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PHYS3009: Force and Function at the Nanoscale  
Week 28 – 10:00am Tuesday – 28 March 2023





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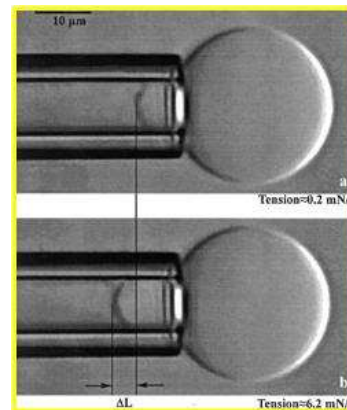
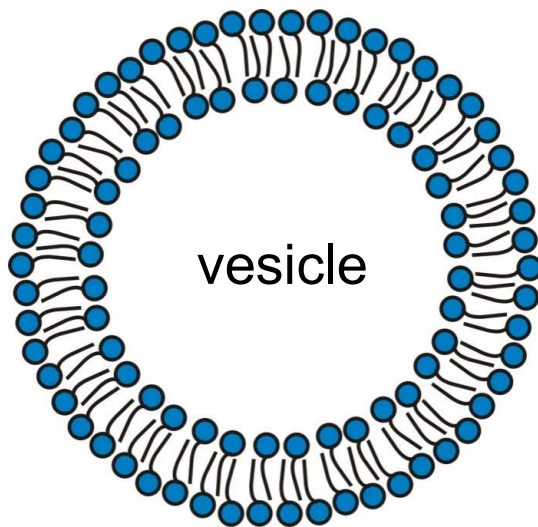
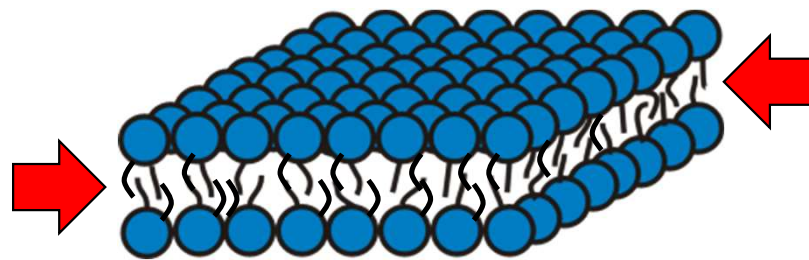
# Vesicles & Membrane Fluctuations

Force & function at the nanoscale



## Vesicle formation

When bi-layers are formed in solution, there is an excess energy associated with the exposed hydrophobic tail groups at the edges of the structure



The bi-layers can offset this energy by folding around to close themselves off and form an isolated shell or vesicle

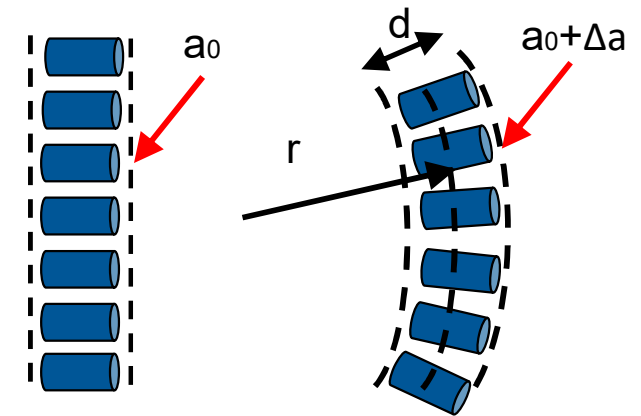


# Elasticity of Bi-layers

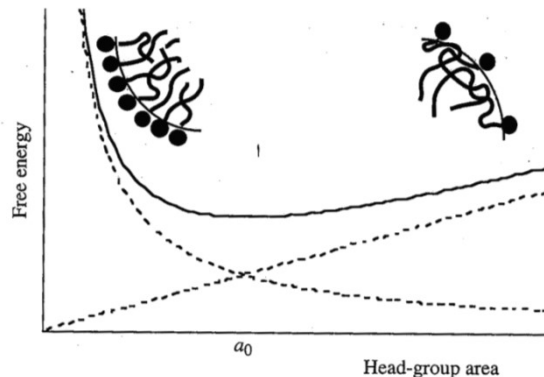
Curvature of bi-layers or membranes into vesicles is governed not only by packing constraints but elasticity of the bi-layer.

Bi-layers lowest state of free energy is that of a flat surface. The problem with bending is that:

- a) On outside heads get pushed apart admitting water
- b) on inside charged head groups are being pushed together.



$$\Delta U/A \approx \frac{1}{2} \frac{\kappa}{R^2}$$



The change in free energy with curvature gives the bi-layer elasticity. This results in a minimum radius of curvature below which vesicles cannot form.

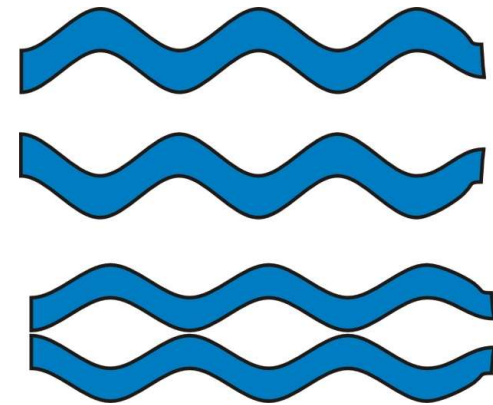
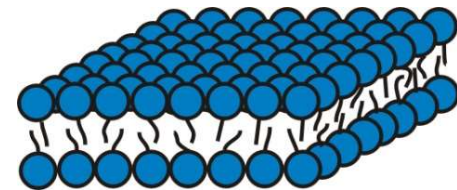


## Membranes (entropic repulsion)

Since membranes do not have high bending moduli they undergo thermally induced bending fluctuations.

When two membranes come into close proximity their motion is restricted (c.f. rod tethered to a surface) and an entropic repulsion force will be generated

$$P_{Undulation} = \frac{(k_B T)^2}{\kappa \pi^2 D^3}$$



$\kappa$  is a bending modulus ( $\sim 10^{-19}$  J)

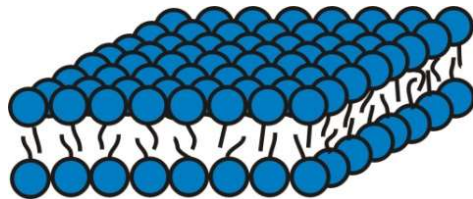


## Membranes (undulation repulsion)

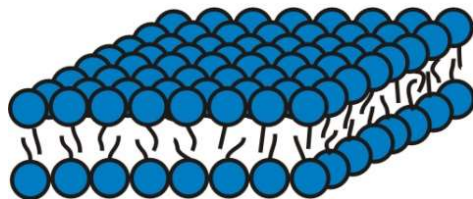
The total pressure is given by

$$P_{tot} = \left[ \frac{(k_B T)^2}{\kappa \pi^2} - \frac{A}{6\pi} \right] \frac{1}{D^3}$$

VdWs



This is largely controlled by the stiffness of the membrane, with flexible membranes repelling each other.



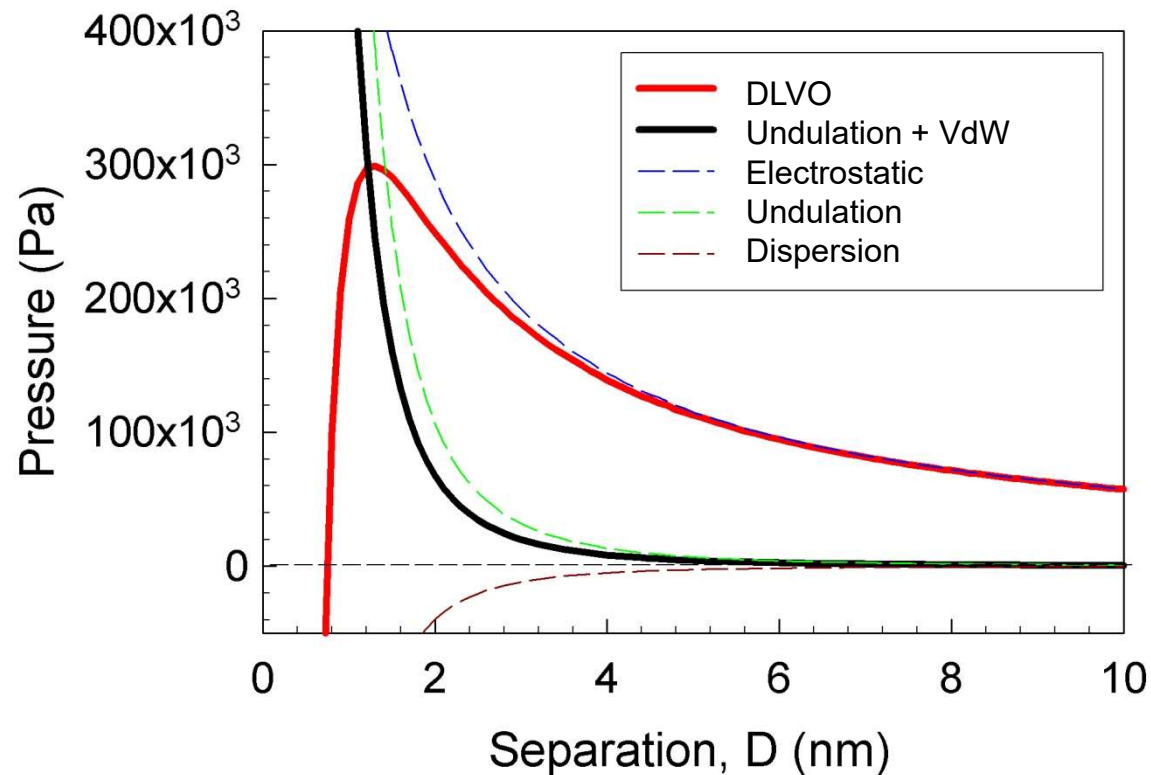
$$\kappa \propto l_c^2 a_0$$

This pressure is either always attractive **or** always repulsive, depending on the sign of the combined terms in the bracket



## Pressure between uncharged membranes

$$A=6 \times 10^{-21} \text{ J}, T=300 \text{ K}, a_0=0.717 \text{ nm}^2, Y=1 \times 10^{-20} \text{ J}$$



**Electrostatic + dispersion** → repulsive at long range,  
attractive at short range

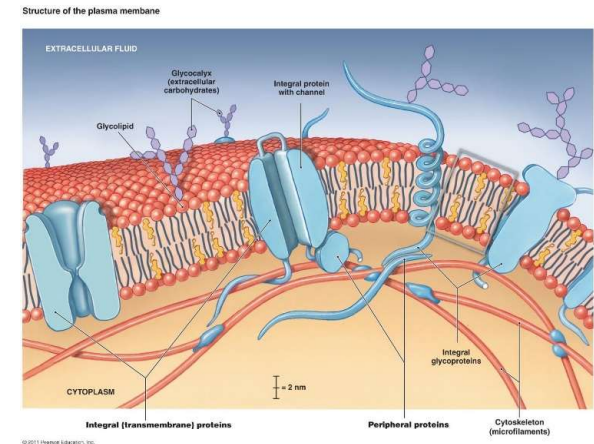
**Undulation + dispersion** → repulsive everywhere



## So how do cells stick together?

If the pressure between 2 bilayers is nearly always repulsive, then it raises the question how cells stick together (something that is fundamental to biological life)

1. The presence of other molecules in the cell wall suppresses the thermal fluctuations of the bi-layer by increasing the stiffness.
2. Special transmembrane proteins provide additional bonds that allow the cells to adhere to one another.



$$P_{tot} = \left[ \frac{(k_B T)^2}{\kappa \pi^2} - \frac{A}{6\pi} \right] \frac{1}{D^3}$$





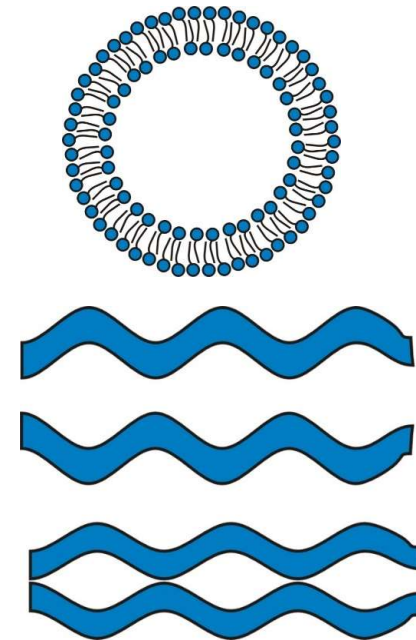
## Summary of key concepts

Membranes / bi-layers often curve into vesicles to prevent the hydrophobic tails being exposed to the liquid.

The elasticity / stiffness of a membrane is proportional to  $1/R^2$  where  $R$  is the radius of curvature of the bend.

As membranes come close together the thermally induced undulations are suppressed.

This gives rise to a repulsive entropic pressure.



$$P_{Undulation} = \frac{(k_B T)^2}{\kappa \pi^2 D^3}$$