The Physics II Student's Guide to Vectors

...and possibly the galaxy.

In three dimensions, a **vector** is a set of three numbers: <x, y, z>

This is different than a **scalar**, which is a single number that represents a quantity.

Position and **Velocity** can both be vector quantities.

Vector Operations

There are *legal* and *illegal* ways of using vectors. Follow the legal operations to travel farther in this course!

Legal Operations

With vectors, you **CAN**:

1. Multiply and Divide by a scaler
2. Take the magnitude
3. Have a unit vector give direction
4. Add and subtract other vectors
5. Differentiate
6. Take the dot product and cross product

Illegal Operations

Vectors **CANNOT**:

1. Be equal to a scalar
2. Be added to or subtracted from a scalar
3. Be in the denominator of an expression (In this case, take the magnitude [see below], which is a scalar!)
4. Be added to or subtracted from other vectors with conflicting units

Position Vectors

- A **position vector** describes a position in 3D space.
- Points from the *origin* to the *location*.
- Can use the right-hand rule:
  - Thumb = +x
  - Pointer = +y
  - Middle = +z
- Each number in the vector (x, y, z) is a **component** of the vector.
  - Components can't be vectors by themselves (as they are only 1 number)
- $\vec{0}$ is a legal vector.
  - This is the position of an object at the origin or the velocity of an object at rest.

Unit Vectors

- A **unit vector** is a vector that has a magnitude of 1 in some direction.
  - If a unit vector = 1, every component in that vector must be *less* than 1.
- There are 3 unit vectors in the Cartesian system that are along the three axes:
  - i-hat ($\hat{i}$) = <1, 0, 0>
  - j-hat ($\hat{j}$) = <0, 1, 0>
  - k-hat ($\hat{k}$) = <0, 0, 1>
- Example: $0.05\hat{i} + (-1.2)\hat{j} + 20\hat{k}$
- Not all unit vectors point along the axis.
- How to find a unit vector:

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{< x, y, z >}{\sqrt{x^2 + y^2 + z^2}}$$

- A vector = another vector if all the components are equal.
- A vector may be factored into the product of a unit vector, multiplied by a scalar equal to the magnitude.
  - Example: <0, 5, 0> = 5 <0, 1, 0>

Drawing Vectors

- For position vectors, the tail is always at the *origin* of a coordinate system.
- The x component of a vector is the difference between the x-coordinate of the tail and the x-coordinate of the tip.
- Length of arrow = distance from origin.
- Direction of arrow = direction of the path from the *initial* position to the *final* position...**displacement**!

Fun Things to do with Vectors

1. **Addition and Subtraction**

   - The magnitude of a vector $\neq$ the sum of the magnitudes of the two original vectors.
   - Vector addition is commutative.
     - $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
   - Vector subtraction is *not* commutative.
     - $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$
   - Vector addition and subtraction are associative.
     - $(\vec{A} + \vec{B}) - \vec{C} = \vec{A} + (\vec{B} - \vec{C})$
   - Application of vector subtraction:
     - Δ = change in quantity = final - initial
       - $\Delta\vec{r} = \vec{r}_f = \vec{r}_i$
   - Graphical addition of vectors:
     A. Draw the first vector.
     B. Add the second vector (without rotating) so the tail is at the tip of the first vector.
     C. Draw a new vector from the tail of the first vector to the tip of the second vector.
   - Graphical subtraction of vectors:
     A. Draw the first vector.
     B. Add the second vector (without rotating) so the tail is at the tail of the first vector.
     C. Draw a new vector from the tip of the first vector to the tip of the second vector.

- Equations:

$$\vec{A} + \vec{B} =< (A_x + B_x), (A_y + B_y), (A_z + B_z) >$$

$$\vec{A} - \vec{B} =< (A_x - B_x), (A_y - B_y), (A_z - B_z) >$$

1. **Multiplying a Vector by a Scalar**

- If a vector is multiplied by a scalar, *each* of the components of the vector is multiplied by the scalar.
  - This scales the vector.
    - Keeps the directions the same, but makes the magnitude larger or smaller.
- Multiplying by a *negative* scalar reverses the direction of the vector.

1. **Magnitude**

- The magnitude of a vector = a scalar.
- Note that the magnitude is always a positive number (remember the absolute value!).
- How to find the magnitude:

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

Scalars

- Scalars:
  - Do not have a direction.
    - Examples: Mass of an object, temperature
  - Cannot be equal to a vector.
  - Cannot be added to a vector.
  - Can be positive, negative, or zero.

References

Chabay, R., & Sherwood, B. A. (2015). Matter and Interactions (4th ed.). John Wiley & Sons.