An Optical Demonstration of Surface Plasmons on Gold

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Citation: American Journal of Physics **42**, 1123 (1974); doi: 10.1119/1.1987955
View online: http://dx.doi.org/10.1119/1.1987955
View Table of Contents: http://aapt.scitation.org/toc/ajp/42/12
Published by the American Association of Physics Teachers

that elementary students will find one of Humphrey's "simple" methods more transparent.

Some insight into these algorithms may be found by expressing the final acceleration in each case as a linear combination of the displacement data points, and comparing the spectrum of the resulting coefficients with that for the least-squares solution[8] (see Fig. 1).

There are two arguments by which one might prefer Humphrey's original method ($k=\frac{1}{4}n$) to the modified method ($k=\frac{1}{6}n$), in spite of the 9% greater probable error. First, it is intuitively more satisfying to "use all the data"—indeed, most students in the elementary lab will have a hard time understanding why they should throw some data away. Second, since a larger statistical sample of independent values of acceleration is generated ($\frac{1}{4}n$ rather than $\frac{1}{6}n$), a more reliable *a posteriori* estimate of the probable error of the mean acceleration is obtained from the mean deviation of the individual values.

I wish to thank N. M. Foote for calling my attention to Edward's paper and for helpful correspondence.[9]

[1] C. L. Humphrey, Am. J. Phys. **41**, 965 (1973).
[2] R. L. Edwards, Am. Phys. Teacher **1**, 36 (1933).
[3] Humphrey's ascription of this method to Millikan *et al.*, is incorrect. The algorithm given in the cited reference is actually Edwards's, with some points discarded.
[4] P. G. Guest, Am. J. Phys. **18**, 324 (1950).
[5] The numerical coefficients of Eqs. (3)–(4) and (7)–(8) assume that the respective submultiples of n are integers and furthermore that the first two denominator factors in Eqs. (2) and (6) are also integers. For typical n-values, say of order ten, this assumption will not normally be true, and the actual standard errors will be somewhat larger. The optimum k-values are those integers closest to the nominal submultiples such that the denominator factors are integers.
[6] M. A. Heald, Am. J. Phys. **37**, 655 (1969), Secs. V and VII.
[7] P. Rudnick, Am. Phys. Teach. **4**, 217 (1936).
[8] E. M. Pugh, Am. Phys. Teach. **4**, 70 (1936); G. J. Cox and M. C. Matuschak, J. Phys. Chem. **45**, 362 (1941). The least-squares coefficients are proportional to $6i^2-6(n+1)i+(n+1)(n+2)$ for $i=1,2,\ldots,n$.
[9] N. M. Foote, Am. J. Phys. **39**, 1122 (1971).

An Optical Demonstration of Surface Plasmons on Gold

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(Received 15 January 1974; revised 4 March 1974)

We have developed a simple but exceedingly effective demonstration of the existence of absorption by surface plasmons at a gold–air interface. Since this absorption occurs near 5100 Å in the visible, it is easily demonstrated visually and a magnified image can be produced using a viewgraph projector. Th demonstration uses a prism coupler which provides an evanescent optical field which probes the metal surface. The understanding of the coupler, therefore, assumes some knowledge of, and illustrates the phenomenon of total internal reflection and the associated evanescent fields which exist near the surface where total reflection is taking place.[1,2] The demonstration has been used successfully with senior undergraduate students and with graduate students who have taken an introductory course in solid state physics.

The discussion of both bulk and surface plasmons is best carried out using the dielectric constant of the metal, $\epsilon_m(\omega)$. Bulk plasmons[3] occur at frequencies which are the solutions of $\epsilon_m(\omega)=0$. For most simple metals there is just one solution to this equation yielding a bulk plasma frequency in the visible or ultraviolet frequency region. When plasmon-like waves are considered at a metal–air interface, the resonance condition $\epsilon_m(\omega)=-1$ is obtained.[4,5] This equation results from setting up fields in the metal and in the adjacent medium (in this case air with $\epsilon=1$) and matching the fields in the usual way at the interface. The above equation is an approximation which holds when retardation is neglected.[5] When the full Maxwell equations are considered so that retardation is included, setting up the fields and matching at the surface gives the dispersion relation[4,5] for the surface plasmon frequency shown in Fig. 1.

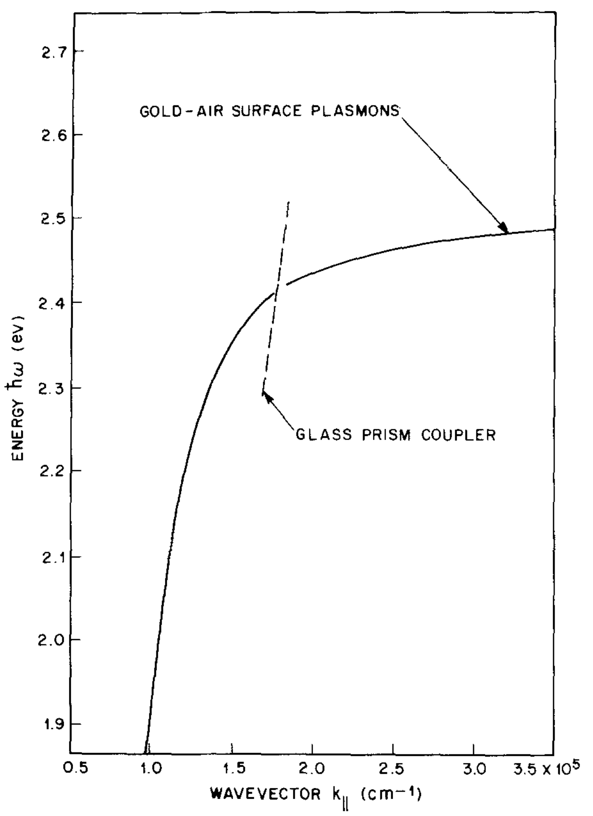

FIG. 1. The dispersion of surface plasmons at a gold–air interface. The simple resonance condition $\epsilon_m = -1$ holds at large $k_{||}$ where the dispersion curve becomes flat and independent of $k_{||}$. The glass coupling prism described in the text projects evenescent fields which follow the dashed curve. Coupling occurs at the crossing of the two curves giving absorption near 2.41 eV (0.51 μ wavelength).

Figure 2 shows the elements of the surface plasmon demonstrator. P is a 90° glass prism[6] which is placed against a flat substrate[6] S which has had a gold coating evaporated onto it. The gold coating must be more than 500 Å thick and may be deposited with almost any general purpose vacuum evaporator. The gold coated flat must be kept reasonably free from dust before assembly. The prism base is placed directly against the gold and the entire assembly is squeezed by the pressure bar and nuts illustrated in the figure. While P and S may appear to be in contact, in fact, natural dust particles will always supply a separation greater than the minimum spacing needed for good coupling of a light beam to the surface plasmons on the gold. This is the reason for the pressure bar and the screws. Light is observed through the prism as illustrated in Fig. 2(b). The light source may be the base plate of an overhead projector or simply a lighted window or distant light colored wall. As long as light reaching the eye travels parallel to the base of the prism, it must undergo one total internal reflection from the base as illustrated in the figure. If light from a white background is being observed,

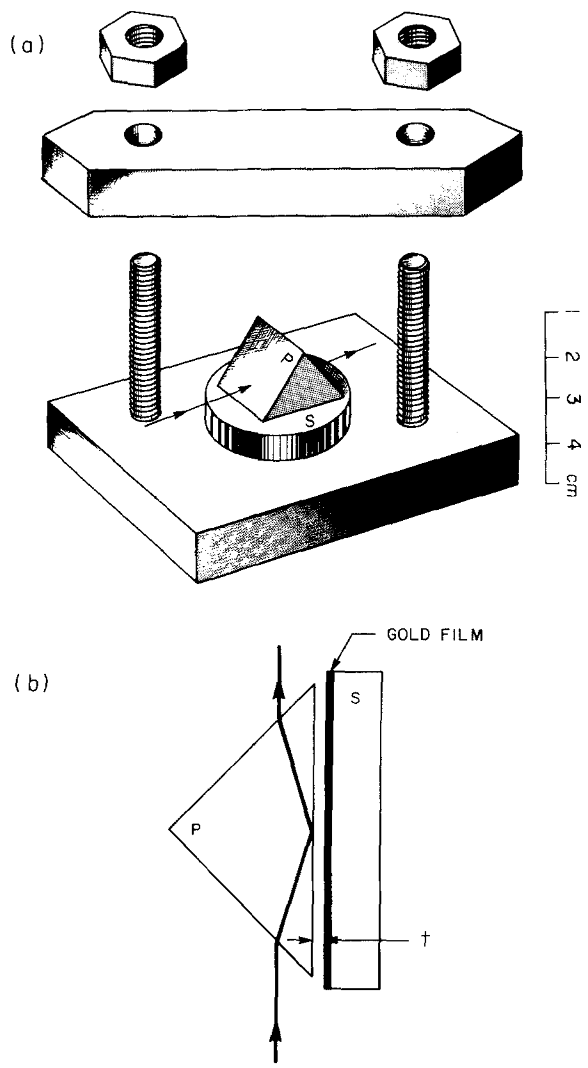

FIG. 2. (a) Surface plasmon demonstrator. The prism P rests on the gold covered substrate S. A metal pressure bar and the two nuts squeeze P and S together to obtain the close coupling distance t which is needed. (b) Schematic of a light beam traversing the coupling prism. The space t contains the evanescent field which probes the gold surface.

then only a white color is perceived. If the screws are now tightened, a pink color begins to appear in some regions of the white field. This is a surface plasmon absorption. Since the coupling occurs near 5100 Å and removes most of the yellow-blue region of the spectrum, an unusual pink color is observed. As the screws are tightened further, the coupling increases and a stronger pink color appears.[7] Figure 3 shows actual absorption spectra calculated for three different coupling distances for the demonstrator.[8] Because there is considerable bending of the prism and the substrate, in fact, central regions of the observed field will correspond to the strongest absorption spectrum and the edges may correspond to a spectrum with considerably larger spacing. To avoid too much bending and thus too great a range in prism-gold spacing we have found it necessary to use a glass substrate (S) at least $\frac{1}{4}$-in. thick and other parts of the size indicated by the scale in Fig. 2. A prism with 90° apex angle and index of refraction 1.52 provides a coupler which probes the dispersion curve along the dashed line shown in Fig. 1. Use of a prism with smaller apex angle or larger index of refraction probes a region of larger $k_{||}$ in Fig. 1 and conversely.

Two points are easily illustrated with the demonstrator set up as in Fig. 2. First a small object (pencil point, etc.) may be inserted between the prism and light source and viewed. The image will be reversed showing that the light reaching the eye (or the projector screen) has in fact undergone one reflection from the base of the prism thus creating an evanescent wave below the prism. Secondly, after tightening the nuts to obtain coupling, a polarizing sheet with the E-vector direction marked on it may also be inserted between the apparatus and the light source to polarize the light. Rotating the polarizer causes the pink absorption region to become stronger or to disappear altogether when the electric vector is parallel to the base of the prism. This demonstrates that the surface plasmon has its electric field perpendicular to the surface and it cannot be excited by a probing electric field parallel to the surface.[7]

The demonstrator should be kept assembled with sufficient pressure on the bar so that the components cannot slide out of place. While the parts must be assembled in a reasonably dust-free

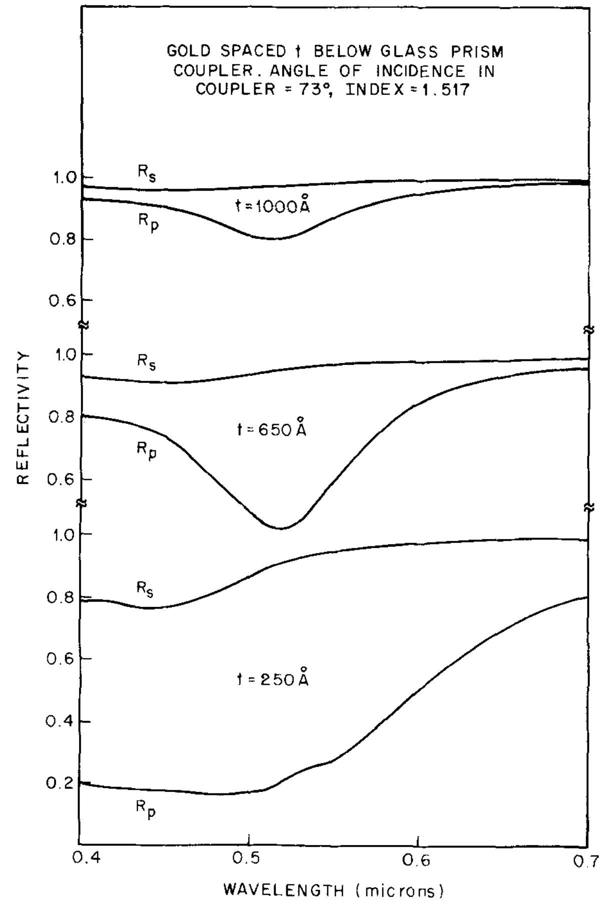

FIG. 3. Absorption spectra for three spacings t. These calculations show the reflection from the prism base for the electric vector parallel to the prism base (R_s) and parallel to the plane of incidence (R_p). With a polarizing sheet, R_p and R_s may be observed separately. For spacings of 1000 Å or 650 Å (light or moderate pressure on the bar of Fig. 2) a white source appears white for R_s but pink for R_p. For heavy pressure on the bar the overcoupling shown at the bottom occurs. The color familiar from normal reflectivity of gold is now observed weakly in R_s and strongly in R_p since the evanescent field no longer exists in any appreciable strength to probe the surface plasmon.

environment, once they are assembled the accumulation of dust on the prism faces and other surfaces does not matter and the entire assembly needs no special precautions as to handling or storing.

As can be seen, the demonstrator may be used to illustrate several points concerning surface plasmons or evanescent fields connected with total internal reflection. Gold has been chosen as the active medium simply to place the phenomenon in the visible range so that the eye serves as a spectrometer in studying the wavelength dependence of the phenomenon. The apparatus is robust enough so that it may be passed around giving students a "hands-on" feeling for the pressure required to bring the prism coupler sufficiently close to the gold to cause surface plasmon absorption.

The author acknowledges a very useful discussion with Professor A. J. Sievers and the capable assistance of J. A. Ditzenberger in constructing several prism coupler assemblies.

[1] F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill, New York, 1957), 3rd ed., Chap. 25.

[2] W J. McDonald, S. N. Udey and P. Hickson, Am. J. Phys. **39,** 1141 (1971).

[3] C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1971), 4th ed., Chap. 8.

[4] A. Otto, Z. Phys. **216,** 398 (1968); N. Marschall and B. Fischer, Phys. Rev. Lett. **28,** 811 (1972); A. S. Barker, Jr., Phys. Rev. Lett. **28,** 892 (1972).

[5] Surf. Sci. **34,** 1 (1973); see review articles beginning on p. 1 of issue 1.

[6] These parts may be obtained from Edmund Scientific Co. or from suppliers of laser components.

[7] For very large forces on the prism the glass–gold spacing becomes too small to support an evanescent field. Coupling is now directly to the bulk gold excitations and the characteristic color of gold appears in place of the pink color. This gold color is observed for both polarizations of the light.

[8] A. S. Barker, Jr., Phys. Rev. A **8,** 5418 (1973), see Sec. II.

Ampère's Law in Symmetric Cases with Non-Steady-State Currents

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(Received 7 June 1973; revised 26 September 1973)

In a standard calculus-based introductory physics course, Ampère's circuital law and the Biot–Savart law are discussed as methods of finding the magnetic field due to a current distribution. The simplest example which has "sufficient symmetry" so that Ampère's law can be used is the infinitely-long straight wire carrying a current i. This yields for the field at a distance R from the wire the usual result

$$B=(\mu_0 i/2\pi R).$$

A common example for which the Biot–Savart law is used is the finite straight wire segment of length L carrying a current i. Here one is asked to find B at a distance R from the segment along a perpendicular bisector (see Fig. 1). It is true that the usual Ampère's law fails in this case, but it is not because of insufficient symmetry. In fact, in going from the infinite case to the finite one, none of the symmetry is lost and the contour integral can be evaluated as before. Of course, the answers are not the same in both cases. Why, then, can Ampère's law not be used for the finite wire?

The reason is that Ampère's law, in its usual form, is valid only for steady-state currents which satisfy $\nabla\cdot\mathbf{J}=0$. In our problem

$$\nabla\cdot\mathbf{J}=-(\partial\rho/\partial t)\neq 0 \quad \text{at} \quad (\pm L/2, 0).$$

The general form of Ampère's law can be obtained by repeating Jackson's derivation[1] without discarding the $\nabla\cdot\mathbf{J}$ term. This yields[2]

$$\oint_C \mathbf{B}\cdot d\mathbf{l}=\mu_0 i_{\text{enc}}+\frac{\mu_0}{4\pi}\int_S\left[\nabla\int\frac{\nabla'\cdot J(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,d^3x'\cdot d\mathbf{S}\right]$$
$$\equiv \mu_0 i_{\text{enc}}+X,$$

where S is any surface bounded by C. The last term, of course, becomes zero for steady-state currents giving the usual Ampère's law. We choose for C the usual circle, center 0 and radius R, and for S the circular disc contained by C.

The extra term becomes

$$X=\frac{\mu_0 i}{4\pi}\int\nabla\left\{\frac{1}{|\mathbf{x}-\mathbf{x}_1|}-\frac{1}{|\mathbf{x}-\mathbf{x}_2|}\right\}\cdot d\mathbf{S}$$