Risk Management: Assignment 7

- Submit through Canvas assignments
- You need to use R for these assignments.
- Assignment 7 (both parts, 7.1 and 7.2) are due on Nov 21^{st}

Assignment 7

Please do this step first for assignment 7

- Draw a random number generated using your gtid as a seed over the interval $(1980, 2010)$. List the seed and your random number at the top of the report.
- Select the corresponding year as the starting year and use (starting year + 6) as ending year for the data (that is say, if 1996 is the random number, use data from 1996-2002)
- Use DSF file for the start year (1996 in this example), create a unique list of PERMNOs for this year and randomly select 100 PERMNO (firms) from this year
- Note, you need to draw the random sample of 100 firms from the firms in existence in the starting year, 1996 in this example. Some of them may not survive till 2001 and you will have a shorter time series for some firms in that case.
- Use this 100 random firms and the random sample period for both assignments 7.1 and 7.2
- In the following steps, I mention (starting year, starting year+ 6 years) or as examples 1996-2002. But each of you should have a different sample based on the random year you drew as starting year based on your GT id and a random sample of 100 firms

Assignment 7.1: Value-at-Risk (VaR) and Expected Shortfall (ES)

As one of the well-known risk measures, value-at-risk (VaR) measures the potential loss from extreme negative returns. VaR is often defined in dollars, denoted by \$VaR, so that the \$VaR loss is implicitly defined from the probability of getting an even larger loss as in

$$\Pr(\$Loss > \$VaR) = p$$

In the case with returns, a VaR is defined as

$$\Pr\left(r > -VaR\right) = p.$$

Thus, the -VaR is defined as the number so that we would get a worse return only with probability p. That is, we are $(1-p)100\%$ confident that we will get a return better than -VaR.

Expected shortfall is an alternative risk measure that is more sensitive to the shape of the loss distribution. Expected shortfall is also called conditional value-at-risk (CVaR) or TailVaR, and defined as

$$ES = -E\left[r|r < -VaR\right]$$

1. Assume that you invest $1,000,000 invested in each of the 100 random stocks (PERMNO) that you generated. Using daily returns in CRSP DSF dataset from *starting year, starting year+ 6 years*, for example January 2001 to December 2007, compute one-day 5% VaR, $VaR, and expected shortfall of your portfolio. (*Hint.* Use a historical distribution of daily returns)

2. For the same portfolio, use daily returns from *January 2000 to December 2010* and compute one-day VaR, $VaR, and expected shortfall of your portfolio. Explain how historical distributions and the risk measures are different from the answers in Question 1.

Assignment 7.2: Volatility modeling

A simple RiskMetrics Model

JP Morgan's RiskMetrics variance model, also known as exponential smoother, considers the variance dynamic model as follows:

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)r_t^2 \tag{1}$$

That is, the RiskMetrics model's forecast for tomorrow's volatility can be seen as a weighted average of today's volatility and today's squared return. When estimating a parameter λ on a large number of assets, RiskMetrics found that the estimates were quite similar across assets, and they simply set $\lambda = 0.94$ for every asset for daily variance forecasting.

3. Compute variances (annualized) of the random sample of 100 firms (generated before and used in 7.1) using daily returns from *(starting year, starting year + 6 years)* and use

them as initial values for RiskMetrics model. That is, using equation (1) with $\lambda = 0.94$ and computed standard deviations as initial values, generate and plot time-series of variance (σ^2_{t+1}) for these 100 random firms from *January 2000 to December 2010.*

GARCH (Generalized Autoregressive Conditional Heteroskedasticity)

The GARCH(h,k) model, introduced by Bollerslev (1986), is of the form

$$\sigma^2_{t+1} = \omega + \sum_{i=0}^{h} \beta_i \sigma^2_{t-i} + \sigma^k_{j=1} \alpha_j r^2_{t-j}. \tag{2}$$

The simplest GARCH model of dynamic variance (GARCH(1,1)) can be written as

$$\sigma^2_{t+1} = \omega + \beta \sigma^2_t + \alpha r^2_t. \tag{3}$$

The RiskMetrics model can be seen as a special case of GARCH(1,1) if we set $\alpha = 1 - \lambda$, $\beta = \lambda$, and $\omega = 0$

4. Using daily returns of these random sample of 100 firms (generated before and used in 7.1) using daily returns from *(starting year, starting year + 6 years)*, estimate parameters α and β for these firms.

5. Using estimated parameters in Question 5 and sample variances for these random sample of 100 firms (generated before and used in 7.1) during *(starting year, starting year + 6 years)*, generate and plot time-series of variance (σ^2_{t+1}) for these stocks from *January 2000 to December 2010.* Compare the plot with the result of Question 3.