The University of Nottingham

SCHOOL OF PHYSICS AND ASTRONOMY

A LEVEL 3 MODULE, RESIT Sample Paper C

FORCE AND FUNCTION AT THE NANOSCALE

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer all questions

Only a calculator from approved list A may be used in this examination.

List A

Basic Models	Aurora HC133	Casio HS-5D	Deli – DL1654	Sharp EL-233
Scientific Calculators	Aurora AX-582 Casio FX82 family Casio FX83 family	Casio FX85 family Casio FX350 family Casio FX570 family	Casio FX 991 family Sharp EL-531 family Texas BA II+ family	Texas Instruments TI-30 family

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

*An indication is given of the approximate weighting of each part of a question by means of a bold figure enclosed by curly brackets, e.g. {**2**}, immediately following that part.*

DO NOT turn examination paper over until instructed to do so

Table of Physical Constants

Speed of light in free space	c	3.00×10^{8} m s^{-1}
Gravitational Constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Planck's Constant	h	6.63×10^{-34} J s
	$\hbar$	1.055×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Mass of the electron	m_e	9.11×10^{-31} kg
Mass of the proton	m_p	1.6726×10^{-27} kg
Mass of the neutron	m_n	1.6749×10^{-27} kg
Boltzmann's constant	k_{B}	1.38×10^{-23} J K^{-1}
Gas constant	R	8.31 J K^{-1} mol^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_{B}	9.27×10^{-24} J T^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Avogadro's number	N_{A}	6.02×10^{23} mol^{-1}

Information you may find useful

$< x^2 > \approx 6Dt$ where $D = \frac{k_B T}{6\pi\mu a}$

$P \propto \exp(-U/k_B T)$

$F(x) = -\frac{dU}{dx}$

$U_{dipole} = -\boldsymbol{p}.\boldsymbol{E}$ where $\boldsymbol{p} = qd\hat{r}$

$\boldsymbol{p} = \alpha \boldsymbol{E_{ext}}$ where $\alpha = 4\pi\epsilon\epsilon_r d^3$

$U(x) = -\frac{\pi n_1 C}{6x^3}$

$A_{12} = n_1 n_2 \pi^2 C$

$W_{adhesion} = \gamma_{13} + \gamma_{23} - \gamma_{12}$

$\Delta P = \gamma \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

$h = \frac{2\gamma\cos(\theta)}{\rho g R}$

$k = \frac{k_B T}{<z^2>} = \frac{3EI}{L^3}$

$< I > = \frac{c\epsilon\epsilon_0 n}{2}\left\langle E^2 \right\rangle$

$\Pi = \left(n_+ + n_- - 2n_0\right) k_B T$

$\frac{-d^2 V}{dx^2} = \frac{\rho}{\epsilon\epsilon_0}$

$P_{Tot} = 4n_0 \frac{z^2 e^2 V_0^2}{k_B T}\exp(-\kappa D) - \frac{A}{6\pi D^3}$

$s = k_B \ln W$

$\Delta U = \Delta H - T\Delta S$

$\Delta U = P_{osm}\Delta V_{excl}$

$\mu = \frac{dU}{dN}$

$P(N = N_c) = \exp(-\Delta U/k_B T)$

$H = \frac{\nu}{l_c a_0}$

$P_{Tot} = \left[\frac{(k_B T)^2}{\kappa\pi^2} - \frac{A}{6\pi}\right]\frac{1}{D^3}$

where the symbols take their usual meanings as used throughout this course.

End

You must answer 3 out of 5 questions.
You should aim to spend about 30 minutes on each question.

1. a) Describe what is meant by the terms
   - i) Covalent bond {**2**}
   - ii) Hydrogen bond {**2**}

   in each case give an estimate of the strength of the bonds (in eV).

   b) Salt (NaCl) forms a crystal structure with a cubic lattice of Na^+ and Cl^- ions in such a way that each Na^+ ion has Cl^- ions as nearest neighbours and each Cl^- ion has Na^+ ions as nearest neighbours. Sketch a diagram of the crystal structure of NaCl, making sure to label the Na^+ and Cl^- ions, the nearest neighbour distance R, and the lattice constant a on your diagram. {**4**}

   c) Use electrostatics to show that the cohesive energy per ion of a NaCl crystal can be written in the form

$$U_{coh} = -\frac{\alpha e^2}{4\pi\varepsilon_r\varepsilon_o R}$$

   where $\alpha = 6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} + \cdots$ is the Madelung constant and ε_r is the relative permittivity of the medium surrounding the crystal. {**8**}

   d) Calculate the cohesive energy per ion (in eV) of a NaCl crystal in

   i) chloroform (ε_r =4.81). {**2**}

   ii) water (ε_r=80). {**2**}

   given that α=1.748 and R=0.282 nm for NaCl.

   e) Use your answer to part d) to suggest why ionic solids are soluble in *polar* solvents at room temperature but not in *apolar* organic solvents. {**5**}

2. a) Amphiphilic molecules with volume v, critical chain length l_c and headgroup area a_0 are added to water. Outline the conditions necessary to form:

   i) Spherical Micelles
   ii) Cylindrical Micelles
   iii) Bi-layers

{**5**}

b) i) Sketch a schematic graph of the free energy U of a bi-layer as a function of the exposed head group area a (where $a = a_0 + \Delta a$). In your graph you should label the equilibrium head group separation a_0. {**3**}

ii) Describe the physical origin of the different contributions to the free energy as the value of a increases and decreases from a_0. {**2**}

c) Show using a Taylor expansion of the free energy about a_0 that as a bi-layer is bent it exhibits an effective elasticity i.e. show that:

$$\Delta U \approx \frac{1}{2} k \Delta a^2$$

where k is a constant. In your answer you should not just provide the derivation but detail any assumptions made. {**6**}

d) Explain the origin of the repulsion between 2 bi-layers as they come close together. {**3**}

e) As 2 bi-layers of stiffness κ = 7x10^{-21}J and Hamaker constant A = 5x10^{-21}J , are brought a distance D = 1nm apart the total pressure between them is given by:

$$P = \left[\frac{(k_B T)^2}{\kappa \pi^2} - \frac{A}{6\pi}\right] \frac{1}{D^3}$$

i) Show that at room temperature (T=20°C) the bi-layers will always be attractive. {**3**}

ii) At what temperature would the bi-layers begin to repel each other? {**3**}

3 a) A small spherical dielectric particle is placed in a vacuum and trapped in the focal plane of a focused laser beam. Show that the potential energy of the particle is given by

$$U = -\frac{2\alpha}{c\varepsilon_0} I$$

where α is the polarizability of the particle, c is the speed of light and I is the intensity of the laser beam. {**5**}

b) If the laser beam has a Gaussian intensity profile, show that in the limit of small displacements from the centre of the beam (i.e. $r << \omega_0$), the force exerted on the particle at position r is given by

$$F \approx \frac{-2\alpha I_0}{c\varepsilon_0 \omega_0^2} r$$

where I_0 is the intensity at the centre of the trap, ω_0 is the radius of the trap, and r is the displacement from the centre of the trap. {**5**} Show that the force on the particle obeys Hooke's law. {**1**}

c) If the polarizability of the particle can be approximated to the expression

$$\alpha = 4\pi\varepsilon_p\varepsilon_0 R^3$$

where ε_p is the relative permittivity and R is the radius of the particle, determine the effective spring constant of a trap formed by a laser beam with a power of 1 W focused down to width of 1 μm if it contains a 100 nm radius particle with a relative permittivity of 10. {**5**}

d) Use your answer above to calculate the displacement of the trapped particle if a force of 1pN is applied to it. {**3**}

e) If the same particle is suspended in water (relative permittivity, ε_w=80 and refractive index, n_w=1.33) and trapped by the same laser beam as above, calculate the displacement of the trapped particle if a force of 1pN is applied to it. {**4**}

f) Describe an experimental application of an optical trap. {**2**}