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PHYS3009: Force and Function at the Nanoscale
Week 23 – 4:00pm Friday – 01 March 2024

valid for 85 minutes from 2:55pm
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Optical Tweezers

Force & function at the nanoscale

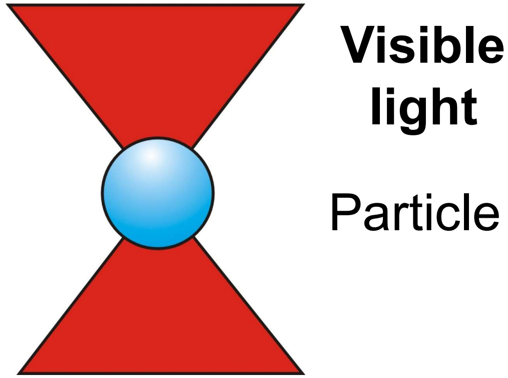
Optical tweezers

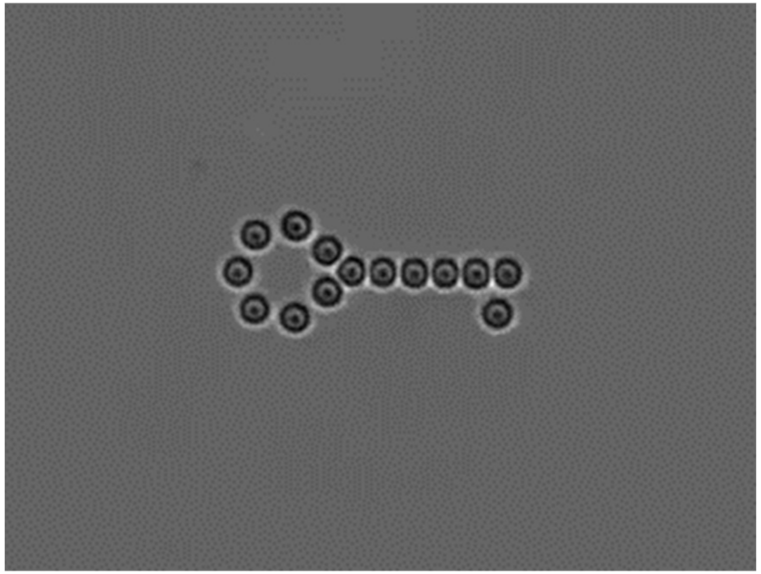
Optical tweezers provide a way of manipulating nano or micron scale objects and measuring very small forces.

Particles are trapped by focussing laser light

Objects can be manipulated by moving the laser beam

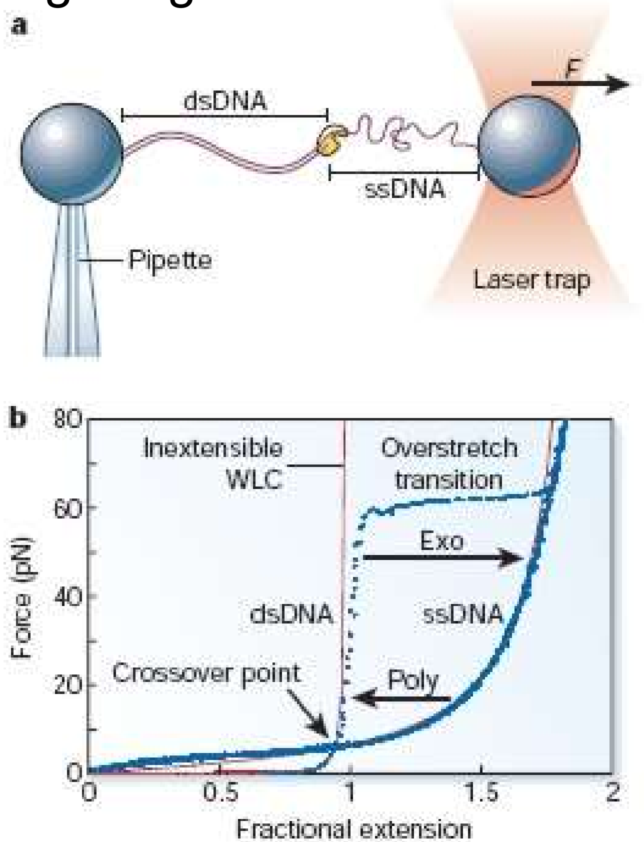
Advanced setups exist which enable the manipulation of many particles simultaneously





Force Measurements

It is also possible to perform more serious measurements on e.g. single molecules



An optical trap and a fine micropipette can be used to measure the forces exerted by individual DNA molecules

Small polymer beads are tethered to the ends of the molecule

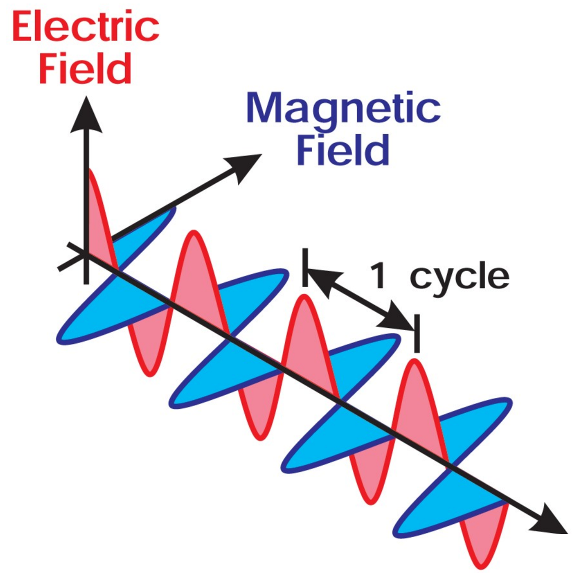
C. Bustamante et al., **Nature**, 421(23), 423 (2003)

The wave nature of light

Light is an electromagnetic wave.

As it propagates through vacuum or a medium it carries energy.

Energy is stored in electric and magnetic fields that oscillate in a direction perpendicular to the direction of travel (more on this in Classical Fields)



Electric field travelling in positive x direction

$$E = E_0 sin(\omega t - kx)$$

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The intensity of light

The time averaged intensity of light, I, can be related to the magnitude of the electric field, E, by the relationship

$$\langle I \rangle = \frac{c\varepsilon\varepsilon_0 n}{2} \langle E^2 \rangle$$

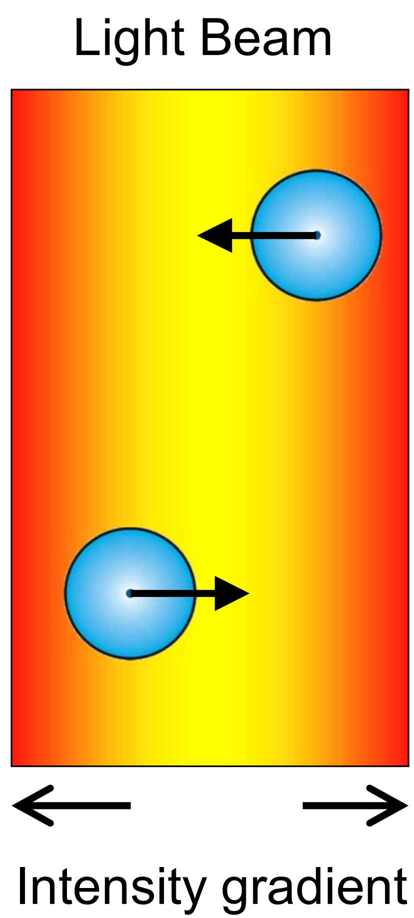
Where c is the speed of light (ms^{-1}) and n is the refractive index of the medium in which it propagates

Note: For a sinusoidal oscillating field, the time average value of $<E^2>$ is not zero. So the intensity is not zero.

Optical trapping

Small dielectric particles experience a force when illuminated with visible light.

If a dielectric particle is placed in an intensity gradient a force is exerted on the particle such that it will move to a region of higher intensity



Calculate the energy change optical tweezers

Suppose we treat the small particle as a dipole with polarizability α.

What is the energy change associated with placing the particle in a laser beam of intensity I?

Recall that:

$$U = -\boldsymbol{p}.\boldsymbol{E} \qquad \boldsymbol{p} = \alpha\boldsymbol{E} \qquad \langle I \rangle = \frac{c\varepsilon\varepsilon_0 n}{2}\langle E^2 \rangle$$

It follows that at any instant:

$$U = -\alpha\boldsymbol{E}.\boldsymbol{E} = -\alpha|\boldsymbol{E}|^2$$

When averaged over time:

$$U = -\frac{2\alpha}{c\varepsilon\varepsilon_0 n}\mathrm{I}$$

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The force on a dipole in an intensity gradient

We can use this potential to calculate the force on the particle

In one dimension, the force on a dielectric particle is

$$\boldsymbol{F} = -\frac{dU}{dx}\boldsymbol{i} = \frac{2\alpha}{c\varepsilon\varepsilon_0 n}\frac{dI}{dx}\boldsymbol{i}$$

Generalising to 3 dimensions we have

$$\boldsymbol{F} = -\frac{dU}{dx}\boldsymbol{i} - \frac{dU}{dy}\boldsymbol{j} - \frac{dU}{dz}\boldsymbol{k} = \frac{2\alpha}{c\varepsilon\varepsilon_0 n}\left(\frac{dI}{dx}\boldsymbol{i} + \frac{dI}{dy}\boldsymbol{j} + \frac{dI}{dz}\boldsymbol{k}\right)$$

So the **force always acts along the direction of increasing intensity gradient. Particles move to regions of higher intensity.**

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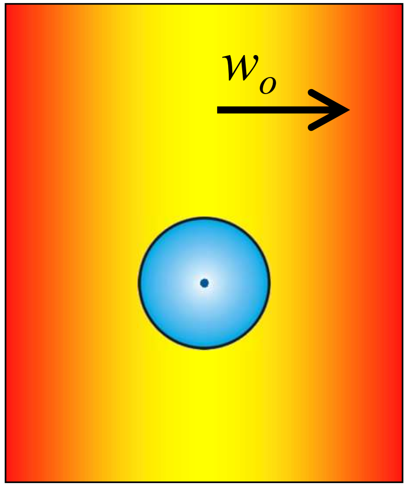
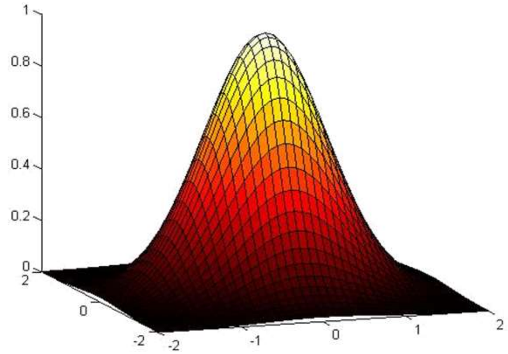
Making a simple optical trap – creating an intensity gradient

Laser Beam



The beam profile of a laser is not uniform - it has a Gaussian profile in the radial direction – this provides trapping in x-y plane

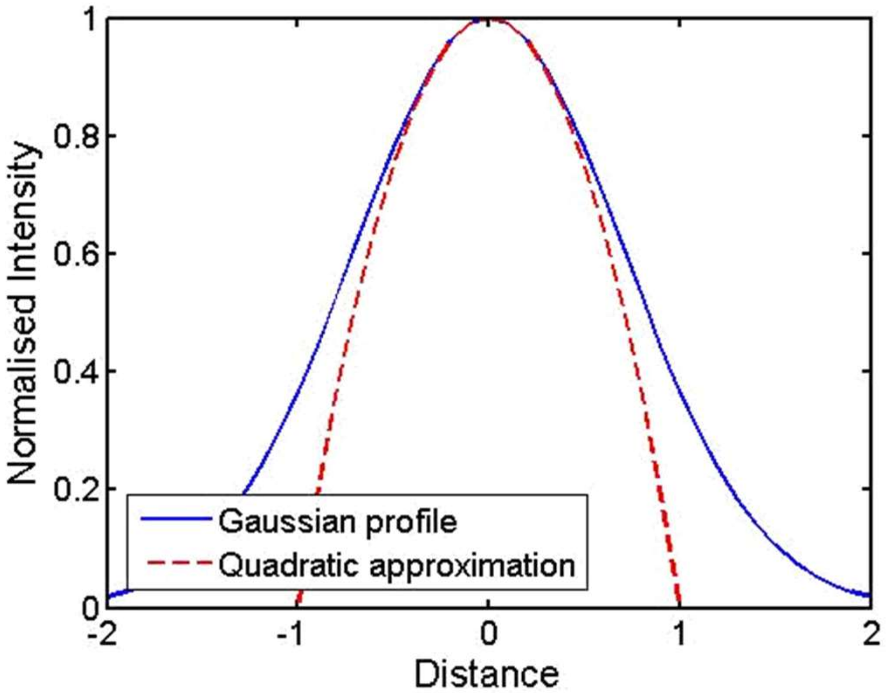
$$I = I_0 exp\left(-\frac{(x^2+y^2)}{2w_0^2}\right) = I_0 exp\left(-\frac{r^2}{2w_0^2}\right)$$



where w_o is the half-width of the beam

10.1 Deriving the optical trap stiffness

For small displacements from the central position we can approximate the Gaussian using a quadratic intensity profile



Laser Intensity

$$I = I_0 exp\left(-\frac{r^2}{2w_0^2}\right) \approx I_0\left(1 - \frac{r^2}{2w_0^2}\right)$$

Gives rise to a force:

$$F = -kr$$

Where the trap spring constant k is:

$$k = \frac{2\alpha I_0}{c\varepsilon\varepsilon_0 n W_0^2}$$

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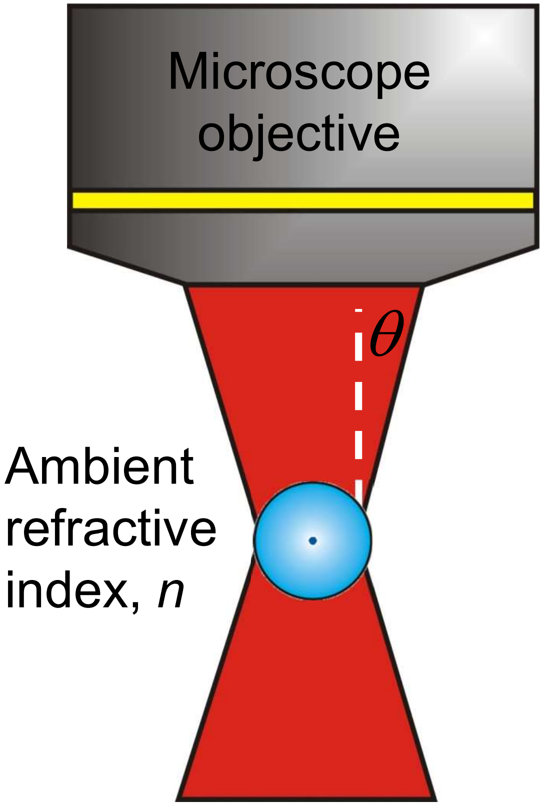
Optical Trapping in 3D



We can use a microscope objective to create a 3D trap!

The changing area of the beam near the focus means that the intensity of the light decreases on either side of the focal point

This traps the particle in z direction also!

$$\boldsymbol{F} = -\frac{dU}{dz}\boldsymbol{k} = \frac{2\alpha}{c\varepsilon\varepsilon_0 n}\frac{dI}{dz}\boldsymbol{k}$$

The maximum gradient in intensity can be obtained by using a lens with a high numerical aperture (NA)

$$NA = n\sin\theta$$

Measuring forces with optical tweezers

As we saw previously, for small displacements, r, the force on a particle in a focussed laser beam is given by $F=-kr$

Where k is the trap stiffness and is given by

$$k = \frac{2\alpha I_0}{c\varepsilon\varepsilon_0 n w_0^2}$$

So if we can measure the displacement we can determine the force.

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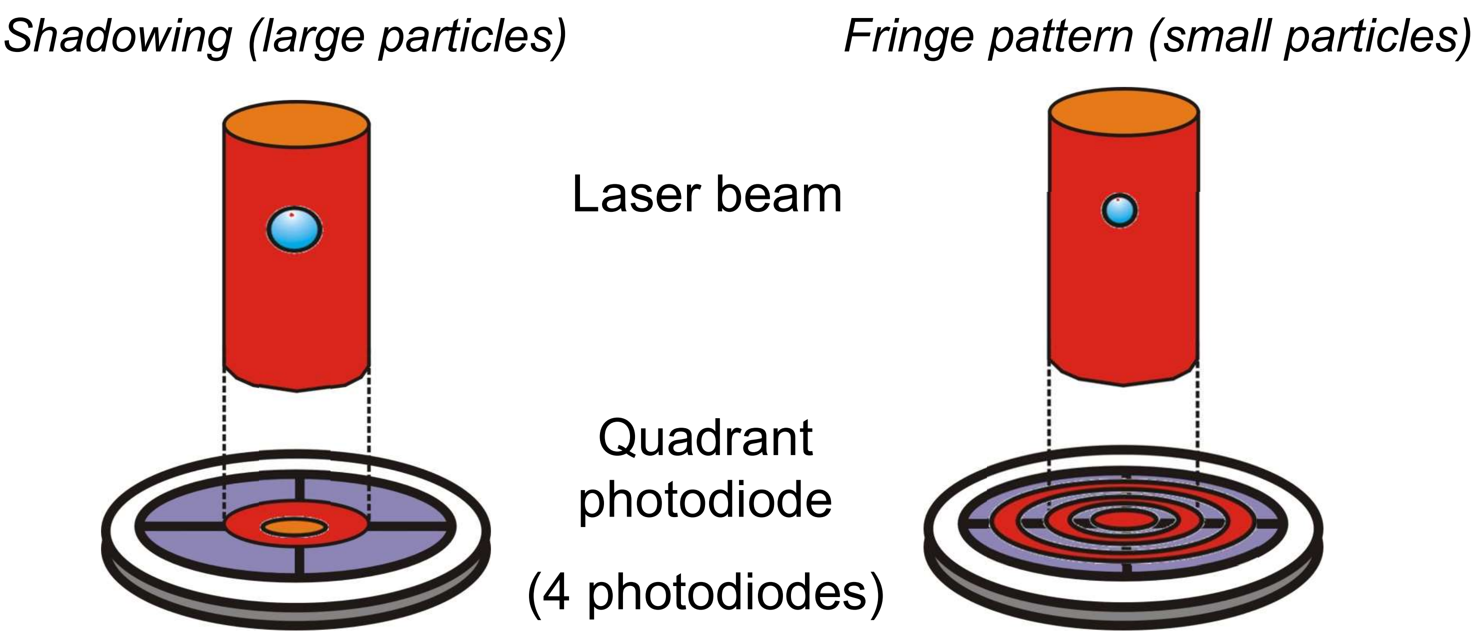
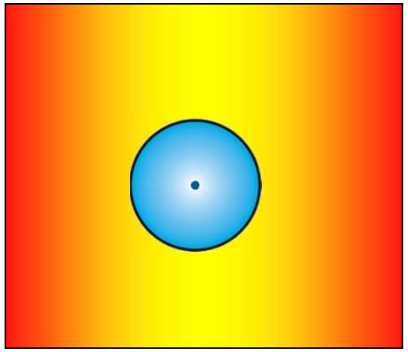
Detecting deflections: Quadrant photodiode

The split photodiode arrangement used in AFM can also be used to detect deflections in optical traps. However a quadrant photodiode is used to detect deflections in both the x and y directions



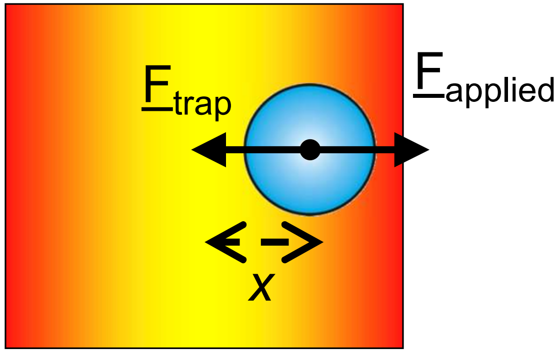
Trap calibration



We can measure the mean square displacement of a trapped particle under the influence of thermal motion

$$k \approx \frac{k_B T}{\langle x^2 \rangle}$$

Or we can measure the displacement of a particle under the influence of known forces



$$k = \frac{F_{applied}}{x}$$

Problem 10.2 : Optical Tweezers

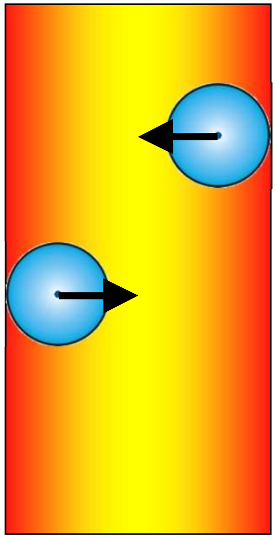
A spherical particle of diameter a=500nm is trapped in a focussed laser beam. The beam is then moved at a constant velocity v=50μms^{-1} in the x direction (perpendicular to the beam) through a liquid of viscosity μ =1mPas. The particle is on average displaced 5nm from the centre of the trap. What is the spring constant of the optical trap?

(Hint the stokes drag on a sphere is F=6πμav)

Summary of key concepts

Particles in a laser beam will experience a force that acts to pull them to regions of higher intensity

$$\boldsymbol{F} = -\frac{dU}{dx}\boldsymbol{i} = \frac{2\alpha}{c\varepsilon\varepsilon_0 n}\frac{dI}{dx}\boldsymbol{i}$$



Lasers have a Gaussian beam profile which naturally lends itself to trapping of particles

Trapping in 3 dimensions can be achieved by focussing the laser using a microscope objective

Trapping forces on nanoscale particles can be measured with pN precision