The University of Nottingham

SCHOOL OF PHYSICS AND ASTRONOMY

A LEVEL 3 MODULE, RESIT Sample Paper A

FORCE AND FUNCTION AT THE NANOSCALE

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer all questions

Only a calculator from approved list A may be used in this examination.

List A

Basic Models	Aurora HC133	Casio HS-5D	Deli – DL1654	Sharp EL-233
Scientific Calculators	Aurora AX-582 Casio FX82 family Casio FX83 family	Casio FX85 family Casio FX350 family Casio FX570 family	Casio FX 991 family Sharp EL-531 family Texas BA II+ family	Texas Instruments TI-30 family

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

An indication is given of the approximate weighting of each part of a question by means of a bold figure enclosed by curly brackets, e.g. {**2**}*, immediately following that part.*

DO NOT turn examination paper over until instructed to do so

Table of Physical Constants

Speed of light in free space	c	3.00×10^{8} m s^{-1}
Gravitational Constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Planck's Constant	h	6.63×10^{-34} J s
	$\hbar$	1.055×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Mass of the electron	m_e	9.11×10^{-31} kg
Mass of the proton	m_p	1.6726×10^{-27} kg
Mass of the neutron	m_n	1.6749×10^{-27} kg
Boltzmann's constant	k_B	1.38×10^{-23} J K^{-1}
Gas constant	R	8.31 J K^{-1} mol^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J T^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Avogadro's number	N_A	6.02×10^{23} mol^{-1}

Information you may find useful

$< x^2 > \approx 6Dt$ where $D = \frac{k_B T}{6\pi\mu a}$

$P \propto \exp(-U/k_B T)$

$F(x) = -\frac{dU}{dx}$

$U_{dipole} = -\boldsymbol{p}.\boldsymbol{E}$ where $\boldsymbol{p} = qd\hat{r}$

$\boldsymbol{p} = \alpha \boldsymbol{E_{ext}}$ where $\alpha = 4\pi\epsilon\epsilon_r d^3$

$U(x) = -\frac{\pi n_1 C}{6x^3}$

$A_{12} = n_1 n_2 \pi^2 C$

$W_{adhesion} = \gamma_{13} + \gamma_{23} - \gamma_{12}$

$\Delta P = \gamma \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

$h = \frac{2\gamma\cos(\theta)}{\rho g R}$

$k = \frac{k_B T}{<z^2>} = \frac{3EI}{L^3}$

$< I >= \frac{c\epsilon\epsilon_0 n}{2} \left\langle E^2 \right\rangle$

$\Pi = \left(n_+ + n_- - 2n_0\right) k_B T$

$\frac{-d^2V}{dx^2} = \frac{\rho}{\epsilon\epsilon_0}$

$P_{Tot} = 4n_0 \frac{z^2 e^2 V_0^2}{k_B T} \exp(-\kappa D) - \frac{A}{6\pi D^3}$

$s = k_B \ln W$

$\Delta U = \Delta H - T\Delta S$

$\Delta U = P_{osm}\Delta V_{excl}$

$\mu = \frac{dU}{dN}$

$P(N = N_c) = \exp(-\Delta U/k_B T)$

$H = \frac{\nu}{l_c a_0}$

$P_{Tot} = \left[\frac{(k_B T)^2}{\kappa\pi^2} - \frac{A}{6\pi}\right] \frac{1}{D^3}$

where the symbols take their usual meanings as used throughout this course.

End

You must answer 3 out of 5 questions.
You should aim to spend about 30 minutes on each question.

1. a) A covalent bond is a "strong interaction" whilst a dispersion interaction is "weak". How does one distinguish between "strong" and "weak" in the context of nanoscale forces? Illustrate your answer with estimates of the approximate strength of a covalent bond and dispersion interaction in units of k_BT and the relative likelihood of their being spontaneously disrupted {**5**}.

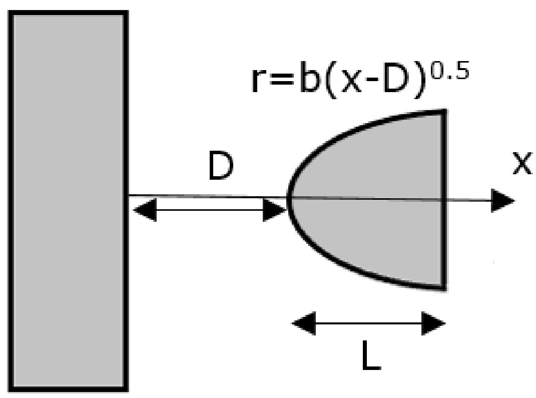
b) Sketch a graph illustrating how the Lennard-Jones potential varies with separation of two molecules, marking the repulsive and attractive parts of the potential and the point corresponding to zero force.

Explain in your own words, the physical origin of the attractive and repulsive parts of the potential. {**7**}

c) An AFM tip is brought close to a semi-infinite slab as shown in the diagram. The tip is aligned with the x axis and has a radial symmetry with profile given by the equation $r = b(x\text{-}D)^{0.5}$. Calculate the potential due to dispersion interactions between the tip and surface. You should write your answer in terms of the Hamaker constant. You may use the fact that the potential between a single atom and a semi-infinite slab is

$$U = \frac{-\pi n C}{6X^3}$$

{**9**}



d) The tip is fixed to the end of a long cantilever which allows the tip to move in x towards and away from the surface. While the AFM tip is far from the surface, the oscillations of the end of the cantilever caused by thermal fluctuations are measured and found to have a mean squared displacement of ($1\text{x}10^{-21}$m). Calculate the deflection of the cantilever when a force of 10nN is applied to the end of the tip. {**4**}

2. a) Why do multiple oil droplets dispersed in water gradually combine into one big droplet? In your answer you should describe the origin of interfacial energy in terms of individual atoms and molecules.{**5**}

b) Why do the same droplets of oil remain separate if mixed with washing up liquid (containing surfactants / amphiphiles)? {**3**}

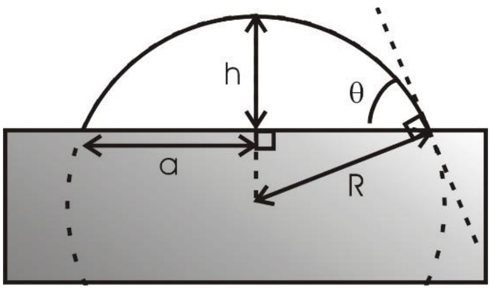
c) Trees have trunks made from wood which consists of a porous structure containing ~20μm diameter holes. Derive an expression for the height liquid would be expected to travel under capillary rise. You may use the fact that the capillary pressure is:

$$\Delta P = \gamma\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

Assume the contact angle=0.

Discuss briefly whether capillary rise offers a reasonable explanation for how water (Surface tension = $72\mathrm{mNm}^{-1}$, density $=1\mathrm{g/cm}^3$) is transferred from the roots to the leaves of a tree of 5m height. {**5**}

d) When it rains water drops land on the leaves of the tree but are not absorbed due to the hydrophobic nature of the leaves. A $1\mathrm{mm}^3$ water droplet lands on the leaf. If the interfacial energy of the leaf-water interface is $10\mathrm{mJm}^{-2}$, the leaf-air interface is $46\mathrm{mJm}^{-2}$ and water-air interface is $72\mathrm{mJm}^{-2}$ what will the basal area (the area of the water in contact with the leaf) be?



You may use the fact that the volume of a spherical cap as shown is given by:

$$V = \frac{\pi h^2}{3}(3R - h)$$

{**9**}

e) Another type of plant known as the Lotus plant has leaves that are superhydrophobic. The interfacial energy of the leaf material in contact with water is similar to other leaves but the contact angle is much higher (~150°). Explain with reference to the leaf structure why the droplet is nearly spherical {**3**}

3. a) Two charged particles are immersed in a liquid containing salt. Explain <u>in terms of the entropy</u> of the ions in the gap in between the particles, why this generates a repulsive force. {**7**}

b) The DLVO potential combines the attractive dispersion interaction with the repulsive electro-osmotic potential discussed above. Sketch 3 curves of the DLVO potential between the two particles for high, medium and low salt concentrations. The salt concentrations should be chosen so that aggregation is:

i) strong and irreversible,
ii) weak and reversible,
iii) almost impossible under thermal fluctuations.

In each case explain why the potential drawn gives rise to the stated type of aggregation.

{**7**}

c) For 3D aggregates the chemical potential is given by the equation:

$$\frac{dU}{dN} = -\mu_B + \frac{8\pi\gamma}{3N^{1/3}}\left(\frac{3v}{4\pi}\right)^{2/3}$$

Derive an expression and hence calculate the critical aggregation number for homogeneous nucleation. The bulk chemical potential is $\mu_B = 10k_BT$, the surface energy $\gamma = 1.5\text{mJm}^{-2}$, volume of a single molecule $v=125\text{nm}^3$ and the temperature is $T=300\text{K}$.

{**5**}

d) When scrambling an egg (during which proteins aggregate), explain why

i) it takes some time before the initial aggregates begin to grow
ii) without heating the saucepan, the egg will never scramble
iii) why the first aggregates to form are often stuck to the spoon or edge of the saucepan.

{**6**}