Quantum Galton Board

Quantum Galton Board is adaptation of the Classical Galton Board. Classical Galton Board is a device that demonstrates how a normal or Gaussian distribution or bell curved distribution by channel balls through a triangular lattice of pegs, with random left or right deflection as they descend down. Quantum Galton Board takes this familiar concept and add quantum mechanics in it. In Quantum Galton board, the balls are replaced by the quantum particles or other quantum states that can exist in the superposition state. Instead of traveling down one specific path determined by random direction, a quantum particle can simultaneously explore all possible paths. It leverages quantum superposition and interference to emulate the branching pathways of classical board with quantum amplitudes influencing final distributions. So, final distribution may be Gaussian distribution or we can create other distribution from it. Due to it, now quantum mechanical Galton Board has been core concept in quantum computing. It has many application in quantum computing like for complex system simulation, quantum machine learning, sampling and search problem, etc. . Now we will see how to create this Quantum Galton Board.

The basic approach to create the QGB is to model the action of individual peg on a physical Galton Board. Here, we will take it as a layer. In ideal classical GB every ball hitting to peg producing a probability of going left or right, then hitting next peg. To use this concept in quantum Galton Board we take initial sate of qubits as ket |0> state and invert middlemost qubit using X gate. We then construct a series of superposed SWAPs gate that this particle can effectively fall through as per action of real ball through array of pegs on the Classical Galton Board. By this model we can get normal or Gaussian distribution as same as in classical Galton board.

Circuit of Quantum Galton Board

Now we see detail for the first layer of Quantum Galton Board for normal distribution later by using its concept we can create n layer of QGB. For it we required three working qubits that are q1, q2, q3 and one control qubits q0 which will control the action of three working qubits. We initialize all qubit to the zero. Then we apply Hadamard gate to the control qubit q0 and place the particle to the middle working qubits q2 using X gate.

Then we perfrom controlled-SWAP operation on q1 and q2 which demonstrate left deflection which give us an eventual state of |q2q1q0> as

1/sqrt(2)(|011> +|100>)

After that we apply invert CNOT on q2 to q0 become

1/sqrt(2)(|011>+|101>)

Given oue control qubit is now astable |1> we can apply a controlled-SWAP gate between q2 and q3 with control qubit q0 to give desired final state of

|q3q2q1q0> = 1/sqrt(2)(|0011> + |1001>)

This module can with some care around the control qubit, be replicated to reproduce each level of the Galton Board successively. We have created first layer of the Galton Board successfully.

Now if we want to continue and create next level of this QGB then ae have to reset means the 2 level then we have to add more working qubit. Now we will required 5 working qubit. First step will be same as we have seen before all qubits in state ket 0. Then we have applied Hadamard gate to control qubit and X gate to the middlemost qubit then we apply controlled swap gate from middlemost qubits as per 1-layer QGB. Now we have to reset controlled qubit and then apply Hadamard gate to controlled qubit. Now our ball can be on two state one left of our initial position or right of our initial position. Now we have to apply control-SWAP then CNOT then again controlled SWAP as per the 1-layer QGB at each position of particle. Only new we have apply again CNOT gate from last qubit we have apply controlled SWQAP gate to the controlled qubit to reset it. For n-layer we have to repeat this process again and again. We can get n-layer QGB.

Now for the biased QGB we have to replaced Hadamard gate by RX gate. Then apply the angle of rotation as per state we required. We can get biased distribution for the Quantum Galton Board