The University of Nottingham

SCHOOL OF PHYSICS AND ASTRONOMY

A LEVEL 3 MODULE, RESIT Sample Paper B

FORCE AND FUNCTION AT THE NANOSCALE

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer all questions

Only a calculator from approved list A may be used in this examination.

List A

Basic Models	Aurora HC133	Casio HS-5D	Deli – DL1654	Sharp EL-233
Scientific Calculators	Aurora AX-582 Casio FX82 family Casio FX83 family	Casio FX85 family Casio FX350 family Casio FX570 family	Casio FX 991 family Sharp EL-531 family Texas BA II+ family	Texas Instruments TI-30 family

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

*An indication is given of the approximate weighting of each part of a question by means of a bold figure enclosed by curly brackets, e.g. {**2**}, immediately following that part.*

DO NOT turn examination paper over until instructed to do so

Table of Physical Constants

Speed of light in free space	c	3.00×10^{8} m s^{-1}
Gravitational Constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Planck's Constant	h	6.63×10^{-34} J s
	$\hbar$	1.055×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Mass of the electron	m_e	9.11×10^{-31} kg
Mass of the proton	m_p	1.6726×10^{-27} kg
Mass of the neutron	m_n	1.6749×10^{-27} kg
Boltzmann's constant	k_{B}	1.38×10^{-23} J K^{-1}
Gas constant	R	8.31 J K^{-1} mol^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_{B}	9.27×10^{-24} J T^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Avogadro's number	N_{A}	6.02×10^{23} mol^{-1}

Information you may find useful

$< x^2 > \approx 6Dt$ where $D = \frac{k_BT}{6\pi\mu a}$

$P \propto \exp(-U/k_BT)$

$F(x) = -\frac{dU}{dx}$

$U_{dipole} = -\boldsymbol{p}.\boldsymbol{E}$ where $\boldsymbol{p} = qd\hat{r}$

$\boldsymbol{p} = \alpha \boldsymbol{E_{ext}}$ where $\alpha = 4\pi\epsilon\epsilon_r d^3$

$U(x) = -\frac{\pi n_1 C}{6x^3}$

$A_{12} = n_1 n_2 \pi^2 C$

$W_{adhesion} = \gamma_{13} + \gamma_{23} - \gamma_{12}$

$\Delta P = \gamma \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

$h = \frac{2\gamma\cos(\theta)}{\rho g R}$

$k = \frac{k_BT}{<z^2>} = \frac{3EI}{L^3}$

$< I > = \frac{c\epsilon\epsilon_0 n}{2}\left\langle E^2 \right\rangle$

$\Pi = \left(n_+ + n_- - 2n_0\right) k_BT$

$\frac{-d^2V}{dx^2} = \frac{\rho}{\epsilon\epsilon_0}$

$P_{Tot} = 4n_0\frac{z^2e^2V_0^2}{k_BT}\exp(-\kappa D) - \frac{A}{6\pi D^3}$

$s = k_B \ln W$

$\Delta U = \Delta H - T\Delta S$

$\Delta U = P_{osm}\Delta V_{excl}$

$\mu = \frac{dU}{dN}$

$P(N = N_c) = \exp(-\Delta U/k_BT)$

$H = \frac{\nu}{l_c a_0}$

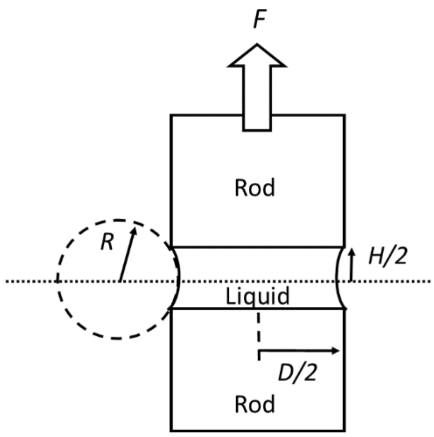
$P_{Tot} = \left[\frac{(k_BT)^2}{\kappa\pi^2} - \frac{A}{6\pi}\right]\frac{1}{D^3}$

where the symbols take their usual meanings as used throughout this course.

End

1. a) Describe the physical origin of interfacial energy in terms of the molecules of a liquid. You should draw a diagram to illustrate your explanation. **{6}**

   An experiment consists of two cylindrical metal rods made from aluminium of diameter *D/2*. The bottom rod is held static and the top rod can be moved upwards. In between the two rods is liquid water. The ends of the two rods are initially H_0 apart. Throughout the experiment the water completely wets both end plates, such that the contact line cannot move.



   The top rod is equipped with a very sensitive force sensor that measures the forces acting on the top plate. The size of the system is sufficiently small that interfacial tension dominates (i.e. gravity can be neglected).

   b) The sides of the liquid column have a shape which is approximately an inverted circular shape as shown. Why is this? **{2}**

   c) Write down the full expression for the Laplace pressure, and use the fact that at the position $H=H_0$ the radius of curvature $|R| << D/2$ to derive an approximate expression for the Laplace pressure in the liquid. **{4}**

   d) At the midpoint between the two rods the liquid bridge has a waist diameter *W*. Obtain an expression for the force measured on the upper plate in terms of the surface tension γ, plate diameter *D*, waist diameter *W* and the gap between the plates *H*. **{7}**

   e) Calculate the measured force if $\gamma=72\text{x}10^{-3}\text{Nm}^{-1}$, H = 2mm, D=50mm, W=49mm. **{2}**

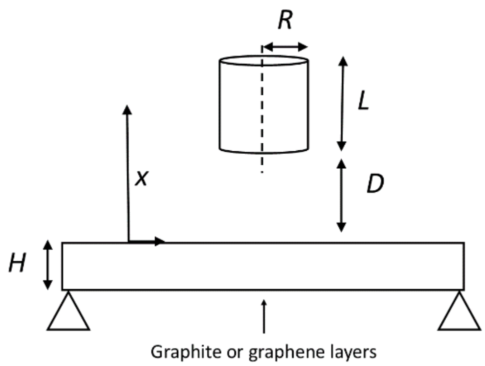
   f) The upper rod is moved so that the gap is $H=H_1$. At this new gap, the sides of the liquid bridge are vertical and it is **not true** that $|R|$ << *D/2*. Derive and calculate the force on the upper plate under these conditions. **{4}**

2. a) Graphene consists of a single layer of carbon atoms held together by covalent bonds.

    I. What is the physical origin of a covalent bond?
    II. Give an estimate of the strength in k_BT. **{3}**

    b) Explain why it is unreasonable to expect a single carbon atom to detach spontaneously from a piece of graphene due to thermal fluctuations. **{4}**

    Graphene is made by removing single layers of material from a piece of graphite. However, this process is difficult and unreliable. A scientist wants to work out whether the pieces of material they are removing are single layers (graphene) or much thicker bits of material. The scientist suspends each piece of material between two supports and then measures the force between an AFM probe and the removed material in order to try and determine if the material is a single or multilayer piece of material.



    The AFM probe is modelled as a cylinder of radius R and length L, which is attached to a cantilever (not shown). The tip of the probe is a distance D from a film of unknown thickness H whose lateral dimensions should be considered infinite. The supports are far enough away from the AFM tip that you can ignore any interaction with them.

    c) The potential of a single atom interacting with a sheet of thickness H is given by the equation:

$$U(x) = \frac{-\pi n_1 C}{6}\left(\frac{1}{x^3} - \frac{1}{(x+H)^3}\right)$$

    Show that the potential between the cylindrical AFM probe and the sheet of material is given by:

$$U(D) = \frac{-AR^2}{12}\left(\frac{1}{(D+H+L)^2} - \frac{1}{(D+H)^2} - \frac{1}{(D+L)^2} + \frac{1}{D^2}\right)$$

    Where A is the Hamaker constant. **{6}**

    d) Derive an approximate expression in the limit that $D << L$. **{2}**

    e) A single layer piece of Graphene is approximately 0.3nm thick. If the AFM tip has a radius R = 5μm and is held D=10nm above the film, and the Hamaker constant A of interaction is 5×10^{-21}J estimate:

    I. The size of the potential for a sheet of graphene **{2}**
    II. The size of the potential for a piece of graphite where H is very large. **{2}**

    f) Describe in your own words the principles by which an AFM measures forces in contact mode. Draw a simple diagram to illustrate your answer **{6}**

3. a) A single colloidal sphere of radius a is placed at the centre of a spherical cavity of diameter d which is filled with a liquid of viscosity η at temperature T.

   I. Describe the motion of a single colloidal particle in a liquid. **{2}**
   II. Derive an expression for the approximate time for the colloidal particle to reach the walls of the cavity. **{3}**
   III. If you observed the motion of the colloidal particle moving in the cavity for long enough what would you expect to see? **{2}**

   b) The single colloidal particle is removed and a large number of much smaller particles of radius r are added to the same cavity. Using the small particles as an example describe what is meant by a microstate and how this is related to the entropy of the system. **{3}**

   c) The single large colloidal particle together with the smaller particles is now returned to the cavity. Explain why the large colloidal particle no longer freely diffuses around the cavity. **{5}**

   An optical tweezer setup is used to try and measure the force between the large colloid and the cavity wall. The refractive indices of the small particles, liquid and cavity wall are all equal which means that the laser beam only interacts with the single large particle.

   d) Initially the large particle is held in the beam far from the wall and the motion due to thermal fluctuations is measured to be $\langle x^2 \rangle = 5 \times 10^{-18}\text{m}^2$. The liquid is maintained at a steady temperature of 290K. What is the spring constant of the optical trap? **{2}**

   The large particle is brought close to the wall, and held by the optical tweezers.

   e) The interaction between the cavity wall and the large particle of radius a is given by the following potential:

$$U(D) = \frac{-n\pi k_B T}{3}[a + 2r - D]^2[2a + r + D]$$

   where r is the radius of the small particles, and D is the distance from the large particle centre to the wall. Derive an expression for the force as a function of distance D from the cavity wall and state the limits over which the expression is valid. **{5}**

   f) The large colloidal particle has a radius a = 5μm, whilst the small particles have a radius r of 50nm and are dispersed at a concentration of 10^{20} particles per unit volume. The temperature T is 290K. If the large colloidal particle centre is held D= 5.05μm from the cavity wall by the optical tweezers, what displacement will it have relative to the centre of the laser beam? **{3}**