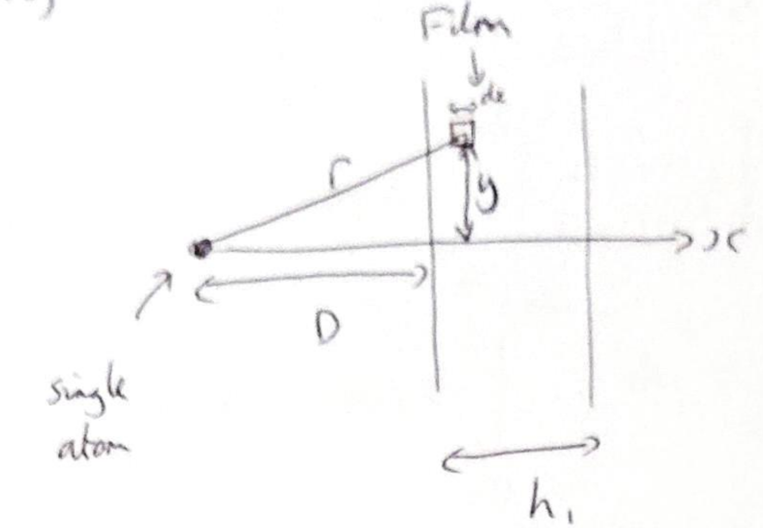
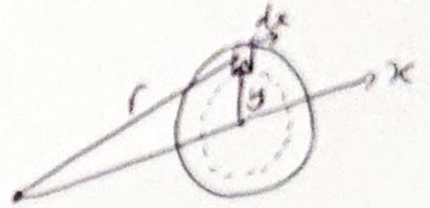
a)





There is a ring of elemental volumes all at the same distance r from the atom.

This ring has a volume $dV_{ring} = 2\pi y \, dy \, dx$

If the atom density is n_1 then there are N_{ring} atoms in the ring.

$$N_{ring} = n_1 \, dV_{ring} = n_1 2\pi y \, dy \, dx.$$

The potential due to the ring is $dU = -2\pi n_1 y \, dy \, dx \frac{C}{r^6}$

Sum up all rings

$$U_{tot} = \int_D^{D+h_1} \int_0^{\infty} \cancel{dxdy} -2\pi n_1 y \cancel{dy} \frac{C}{r^6} \, dy \, dx$$

r is a function of x + y $\quad r^2 = x^2 + y^2$

change variables $w = x^2 + y^2 \Rightarrow \frac{dw}{dy} = 2y$

$$U_{tot} = \int_D^{D+h_1} \int -\frac{\pi n_1 C \, dW}{W^3} \, dx$$

$$u_{tot} = \int_D^{D+h_1} -\pi n_1 C \left[\frac{-1}{2w^2}\right] dx$$

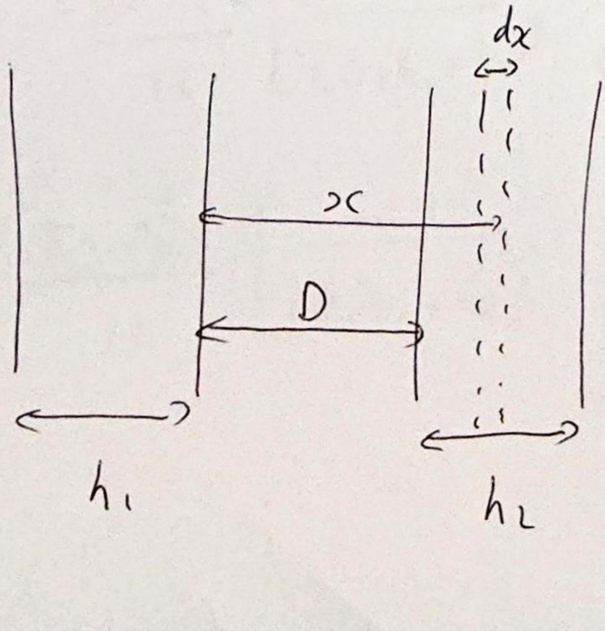
$$= -\pi n_1 C \int_D^{D+h_1} \left[\frac{-1}{2(x^2+y^2)^2}\right]_0^\infty dx$$

$$= -\pi n_1 C \int_D^{D+h_1} \left[0^- - \frac{1}{2x^4}\right] dx$$

$$= \frac{\pi n_1 C}{2} \left[\frac{-1}{3x^3}\right]_D^{D+h_1}$$

$$u_{tot} = -\frac{\pi n_1 C}{6}\left[\frac{1}{D^3} - \frac{1}{(D+h_1)^3}\right]$$

b) $$\frac{dU}{A} = \underbrace{n_2\,dx}_{\text{No. atoms/unit area in film 2 in infinitely thin slice}} \times \underbrace{-\frac{\pi n_1 C}{6}\left[\frac{1}{x^3} - \frac{1}{(x+h_1)^3}\right]}_{\text{Potential of 1 atom with whole of film 1.}}$$ Substitute x for D.



$$\frac{U_{tot}}{A} = \int_{D}^{D+h_2} \frac{-\pi n_1 n_2 C}{6}\left(\frac{1}{x^3} - \frac{1}{(x+h_1)^3}\right) dx$$

integrate over all slices in film 2.

$$= \frac{-\pi n_1 n_2 C}{6}\left[\frac{-1}{2x^2} + \frac{1}{2(x+h_1)^2}\right]_{D}^{D+h_2}$$

$$= \frac{-\pi n_1 n_2 C}{12}\left[\frac{1}{(D+h_1+h_2)^2} - \frac{1}{(D+h_1)^2} - \frac{1}{(D+h_2)^2} + \frac{1}{D^2}\right]$$

$$\text{Pressure} = \frac{F}{A} = \frac{-1}{A}\frac{dU}{dD}$$

$$\frac{d}{dD}\left(\frac{1}{(D+a)^2}\right) = \frac{dw}{dD}\frac{d}{dw}\left(\frac{1}{w^2}\right) = \frac{-2}{w^3} = \frac{-2}{(D+a)^3}$$

General form of $D+h_1+h_2$ set $w = D+a$

or $D+h_1$

or D with $a=0$

$$\text{Pressure} = \frac{-\pi n_1 n_2 C}{12}\left[\frac{-2}{(D+h_1+h_2)^3} - \frac{-2}{(D+h_1)^2} - \frac{-2}{(D+h_2)^2} + \frac{-2}{D^3}\right]$$

$$P = \frac{\pi n_1 n_2 C}{\cancel{12}\ 6}\left[\frac{-1}{(D+h_1+h_2)^3} + \frac{1}{(D+h_1)^3} + \frac{1}{(D+h_2)^3} - \frac{1}{D^3}\right]$$