



University of
Nottingham

UK | CHINA | MALAYSIA

PHYS3009: Force and Function at the Nanoscale
Week 23 – 9:00am Wednesday – 26 February 2025



valid for 15 minutes from 8:55am
generated 2025-01-04 03:10



University of
Nottingham

UK | CHINA | MALAYSIA

Atomic Force Microscopy

Force & function at the nanoscale

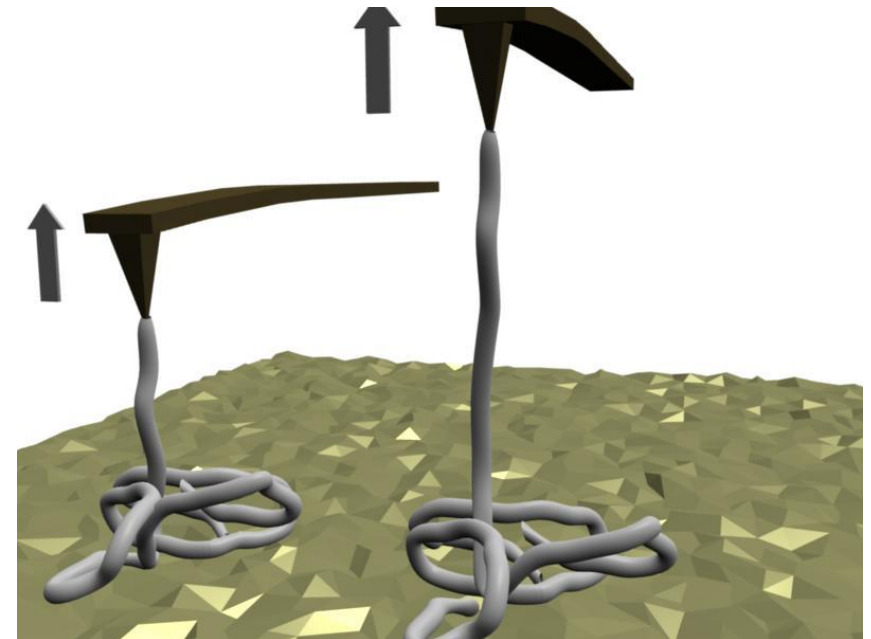
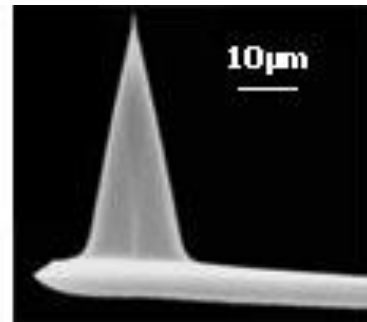
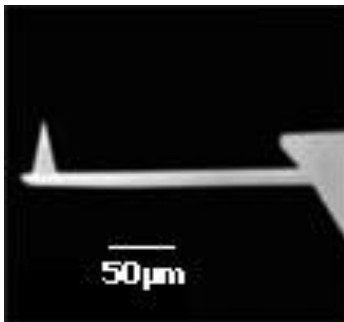
In this lecture...

How do we measure nanoscale forces?

Atomic Force Microscopy (AFM)

Cantilever deflection

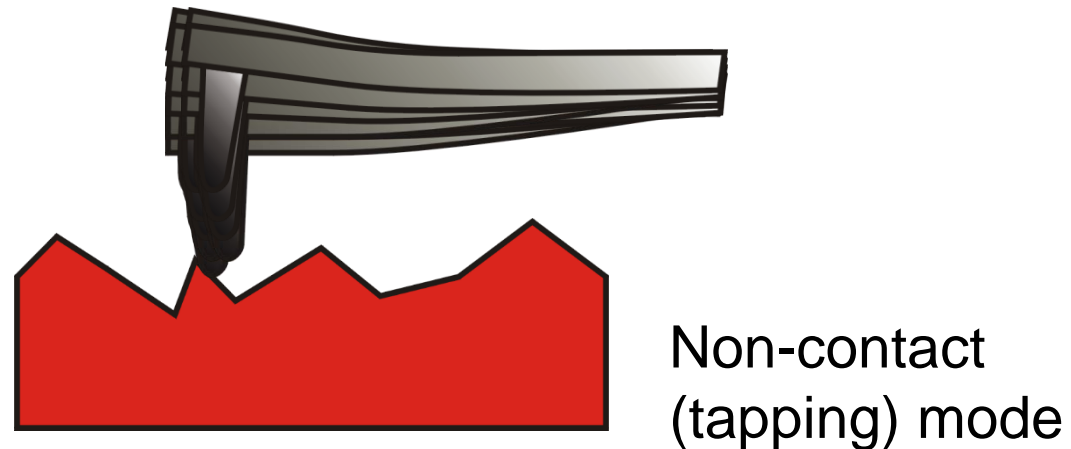
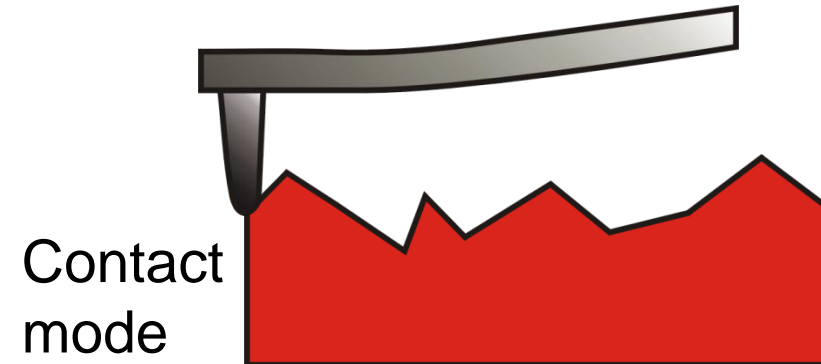
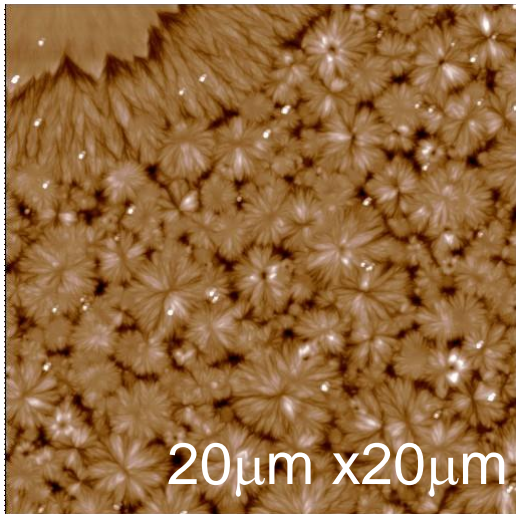
Detection





Atomic force microscopy (AFM)

Atomic force microscopy can be used to image the surface of a sample



Forces at the nanoscale

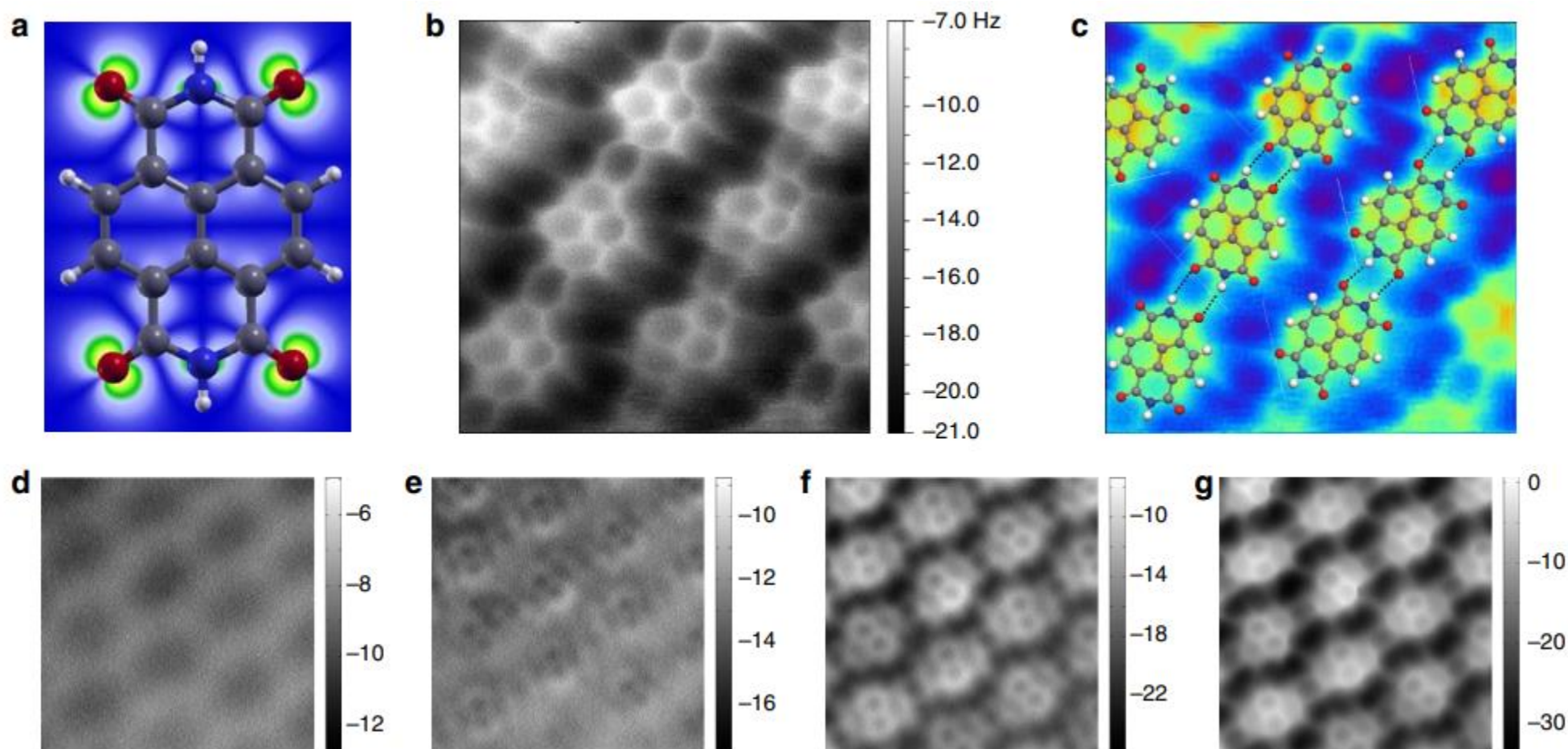
Hook's law tells us we can measure a force from the stretch of a spring of known stiffness.



Challenges

1. How stiff is the cantilever?
2. Measure nanoscale deflections
3. Position cantilever with sub nanometre precision

Imaging bonds with Atomic Force Microscopy

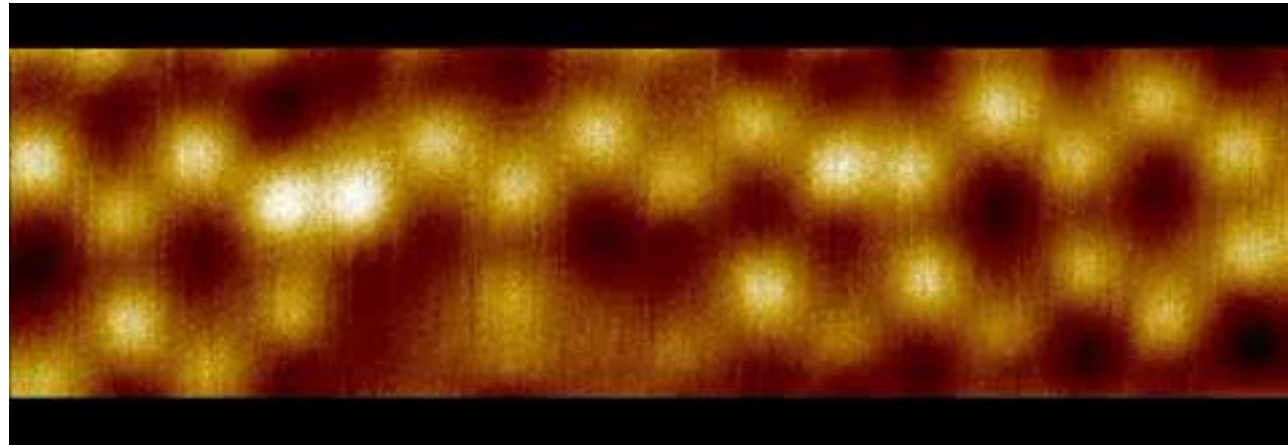
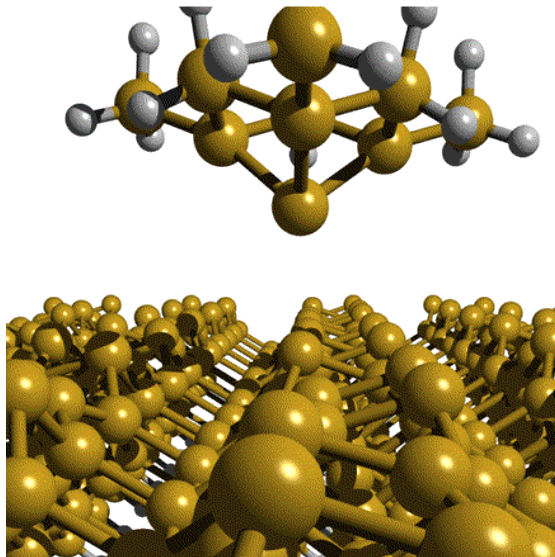


“Mapping the force field of a hydrogen-bonded assembly”
Nature Comms. 5:3931 (2014) Sweetman et al



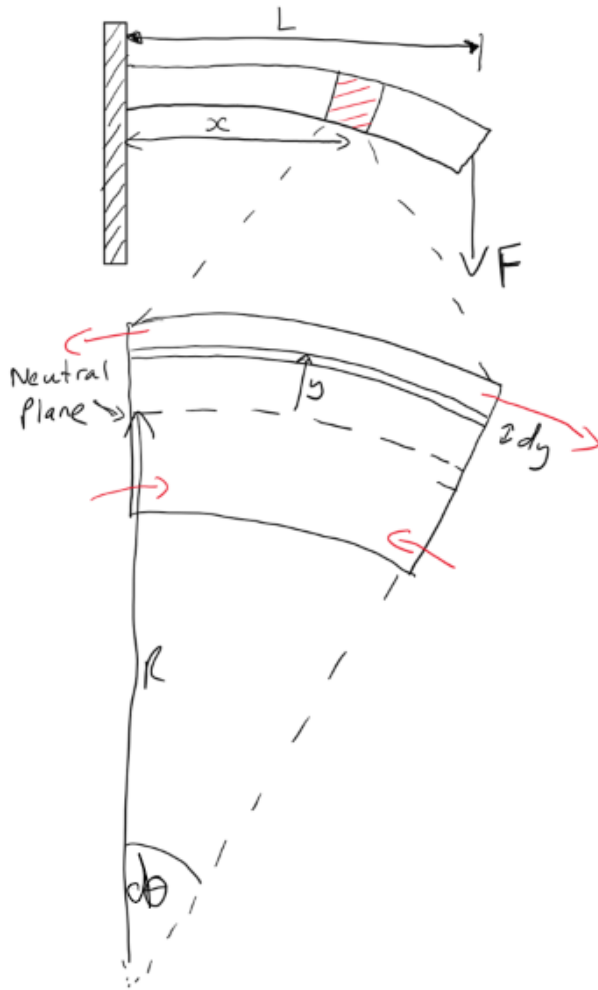
Manipulating atoms

Switching individual silicon atoms
between two positions





Derivation spring constant



$$k = \frac{3EI}{l^3}$$

where

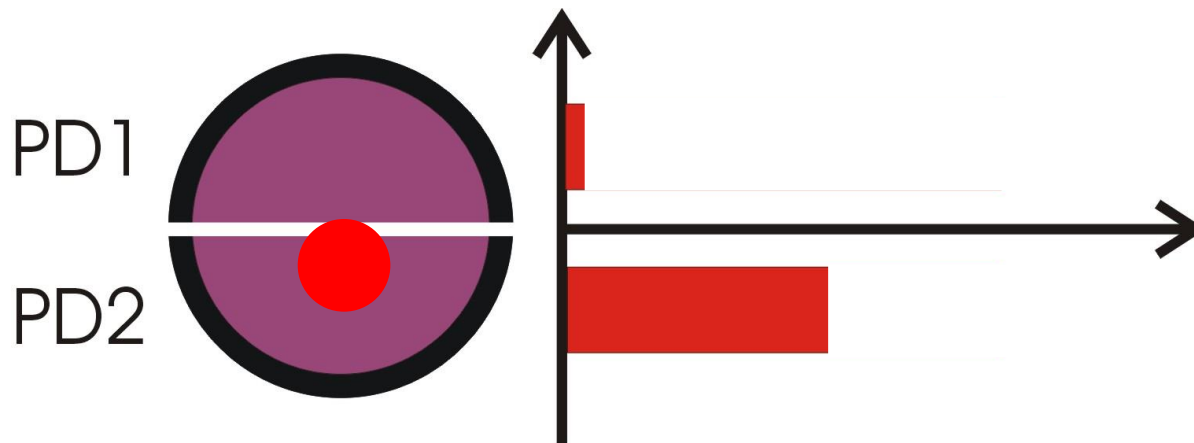
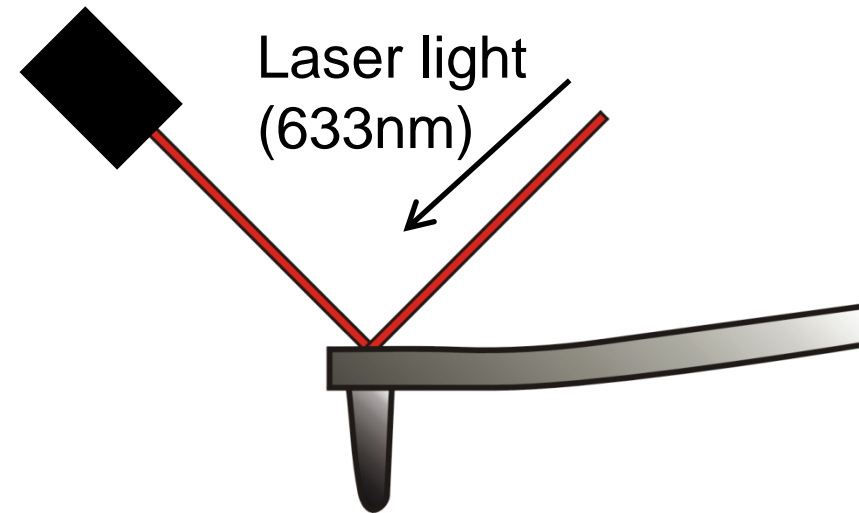
E = Youngs Modulus of cantilever (Pa)

I = Geometric moment of inertia (m^4)

2. How do we detect the deflection of the cantilever?

The deflection is detected by reflecting a laser beam off the back of the cantilever.

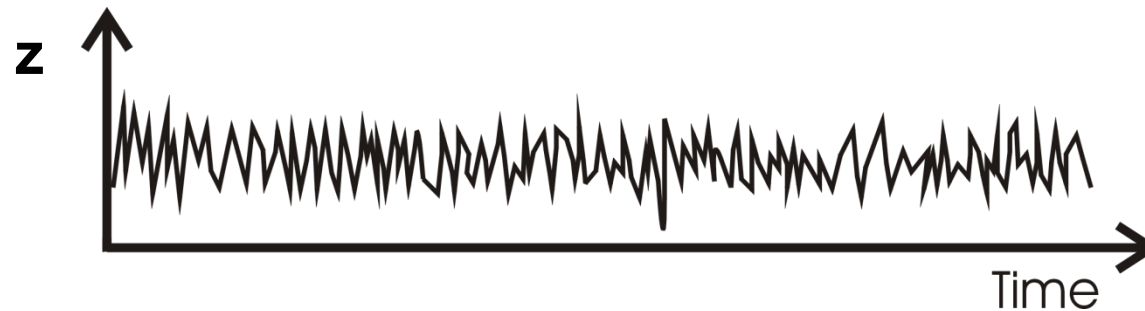
The light is collected on a split photodiode detector



Photodiode
current

Calibrating the cantilever spring constant: thermal tuning

The spring constant of the cantilever is determined from measurements of the natural response of the cantilever to thermal fluctuations.



The **mean square displacement of the cantilever $\langle z^2 \rangle$** is **obtained from the noise signal** and used in combination with equipartition theory (from Thermal Physics) to give;

$$\frac{1}{2} k \langle z^2 \rangle = \frac{1}{2} k_B T \qquad k = \frac{k_B T}{\langle z^2 \rangle}$$



9.2: AFM cantilever

An AFM cantilever has a large particle stuck to its end. Far from the surface, the measured mean squared height fluctuations were 2.3nm^2 . Assuming that the tip contains most of the mass (1ng), what approximately is the resonant frequency of the cantilever?

You may assume $(2\pi f)^2 = k/m$.

$$k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}, T = 300\text{K}$$

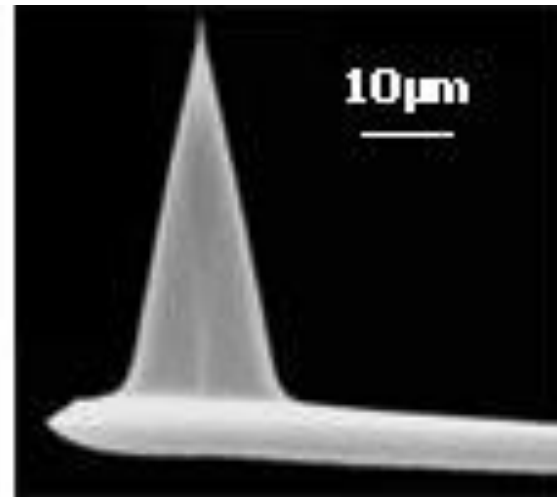
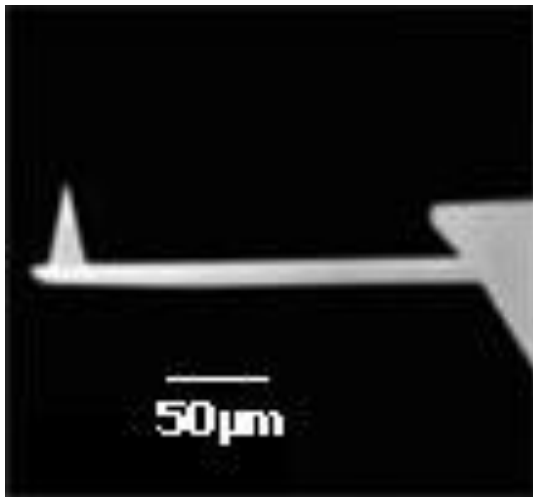


How small a force can we detect?

Typical cantilever spring constant: $0.01\text{-}50\text{ Nm}^{-1}$

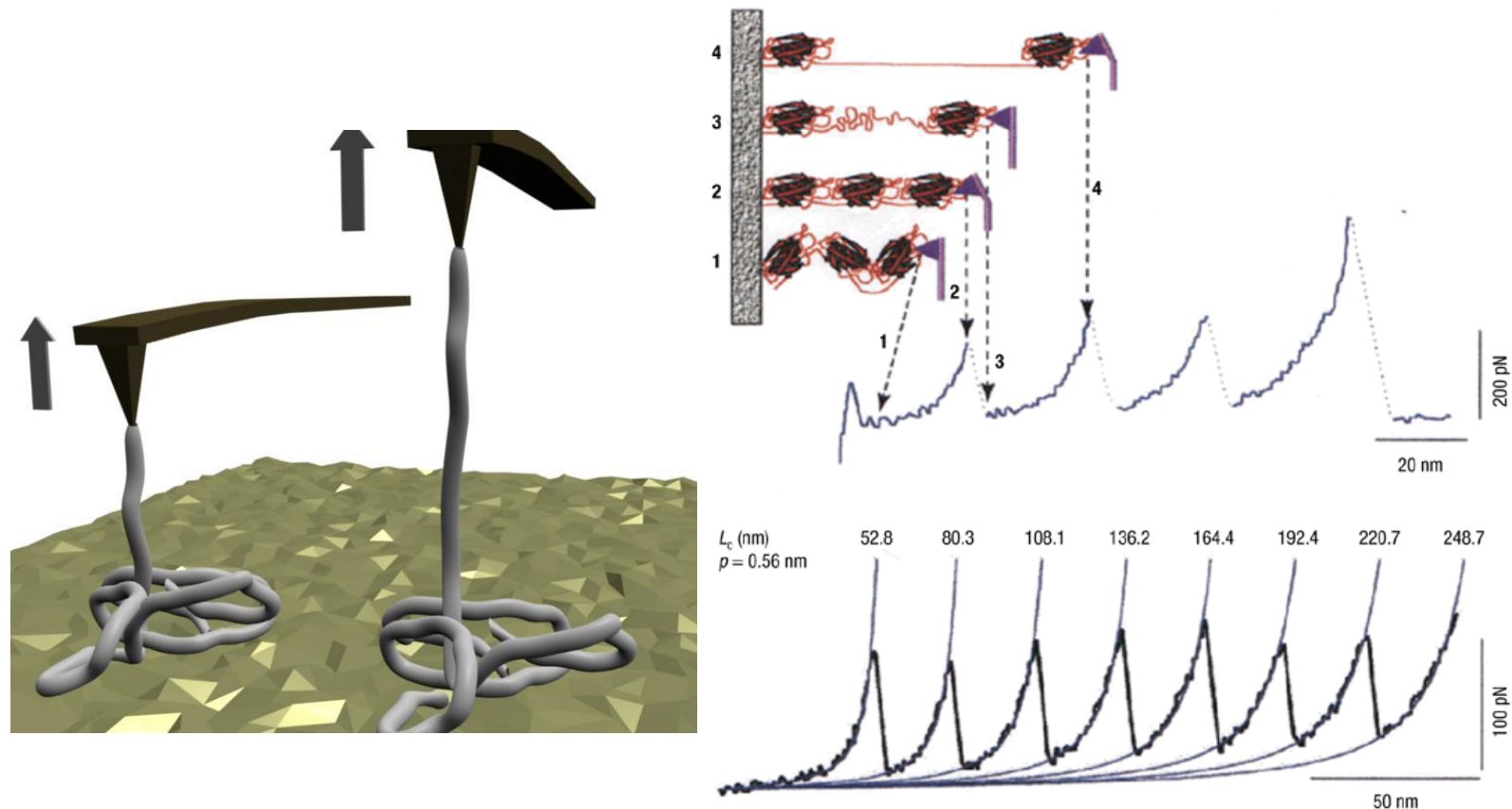
We can routinely measure cantilever displacements of $\sim 0.1\text{ nm}$

We can therefore measure forces $> 10\text{ pN}$ with relative ease using AFM



Measuring Force-Distance curves: Protein unfolding

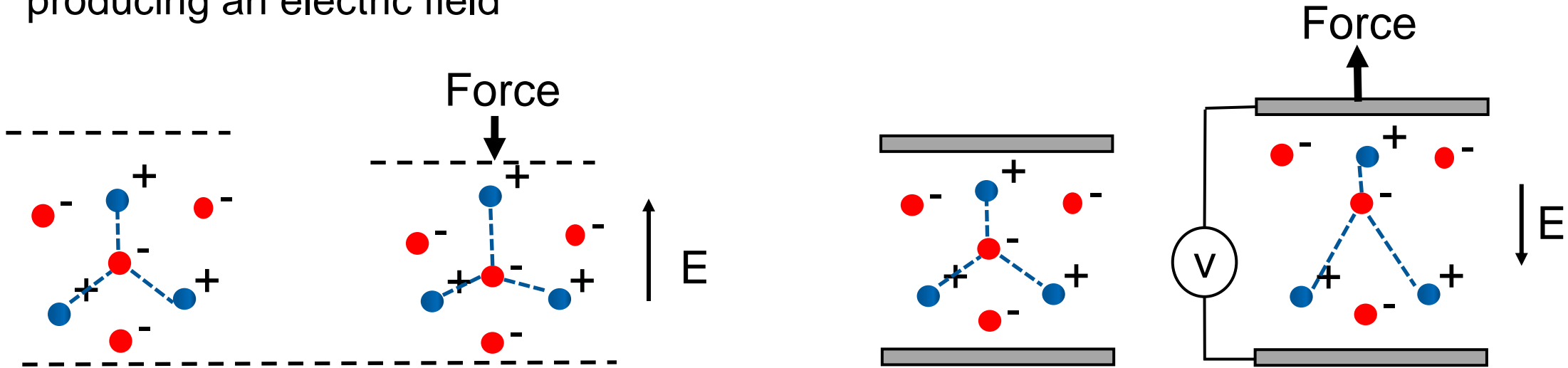
AFM is routinely used to measure the forces associated with the folding and unfolding of individual protein molecules



G. Bao & S. Suresh, *Nature Materials* 2, 715 - 725 (2003)

3. Nanoscale positioning of the tip - Piezo-electric motors

When a stress / strain is applied to a “Piezo-electric” crystal it becomes polarized producing an electric field



Applying an electric field can reverse this effect deforming the crystal by a small amount

By combining layers of crystal into a stack enables motion well below a nm.

$$P = -d\sigma + \alpha E$$

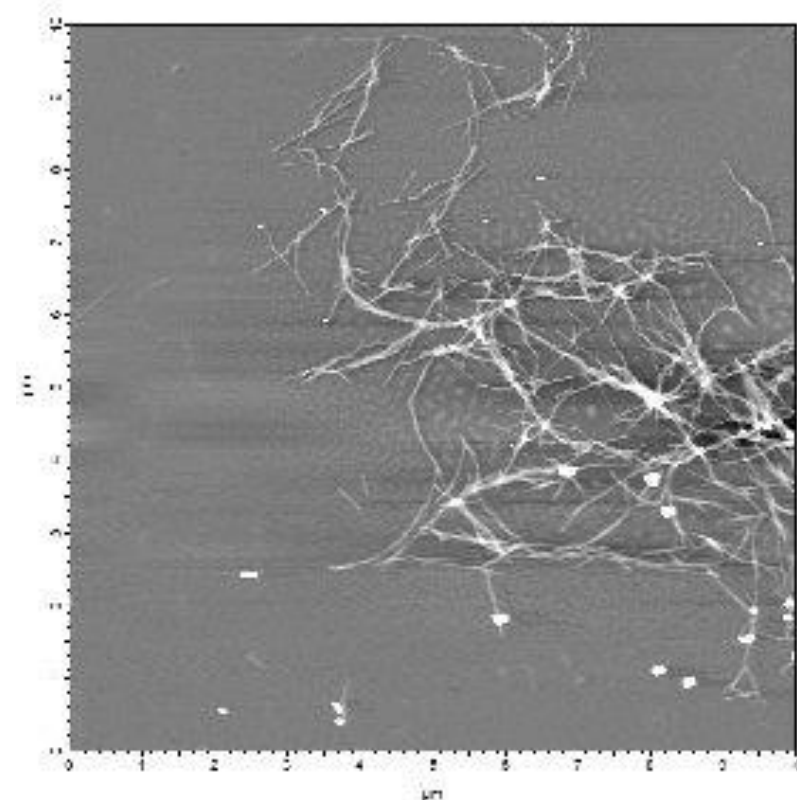
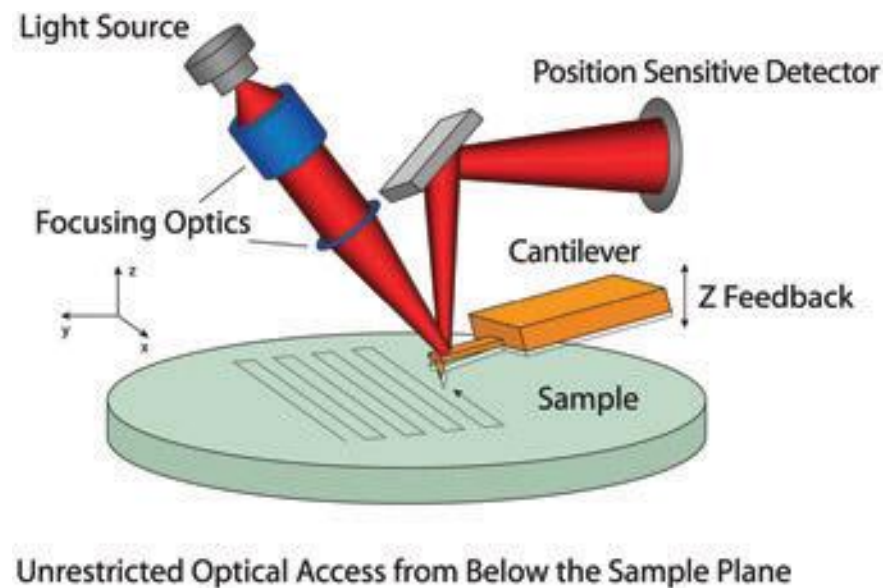
$$\epsilon = \sigma/Y + dE$$



P = Polarization (dipole moment / volume), d = piezoelectric constant, σ = Stress, α = Polarizability E = Electric Field, ϵ = strain, Y = Young's modulus,

Mapping forces: Imaging

AFM can be used to image samples on surfaces. The cantilever is raster scanned across the surface. As the tip encounters an object the intermolecular forces deflect the cantilever.



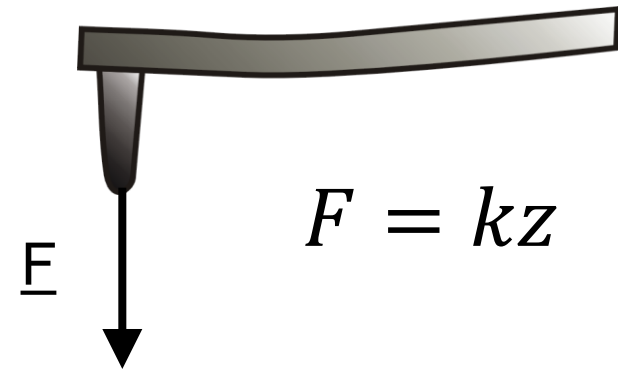
Amyloid Fibrils

Summary of key concepts

Atomic force microscopes can be used to measure forces

The spring constant of an AFM cantilever is determined by its material properties and its physical dimensions

A split photodiode arrangement is used to detect deflections and to measure forces with $>10\text{pN}$ precision



$$k = \frac{3EI}{l^3}$$

