2-Dimensional Fluid Flow Analysis

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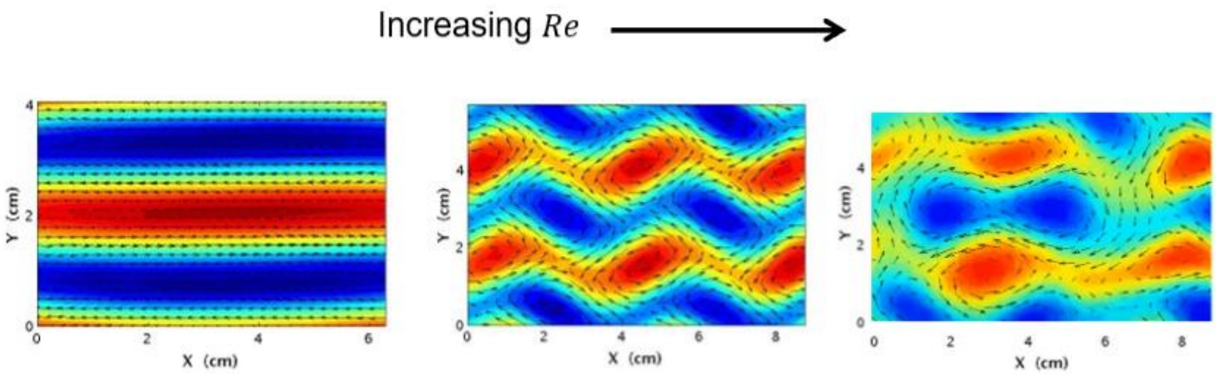
Abstract

The goal of this experiment was to use digital image analysis to study and quantify 2-dimensional fluid flow within an electromagnetic field. Fluid flow is highly unpredictable, so studying it on a quasi-2D plane makes everything much easier, from experimental setup to supporting calculations.

Introduction

Fluids, by definition, can flow and transfer energy. They also possess some degree of randomization. The instability of a fluid can be quantitatively measured through a Reynolds number, a unitless value determining if the fluid's motion is predictable and laminar, or turbulent and unstable. The Reynolds number compares the viscous forces of a fluid against the inertial forces applied to the material. This experiment helped prove that the Reynolds number is related to the velocity of the fluid. While many forms of 3-dimensional fluid flow exist, two-dimensional digital image correlation is used to analyze only surface characteristics of a system. The process involves taking a video, uploading it to a python-based software known as openPIV, and obtaining a vector field. Our system is made with an acrylic tray containing electrolytic fluid consisting of Copper Sulfate, Glycerol, and water, an array of neodymium magnets (of alternating magnetic poles) underneath, and with an applied voltage on either side. We hypothesize that the more voltage we add to this system, the higher the average fluid velocity will be, and therefore the higher the Reynolds number will be. At a certain Reynolds number, the flow stops moving predictably and becomes turbulent flow.



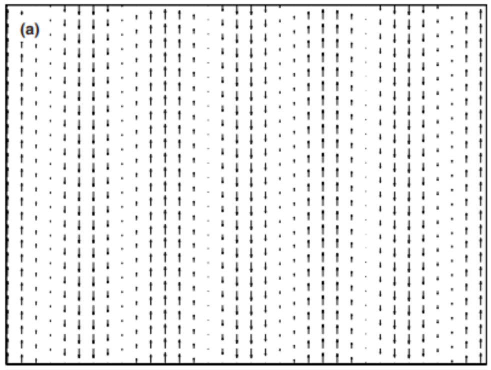
Theory

The objective of this lab is to use Digital Image Correlation to analyze the flow of an electrolytic solution with an applied current. To do this we need to understand fluid dynamics, and Digital Image Correlation. D.I.C. applies a correlation function to each frame of a video taken of the surface of a system. It then can reproduce a velocity vector field. With the measured velocity and characteristics of the fluid we can come up with a Reynolds number at that voltage. Reynold's number is a ratio of inertial forces to viscous forces. The higher the Reynolds number the more turbulent flow, lower Reynold's number relates to laminar flow. Our objective is to test at what Reynold's number can we see the fluid transition from laminar to turbulent flow. This question arose when looking at our hypothesis. We expect the Reynolds number to increase directly with voltage, we hope to see at what voltage range our flow becomes turbulent.

When calculating the average Reynolds number, four variables are considered, following the equation below:

$$Re = \rho V L / \mu$$

Where ρ is the density of the fluid, V is the average speed, L is the characteristic length of flow (in our experiment the length of the magnets), and μ is the dynamic viscosity of the material, which is constant for a uniformly mixed fluid. From our sources, we found our fluid has a density of 1.2 g/mL, a characteristic length of 10.5 cm, and a dynamic viscosity of 4.9 centiStokes. Thus, our average Reynolds number is linearly proportional to the average speed of the fluid. The velocity is not constant throughout our fluid is not constant, however. We have alternating bands of fluid flow, as shown in the diagram below.



OTypically this specific orientation follows a category of fluid dynamics called Kolomogorov flow, where our velocity is determined by the equation:

$$\boldsymbol{F} = F_0 \sin \frac{2\pi x}{L} \hat{z}.$$

Where F0 is the constant force on the fluid, and L/2 is the width of the flow band. The flow will be expected to have alternating maximum and minimum speeds in opposite directions, due to the magnets having alternating poles. Tying back to our Reynolds number, we cannot find a Reynolds number for the system using only one point. Instead, we will calculate the average speed by evaulating the speed of each surface particle via 2-dimensional visual analysis. Our program in python will evaluate the average speed of several groups of pixels in a .avi file, approximating the average speed of that portion of the fluid.

We may also approximate the Reynolds number of our system by using the equation:

$$\mathrm{Re} = \frac{CL^2}{\nu} \sqrt{\frac{NIB_0 e^{-z/\zeta}}{\rho V}},$$

Where Re is the Reynold's number, C is the correction factor (calculated at .42), L is the width of each magnet at .5 inches, v is the dynamic viscosity of our fluid calculated at 4.9 centistokes, V is the volume of our fluid, I is the current travelling through the volume, N is the number of Magnets used, B is the magnitude of the magnetic field of each magnet, z is the depth of the fluid, and d is half the length of the magnet, or width of the flow in view of the camera.

Experimental Setup

The first step in building our experimental setup was to buy the required materials. We used a 10”x10” acrylic tray, 8ft of ¾" dowel, 4 hose clamps, several 3d printed parts, 4 folders, zip ties, black spray-paint, ¼" wood sheet 12”x12”, acrylic plate 12”x6”, 12 strong neodymium magnets 6”x0.5” each, copper sheet, led lights, breadboards and wire leads, and several power supplies. Also, we needed distilled water, copper sulfate, and glycerol to make the working fluid, as well as glass microspheres to sprinkle onto the fluid.

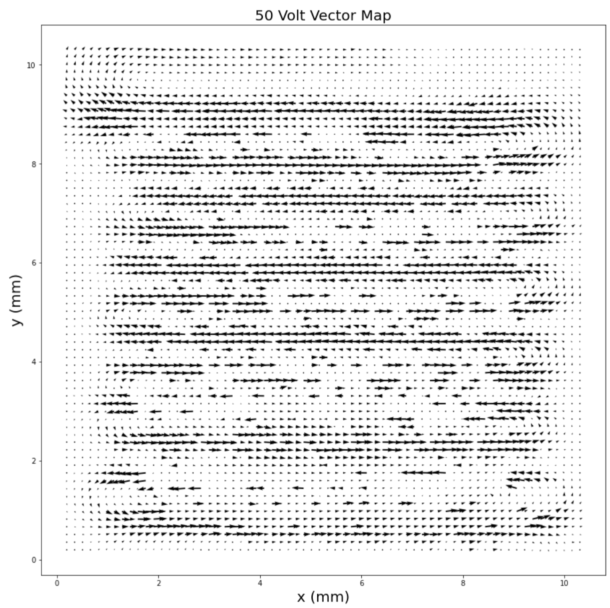
Many of the materials listed were obtained after initial testing without them, but if we had to do it again, we would have collected everything all at once to speed up the process. To calculate the magnetic field for each magnet we used the magnetometer on one of our smartphones to try and get a measured value. We were getting a reading of only a few hundred microTesla, which we believe was due to the orientation of our magnets. According to the manufacturer of the magnets, each magnet has a B-field of 1.3 Teslas, which is the value we used for all our calculations. Assembly of the experiment is straightforward. The tools we used were a mill to cut the depths of the magnets into the acrylic board, a drill press to cut holes in the wooden board, a hand saw to cut the dowel to length, a Dremel to clean up the 3d prints, and a crimping tool to add DuPont clips to the ends of the LED’s.

The wiring is as simple as plugging in the LED DuPont's into a breadboard and powering it with a power supply with DuPont's clipped into the alligator clips on the ends of the power supply leads. As for powering the copper electrodes in the working fluid, we just used two more alligator clips from another power supply clipped onto the acrylic tray. (Eventually during testing, we determined that one power supply did not supply enough voltage to observe turbulent fluid flow in the system, so we daisy-chained up to 4 together for around 110 volts, but for initial tests one sufficed.)

After the experiment was set up, we created our working fluid using 100 ml of distilled water, 28 grams of copper sulfate, and 190 ml of glycerol. Next, we sprinkled glass microspheres onto the surface, and set up the phone to record the results. In order to ensure the video framing was consistent, we traced the outline of the phone being used with a pencil. The videos were taken in 5- or 10-volt increments and were each about 2 seconds long. To give the fluid time to adjust to the next voltage, we gave the fluid about 30 seconds in between voltage adjustments. The process of setting up the video was to first crop the raw video file to a square in the phone gallery app, then convert it to an .avi file with a free video conversion app, within this app renaming the video file to the iteration of that voltage, then the voltage itself, then the amperage (eg: “(2)50v,0.356a”), and finally uploading the video to Microsoft teams so everyone in the group could download and analyze the video themselves. After downloading the video, you must first copy the file name. Then in the OpenPIV code, paste the file name into the required field as well as the voltage and run the code. The code would then create two histograms pertaining to the horizontal and vertical particle quantities that were detected. These histograms allowed us to fine tune the parameters of the code to get cleaner looking vector maps. Usually, only minor adjustment to the U-range and V-range values are required to clean up the maps and resulting velocity calculations.

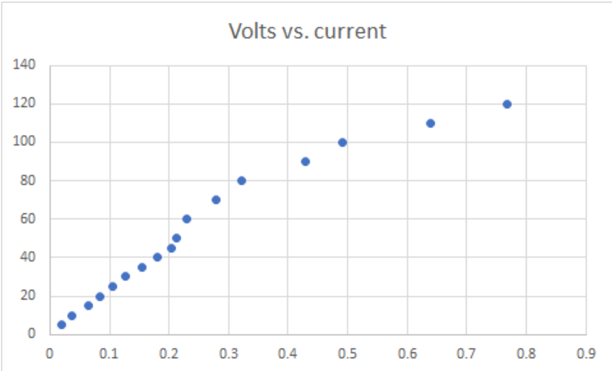
Data Analysis and Results

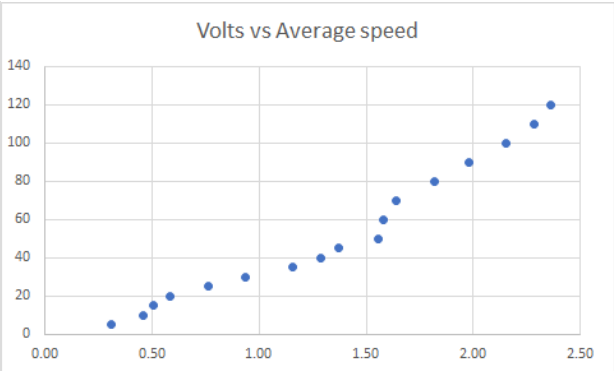
Our setup was run for working voltages of 5V to 60V in 5V increments and 60-120V in 10V increments. The viscosity and density of our fluid were given to us from our references and are constant throughout our experiment. The velocity field was calculated using OpenPIV software, which takes the difference between positions of identified particles per unit of time to find approximate velocity values of the particles. A velocity map was generated for each voltage value tested with this system. An example is below:



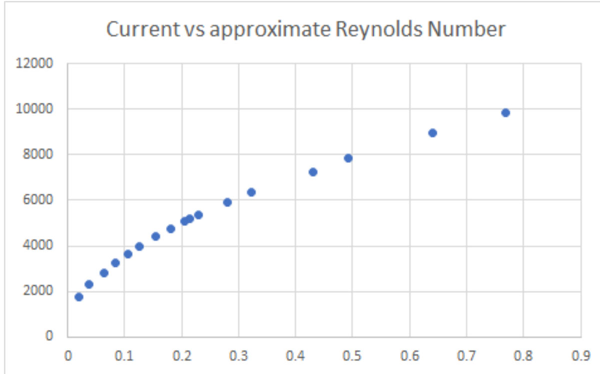
The program filters out velocities outside the expected range of the voltage. We often identified clumping of particles as a cause of inaccuracies in the data, as the program identifies a clump as many particles moving synonymously in one direction, shifting the average velocity of the portion dramatically. Once we obtained velocity maps of each voltage (and an average speed of the fluid), we compared the current, approximated Reynolds number, average speed, and average Reynolds number.

Our results were placed in the charts below:



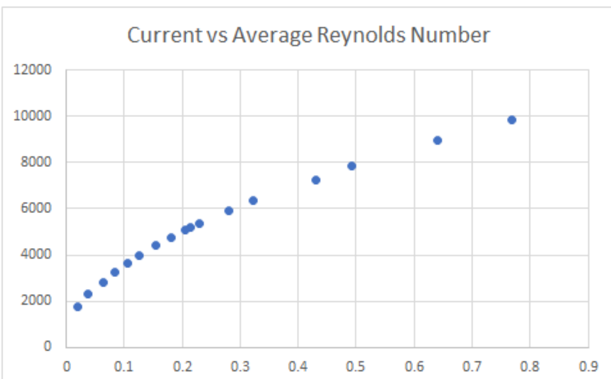


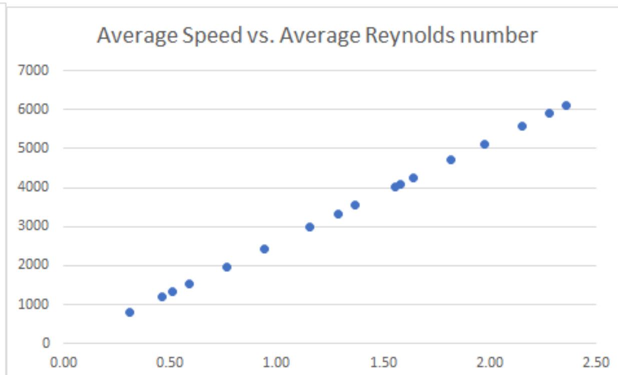
These charts identified the relation between the average speed of the particles, the voltage, and the Reynolds number. Just as we predicted, the particles had their average speed increase linearly with the increase in voltage, and that there was a direct relationship between the magnitude of the average speed and the Reynolds number. The results are below.



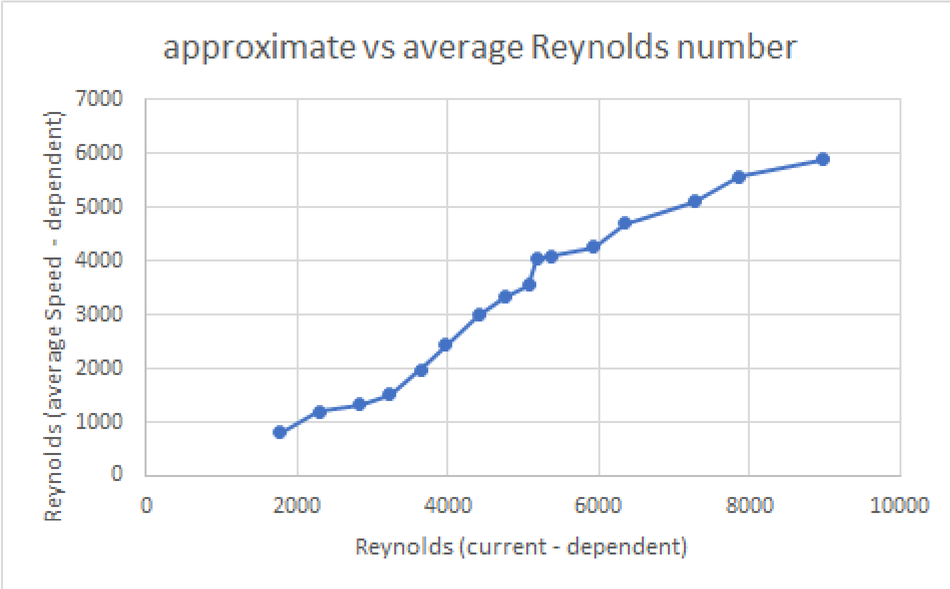
Comparing the current and approximated Reynolds number of the system using current as our independent variable, we found that the Reynolds number is related to the square root of the current.

Using the average speed and current, we found the following trends:





The results match our expectations that the average Reynolds number is related to the square root of the applied current. Finally, to assure that our two approaches to calculating the Reynolds number aligns, we compared the numerical values of each reynolds number below:



Our final set of data is posted below:

Voltage	Current	Avg speed	Reyn(I)	Avg. Reyn
5	0.02	0.31	1772.997778	799.29620
10	0.037	0.46	2298.067772	1190.7934
15	0.064	0.51	2836.796445	1321.7529
20	0.083	0.59	3230.555699	1517.6851
25	0.106	0.77	3650.825873	1981.2133
30	0.126	0.94	3980.37014	2433.2702
35	0.155	1.16	4414.728866	2990.3315
40	0.18	1.29	4757.452268	3339.7789
45	0.205	1.37	5077.092987	3549.7561
50	0.213	1.56	5175.210137	4029.9871
60	0.23	1.58	5377.768526	4087.9491
70	0.28	1.64	5933.585473	4251.2156
80	0.322	1.82	6363.061531	4704.0348
90	0.43	1.98	7267.128377	5122.6244
100	0.492	2.15	7865.398635	5571.0907
110	0.64	2.28	8970.738026	5903.2073
120	0.768	2.36	9826.951148	6103.8844

Conclusion

In conclusion, our group found that the Reynolds number of our electrolytic Copper sulfate solution is linearly related to the average speed and proportional to the current of the system. While there are some discrepancies in our data from 45V data to our 60V data points, we believe these indicate the presence of a transition phase of our fluid as it approaches turbulent flow. Our data further suggests that we may find even more chaotic flow at higher voltages, though we advise not testing these without stricter safety measures. The data implies that both approaches to approximating the Reynolds number of the system are valid and are related to each other by some coefficient. To generalize, the Reynolds number of 2-dimensional fluid flow of an electrolyte in a magnetic field may be calculated using either the current applied to the system or the average speed of the fluid.

Bibliography

[0] Kelley, Douglas H., and Nicholas T. Ouellette. “Using Particle Tracking to Measure Flow Instabilities in an Undergraduate Laboratory Experiment.” *American Journal of Physics*, vol. 79, no. 3, 2011, pp. 267–273., https://doi.org/10.1119/1.3536647.

[1] Purcell, E. M. “Life at Low Reynolds Number.” *AIP Conference Proceedings*, 1976, https://doi.org/10.1063/1.30370.