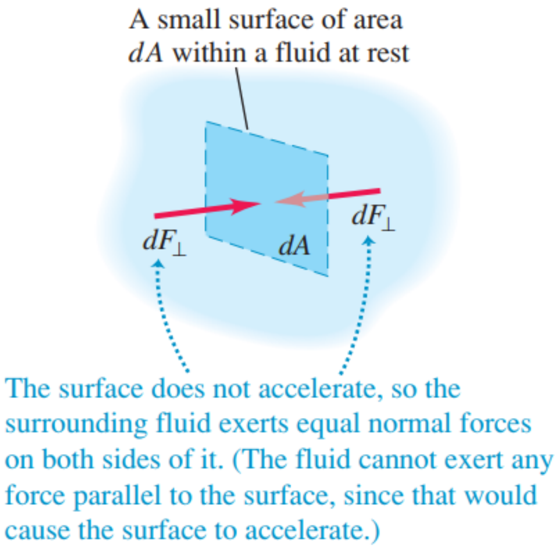
$$1\ \mathrm{g/cm^3} = 1000\ \mathrm{kg/m^3} \tag{2}$$

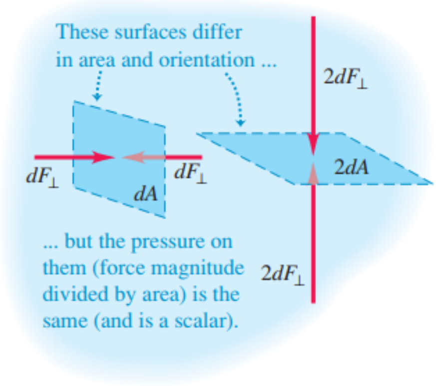
The density of some materials varies from point to point within the material. One example is the material of the human body, which includes low-density fat and high-density bone. Two others are the earth's atmosphere (which is less dense at high altitudes) and oceans (which are denser at greater depths). For these materials, Equation (1) describes the **average density**. In general, the density of a material depends on environmental factors such as temperature and pressure.

Pressure

When a fluid (either liquid or gas) is at rest, it exerts *a force perpendicular to any surface in contact with it,* such as a container wall or a body immersed in the fluid. This is the force you feel pressing on your legs when you dangle them in a swimming pool.

If we think of an imaginary surface within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface. Consider a small surface of area dA centered on a point in the fluid; the normal force exerted by the fluid on each side is $dF_\perp$.





The **pressure** p at that point is the normal force per unit area,

$$p = \frac{dF_{\perp}}{dA} \tag{3}$$

If the pressure is the same at all points of a finite plane surface with area A, then,

$$p = \frac{F_{\perp}}{A} \tag{4}$$

where $F_{\perp}$ is the net normal force on one side of the surface. The SI unit of pressure is the **pascal**, where

$$1 \text{ pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2$$

Atmospheric pressure

Atmospheric pressure p_a is *the pressure of the earth's atmosphere*, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation.

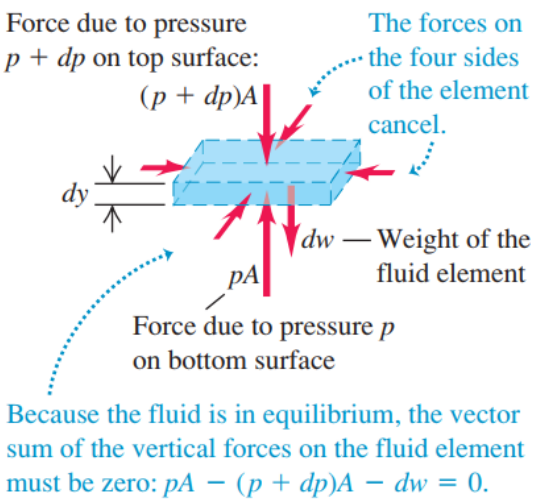
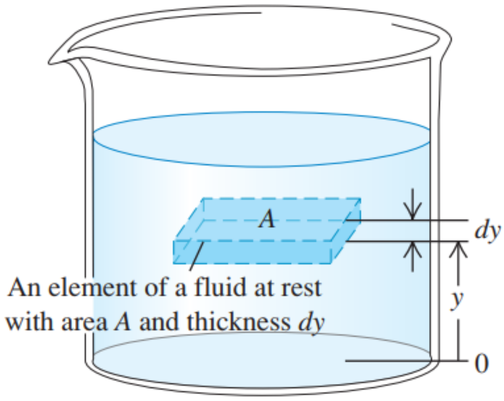
Normal atmospheric pressure at sea level (an average value) is 1 *atmosphere (atm)*, defined to be exactly $101,325$ Pa. To 4 significant figures,

$$\begin{aligned}(p_a)_{av} &= 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} \\ &= 1.013 \text{ bar} = 1013 \text{ millibar} = 14.70 \text{ lb/in}^2\end{aligned}$$

Pressure, Depth, and Pascal's Law

When you dive into deep water, your ears tell you that the pressure increases rapidly with increasing depth below the surface. There is an equation that expresses the relationship between the pressure at any point in a fluid at rest and the elevation of any point. The equation is derived from the total y-component of all the forces acting on an element of a fluid. This is given by,

$$\frac{dp}{dy} = -\rho g \tag{5}$$

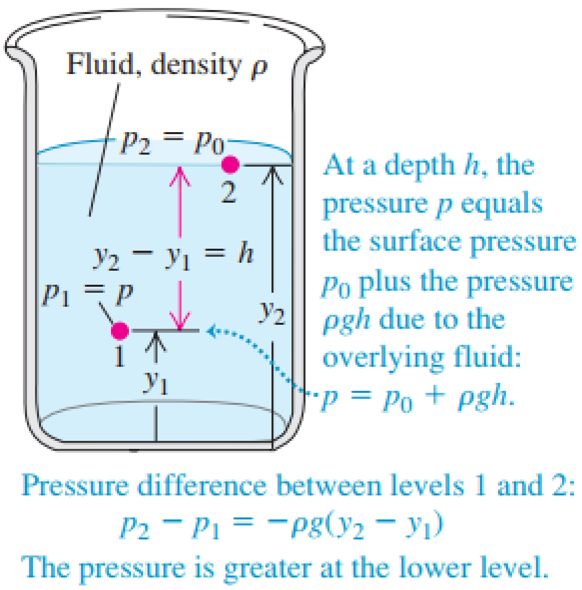
Different Forces acting on an element of fluid	An element of a fluid
	

Equation (5) shows that when y increases, p decreases; that is, as we move upward the fluid, pressure decreases, as we expect.

How pressure varies with depth in a fluid with uniform density:

If p_1 and p_2 are the pressures at elevations y_1 and y_2, respectively, and if ρ and g are constant, then

$$p_2 - p_1 = -\rho g(y_2 - y_1) \tag{6}$$



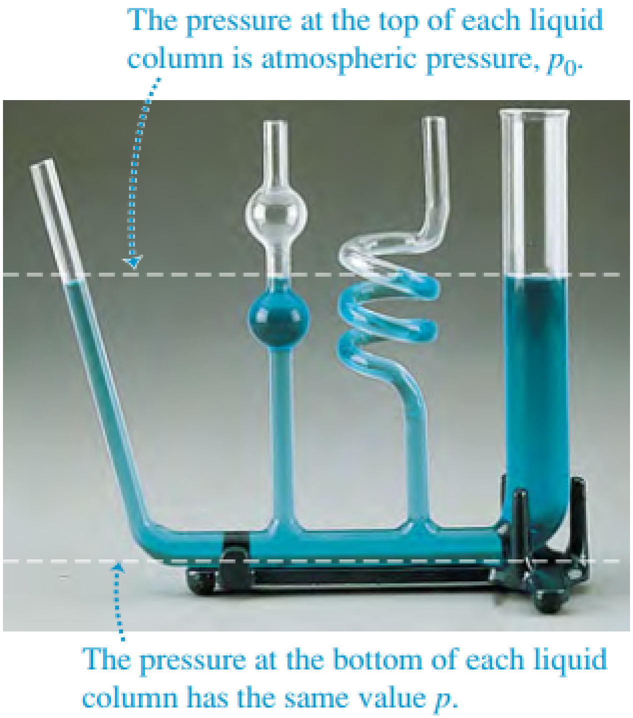
It's often convenient to express Equation (6) in terms of the depth below the surface of a fluid (refer to the figure above). The depth of point 1 below the surface is $h = y_2 - y_1$, and Equation (6) becomes

$$p_0 - p = -\rho g(y_2 - y_1) = -\rho gh \tag{7}$$

or

$$p = p_0 + \rho gh \tag{8}$$

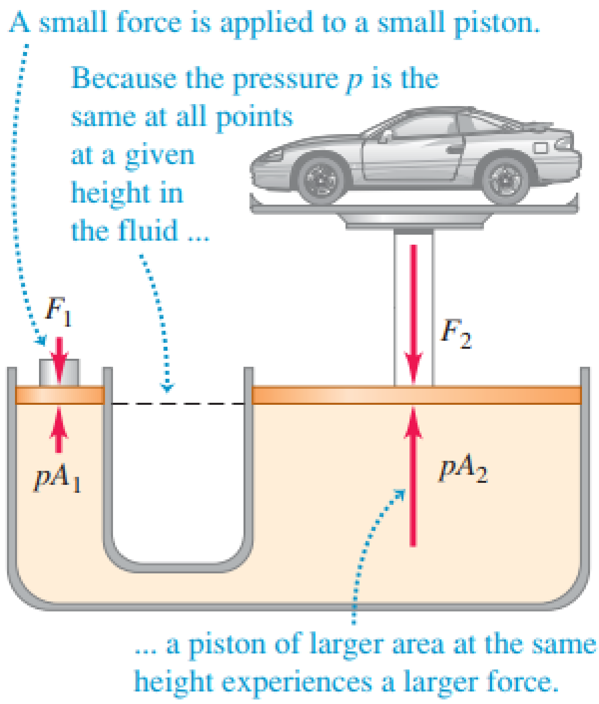
The pressure at a depth h is greater than the pressure p_0 at the surface by an amount $\rho g h$. Note that the pressure is the same at any points at the same level in the fluid. The shape of the container does not matter.



Equation (8) shows that if we increase the pressure p_0 at the top surface, possibly by using a piston that fits tightly inside the container to push down on the fluid surface, the pressure p at any depth increases by exactly the same amount. This fact was recognized in 1653 by the French scientist *Blaise Pascal* and is called **Pascal's law**.

Pascal's Law

The hydraulic lift shown below illustrates Pascal's law.



A piston with small cross-sectional area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $p = F_1/A$ is transmitted through the connecting pipe to a larger piston of area A_2, that is,

$$\begin{aligned} p &= \frac{F_1}{A_1} = \frac{F_2}{A_2} \\ F_2 &= \frac{A_2}{A_1} F_1 \end{aligned} \tag{9}$$

The hydraulic lift is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lift and jacks, many elevators, and hydraulic brakes all use this principle.

Absolute Pressure and Gauge Pressure

If the pressure inside a car tire is equal to atmospheric pressure, the tire is flat. The pressure has to be greater than atmospheric to support the car, so the significant quantity is the difference between the inside and outside pressures.

When we say that the pressure in a car tire is "32 pounds" (220kPa or 2.2×10^5 Pa), we mean that it is greater than atmospheric pressure by this amount. The total pressure in the tire is then 47 lb/in^2 or 320 kPa. The excess pressure above atmospheric pressure is usually called **gauge pressure**, and the total pressure is called **absolute pressure**. If the pressure is less than atmospheric, as in a partial vacuum, the gauge pressure is negative.

This is shown as follows,

$$\begin{aligned} \text{Gauge pressure} &= 2.2 \times 10^5 \text{ Pa} - \underbrace{1.013 \times 10^5 \text{ Pa}}_{\text{atmospheric pressure}} = 1.2 \times 10^5 \text{ Pa} \\ \text{Absolute pressure} &= 2.2 \times 10^5 \text{ Pa} + \underbrace{1.013 \times 10^5 \text{ Pa}}_{\text{atmospheric pressure}} = 3.2 \times 10^5 \text{ Pa} \end{aligned} \tag{10}$$

Engineers use the abbreviations psig and psia for "pounds per square inch gauge" and "pounds per square inch absolute", respectively.

Buoyancy

Buoyancy is a familiar phenomenon: a body immersed in water seems to weigh less than when it is in air. When the body is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air.

> **Archimedes principle**: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

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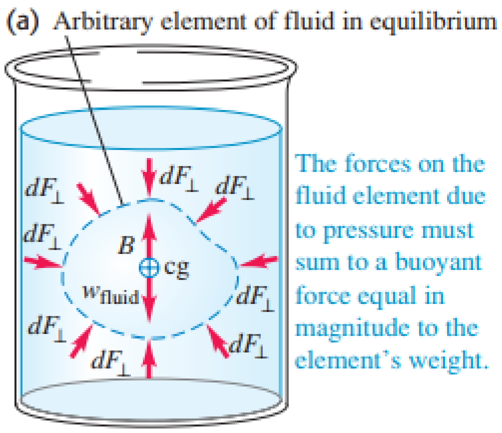
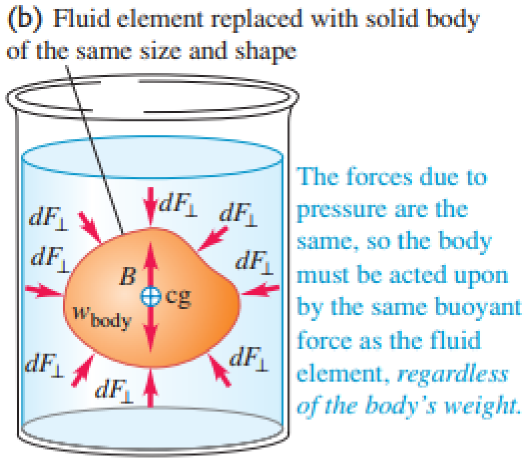
<tr><th>Fluid without the solid body</th><th>Fluid with the solid body</th></tr>
<tr><td>
</td><td>
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For a static fluid, the upward forces acting on an element of a fluid should be the same, equal in magnitude to the weight mg of the fluid displaced to make way for the body. We call this upward force the **buoyant force** on the solid body. The buoyant force is equal to the weigh of fluid displaced by the object:

$$B = \rho_{fluid} g V_{disp} \tag{11}$$

In equilibrium the forces acting on the body is given as follows,

$$\begin{gathered} \sum F_y = B - w_{body} = 0 \\ \rho_{fluid} g V_{disp} = m_{body} g \\ \rho_{fluid} V_{disp} = \rho_{body} V_{body} \end{gathered} \tag{12}$$

Remarks:

- If buoyancy exceeds the weight of the immersed object, the object tends to rise.
- If the object's weight exceeds the buoyancy, the object tends to sink.

A body whose average density is less than that of a liquid can float partially submerged at the free upper surface of the liquid.

A fish's flesh is denser than water, yet a fish can float while submerged because it has gas-filled cavity within its body. This makes the fish's average density the same as water's, so its net weight is the same as the weight of the water it displaces.

Implementation

The weight of a roomful of air

> Find the mass and weight of the air at 20 degrees Celsius in a living room with a $4.0 \text{ m} \times 5.0 \text{ m}$ floor and a ceiling 3.0 m high, and the mass and weight of an equal volume of water.

The volume of the room is: $V = (4.0m \times 5.0m \times 3.0m) = 60m^3$, so

$$\begin{aligned} m_{air} &= \rho_{air} V = (1.20 \text{kg}/\text{m}^3 (60 \text{ m}^3)) = 72 \text{ kg} \\ w_{air} &= m_{air} g = (72 \text{kg})(9.8 \text{ m}/\text{s\^{}2}) = 700 \text{ N} = 160 \text{ lb} \end{aligned} \tag{13}$$

The mass and weight of an equal volume of water are

$$\begin{aligned} m_{water} &= \rho_{water} V = (1000 \text{kg}/\text{m}^3 (60 \text{ m}^3)) = 6.0 \times 10^4 \text{ kg} \\ w_{water} &= m_{water} g = (6.0 \times 10^4 \text{kg})(9.8 \text{ m}/\text{s\^{}2}) = 5.9 \times 10^5 \text{ N} = 1.3 \times 10^5 \text{ lb} \end{aligned} \tag{14}$$

Finding absolute and gauge pressures

> Water stands 12.0 m deep in a storage tank whose top is open to the atmosphere. What are the absolute and gauge pressures at the bottom of the tank?

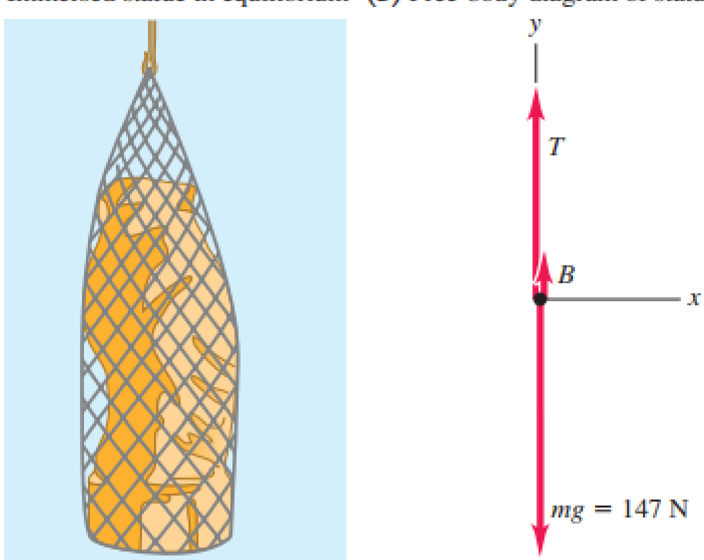
From definition: *The excess pressure above atmospheric pressure is usually called **gauge pressure**, and the total pressure is called **absolute pressure**.*

$$\begin{aligned} p &= p_0 + \rho g h \\ p_{abs} &= 1.01 \times 10^5 \text{Pa} + (1000 \text{kg}/\text{m}^3)(9.80 \text{m}/\text{s}^2)(12.0 \text{ m}) \\ &= 2.19 \times 10^5 \text{Pa} = 2.16 \text{ atm} \\ p_{gauge} &= p - p_0 = \rho g h = (1000 \text{kg}/\text{m}^3)(9.80 \text{m}/\text{s}^2)(12.0 \text{ m}) \\ &= 1.18 \times 10^5 \text{Pa} = 1.16 \text{ atm} \end{aligned}$$

Buoyancy

> A 15.0 kg solid gold statue is raised from the sea bottom. What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

(a) Immersed statue in equilibrium (b) Free-body diagram of statue



a The statue is at rest, sot the net external force acting on it is zero.

$$\begin{aligned} \sum F_y &= T + B - (15 \text{ kg})(9.8 m/s^2) = 0 \\ T &= 147 \text{N} - B \end{aligned} \tag{15}$$

To calculate for the volume of the gold, we use equation (1),

$$V_g = \frac{m_{goldstatue}}{(\rho_{gold})} \tag{16}$$

$$V_g = \frac{15 \text{ kg}}{19.3 \times 10^3 kg/m^3} = 7.77 \times 10^{-4} m^3$$

The buoyant force equals the weight of this same volume of seawater.

$$\begin{aligned} B &= w_{sw} = m_{sw} g = \rho_{sw} V_g \\ &= (1.03 \times 10^3)(7.77 \times 10^{-4})(9.80 m/s^2) \\ &= 7.84N \end{aligned}$$

Therefore, the tension is, $T = 147N - 7.84N = 139N$.

b The density of air is about $1.2\mathrm{kg/m}^3$, so the buoyant force of air on the statue is

$$\begin{aligned} B_{air} &= \rho_{air} V_g = (1.2\mathrm{kg/m}^3)(7.77 \times 10^{-4}\mathrm{m}^3)(9.80\mathrm{m/s}^2) \\ &= 9.1 \times 10^{-3} \end{aligned}$$

This is negligible compared to the statue's actual weight $m_{goldstatue} g = 147N$. So within the precision of our data,the tension in the cable with the statue in air is $T = 147N$.

Perspective

- Don't confuse pressure and force. In everyday language the words "pressure" and "force" mean pretty much the same thing. In fluid mechanics, however, these words describe distinct quantities with different characteristics.
- Fluid pressure acts perpendicular to any surface in the fluid, no matter how that surface is oriented. Hence pressure has no intrinsic direction of its own; its a scalar.