Explore the magnetic field and application of Halbach array magnets

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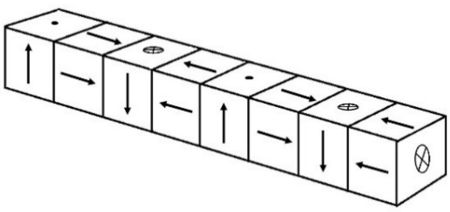
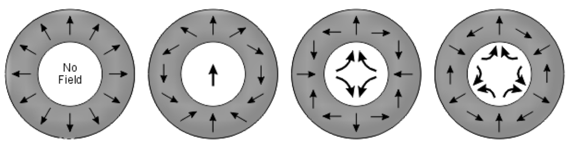
Abstract: this paper discusses the particularity of Halbach magnetic field, the composed of n magnet ring Halbach of the structure of the magnetic field are calculated, the magnetic field distribution are analyzed, and the computer simulation and experiment, the magnetic field distribution of credible data, intuitive embodies the characteristics of the magnetic field distribution, and the application are discussed in this paper.

Key words: magnetic ring; Magnetic field; Concentrated; simulation

A Halbach array is a magnet structure designed to produce a relatively concentrated and strong magnetic field with fewer magnets. That is to say, the magnet magnetized ability itself has certain case, through the study of the arrangement of multiple magnet configuration (for example, we have eight magnets, we can let them toward the north and south poles, the same phase in turn have together, also can arrange them like figure 1, the formation of Halbach array configuration mode), can be very good in some areas to produce strong magnetic field while some regional magnetic is very small. Because it concentrates the magnetic field well and requires fewer magnets, this structure is necessary when the element desired to provide the magnetic field is not large. For example, refrigerator magnets and so on, we want magnets not too much volume, but at the same time can ensure that there is a certain amount of suction. Of course, the Halbach array can also be arranged in a ring (Figure 2). As shown in Figure 2, 12 magnets are arranged in a ring in different ways. As shown in Figure 2(a), the North Pole of all magnets points to the radial direction, so there is no magnetic field inside them. This paper mainly discusses this "magnetic ring" (as Halbach's ring array is called).



Graph 1（线性）

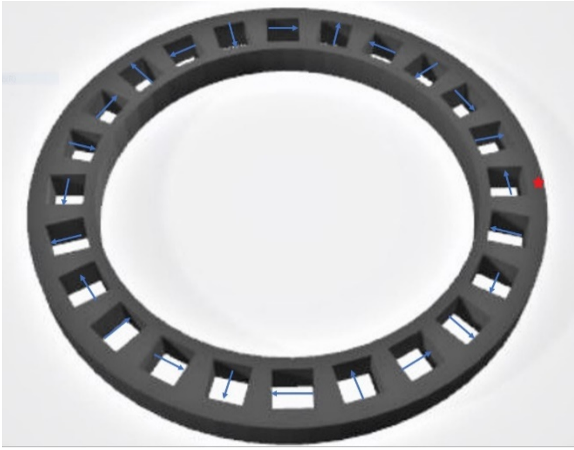


(a) (b) (c) (d)

Graph2

1 theory

The model of the "magnetic ring" in this paper is a Halbach array composed of n square permanent magnets. The conceptual diagram is as follows, n=24 as shown in Figure 3. The empties in the annular object can be placed magnets, we know that the magnet poles is the direction of the magnet internal magnetization vector, then for rectangular magnets in the empty can have six orientations, so there will be many possible arrangements in the magnet ring array. what kind of arrangement will have the best concentration of magnetic field? The Halbach array in Figure 3 can achieve this purpose. The blue arrows in the figure represent the direction of the magnetization vector of each magnet. Let's start with the magnet with the red asterisk. The direction of polarization of this magnet is left-handed tangential. Look at the next magnet counterclockwise, and its magnetization direction can be seen as the magnetic intensity vector attached to the iron (the whole vector) first revolves clockwise around the center of the circle $\frac{2\pi}{24}$ degree, turn to the next magnet and rotate 90 degrees along the needle to form the radial outward direction shown in the figure. In the same way, the next magnet in the counterclockwise direction is placed in the magnetization direction.



Grape 3（conceptual diagram, as n=24 for example ）

1 .1 magnetic field in one single magnet

The permanent magnet is regarded as uniformly magnetized, with the same internal M everywhere and side length a. The model type is a side-by-side square current, as shown in Fig. 4. First, calculate the space magnetic field of a square current loop, as shown in Fig. 5:

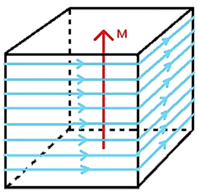


图 4

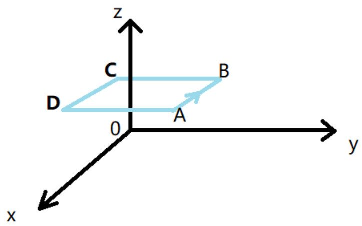


图 5

Magnet field originated from AB electric current in the point (x', y', z') is:

$$\vec{B} = \int_{\frac{a}{2}}^{-\frac{a}{2}} \frac{u_0 I(-dx,0,0)\times(x'-x, y-\frac{a}{2}, z'-z)}{4\pi[(x'-x)^2+\left(y'-\frac{a}{2}\right)^2+(z'-z)^2]^{\frac{3}{2}}} \tag{1}$$

integral:

$$B_{x1} = 0 \tag{2}$$

$$B_{y1} = \frac{u_0 I(z'-z)}{4\pi t^2}\left[\frac{-x'-\frac{a}{2}}{\sqrt{\left(x'+\frac{a}{2}\right)^2+t^2}} - \frac{\frac{a}{2}-x'}{\sqrt{(x'-\frac{a}{2})^2+t^2}}\right] \tag{3}$$

$$B_{z1} = \frac{u_0 I(\frac{a}{2}-y\prime)}{4\pi t^2}[\frac{-x'-\frac{a}{2}}{\sqrt{\left(x'+\frac{a}{2}\right)^2+t^2}} - \frac{\frac{a}{2}-x\prime}{\sqrt{(x'-\frac{a}{2})^2+t^2}}] \tag{4}$$

With $t^2 = (y' - \frac{a}{2})^2 + (z' - z)^2$

In the same way, we can get magnet field originated from BC electric current (B_{x2},B_{y2},B_{z2}), CD:(B_{x3},B_{y3},B_{z3}), DA:(B_{x4},B_{y4},B_{z4})

For side-by-side currents,

$$B_x = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{(B_{x2}+B_{x4})dz}{a} \tag{5}$$

$$B_y = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{(B_{y1}+B_{y3})dz}{a} \tag{6}$$

$$B_z = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{(B_{z1}+B_{z2}+B_{z3}+B_{z4})dz}{a} \tag{7}$$

integral,

$$\begin{aligned} B_x = -\frac{u_0 I}{2a\pi}(\Big(&\left(y'+\frac{a}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{\left(x'+\frac{a}{2}\right)^2+\left(y'+\frac{a}{2}\right)^2}-\sqrt{(z-z')^2+\left(x'+\frac{a}{2}\right)^2+\left(y'+\frac{a}{2}\right)^2}}{\left|x'+\frac{a}{2}\right|-\sqrt{(z-z')^2+\left(x'+\frac{a}{2}\right)^2}}\right) \\ &+\left(y'-\frac{a}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{\left(x'+\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2}-\sqrt{(z-z')^2+\left(x'+\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2}}{\left|x'+\frac{a}{2}\right|-\sqrt{(z-z')^2+\left(x'+\frac{a}{2}\right)^2}}\right) \\ &+\left(y'+\frac{a}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{\left(y'+\frac{a}{2}\right)^2+\left(x'-\frac{a}{2}\right)^2}-\sqrt{(z-z')^2+\left(y'+\frac{a}{2}\right)^2+\left(x'-\frac{a}{2}\right)^2}}{\left|x'-\frac{a}{2}\right|-\sqrt{(z-z')^2+\left(x'-\frac{a}{2}\right)^2}}\right) \\ &+(y'-\frac{a}{2})\operatorname{atanh}\left(\frac{\sqrt{(y'-\frac{a}{2})^2+(x'-\frac{a}{2})^2}-\sqrt{(z-z\prime)^2+(y'-\frac{a}{2})^2+(x'-\frac{a}{2})^2}}{\left|x'-\frac{a}{2}\right|-\sqrt{(z-z')^2+(x'-\frac{a}{2})^2}}\right))\Big|_{-\frac{a}{2}}^{\frac{a}{2}} \end{aligned} \tag{8}$$

$$\begin{aligned} B_y = -\frac{u_0 I}{2a\pi}(\Big(&\left(-x'-\frac{a}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{\left(x'+\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2}-\sqrt{(z-z')^2+\left(x'+\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2}}{\left|y'-\frac{a}{2}\right|-\sqrt{(z-z')^2+\left(y'-\frac{a}{2}\right)^2}}\right) \\ &+\left(x'-\frac{a}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{\left(x'-\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2}-\sqrt{(z-z')^2+\left(x'-\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2}}{\left|y'-\frac{a}{2}\right|-\sqrt{(z-z')^2+\left(y'-\frac{a}{2}\right)^2}}\right) \\ &+\left(-x'-\frac{a}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{\left(y'+\frac{a}{2}\right)^2+\left(x'+\frac{a}{2}\right)^2}-\sqrt{(z-z')^2+\left(y'+\frac{a}{2}\right)^2+\left(x'+\frac{a}{2}\right)^2}}{\left|y'+\frac{a}{2}\right|-\sqrt{(z-z')^2+\left(y'+\frac{a}{2}\right)^2}}\right) \\ &+(x'-\frac{a}{2})\operatorname{atanh}\left(\frac{\sqrt{(y'+\frac{a}{2})^2+(x'-\frac{a}{2})^2}-\sqrt{(z-z\prime)^2+(y'+\frac{a}{2})^2+(x'-\frac{a}{2})^2}}{\left|y'+\frac{a}{2}\right|-\sqrt{(z-z')^2+(y'+\frac{a}{2})^2}}\right))\Big|_{-\frac{a}{2}}^{\frac{a}{2}} \end{aligned} \tag{9}$$

For magnetic field in Z axis, we consider the integral below firstly:

$$\int \frac{du}{\sqrt{c^2+u^2}(u^2+b^2)}$$

get,

$$\sqrt{\frac{1}{(c^2-b^2)b^2}}\arctan\left(\frac{u}{\sqrt{u^2+c^2}}\sqrt{\frac{c^2-b^2}{b^2}}\right)$$

then， we get magnetic field in Z axis,

$$B_z=\frac{u_0 I}{4\pi}\Bigg[\left(y'-\frac{a}{2}\right)\left(x'+\frac{a}{2}\right)\sqrt{\frac{1}{({c_1}^2-{b_1}^2){b_1}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_1}^2}}\sqrt{\frac{{c_1}^2-{b_1}^2}{{b_1}^2}}\right)+\left(y'-\frac{a}{2}\right)\left(-x'+\frac{a}{2}\right)\sqrt{\frac{1}{({c_2}^2-{b_1}^2){b_1}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_2}^2}}\sqrt{\frac{{c_2}^2-{b_1}^2}{{b_1}^2}}\right)+\left(y'+\frac{a}{2}\right)\left(-x'-\frac{a}{2}\right)\sqrt{\frac{1}{({c_3}^2-{b_2}^2){b_2}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_3}^2}}\sqrt{\frac{{c_3}^2-{b_2}^2}{{b_2}^2}}\right)+\left(y'+\frac{a}{2}\right)\left(x'-\frac{a}{2}\right)\sqrt{\frac{1}{({c_4}^2-{b_2}^2){b_2}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_4}^2}}\sqrt{\frac{{c_4}^2-{b_2}^2}{{b_2}^2}}\right)+\left(y'+\frac{a}{2}\right)\left(-x'+\frac{a}{2}\right)\sqrt{\frac{1}{({c_4}^2-{b_3}^2){b_3}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_4}^2}}\sqrt{\frac{{c_4}^2-{b_3}^2}{{b_3}^2}}\right)+\left(y'-\frac{a}{2}\right)\left(x'-\frac{a}{2}\right)\sqrt{\frac{1}{({c_2}^2-{b_3}^2){b_3}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_2}^2}}\sqrt{\frac{{c_2}^2-{b_3}^2}{{b_3}^2}}\right)+\left(-y'-\frac{a}{2}\right)\left(x'+\frac{a}{2}\right)\sqrt{\frac{1}{({c_3}^2-{b_4}^2){b_4}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_3}^2}}\sqrt{\frac{{c_3}^2-{b_4}^2}{{b_4}^2}}\right)+\left(y'-\frac{a}{2}\right)\left(x'+\frac{a}{2}\right)\sqrt{\frac{1}{({c_1}^2-{b_4}^2){b_4}^2}}\arctan\left(\frac{z-z'}{\sqrt{(z-z')^2+{c_1}^2}}\sqrt{\frac{{c_1}^2-{b_4}^2}{{b_4}^2}}\right)\Bigg]\Bigg|_{-\frac{a}{2}}^{\frac{a}{2}} \quad (10)$$

with,

$$c_1^2=\left(x'+\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2;\ b_1^2=\left(y'-\frac{a}{2}\right)^2$$

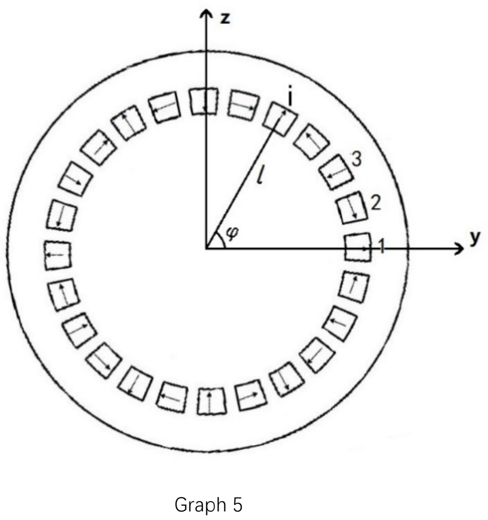
$$c_2^2=\left(x'-\frac{a}{2}\right)^2+\left(y'-\frac{a}{2}\right)^2;\ b_2^2=\left(y'+\frac{a}{2}\right)^2$$

$$c_3^2=\left(x'+\frac{a}{2}\right)^2+\left(y'+\frac{a}{2}\right)^2;\ b_3^2=\left(x'-\frac{a}{2}\right)^2$$

$$c_4^2=\left(x'-\frac{a}{2}\right)^2+\left(y'+\frac{a}{2}\right)^2;\ b_4^2=\left(x'+\frac{a}{2}\right)^2$$

1 .2 A superposition of magnet fields from n magnets

Back to the original Halbach array, the circular array baker center as the origin of coordinates, build coordinate frame as shown in figure, just have a magnet in the magnetic field distribution, for n magnets, by using the superposition principle, it is important to note that the magnetic field distribution of each magnet, to coordinate frame of now, should be multiplied by a rotation matrix and translation transformation.



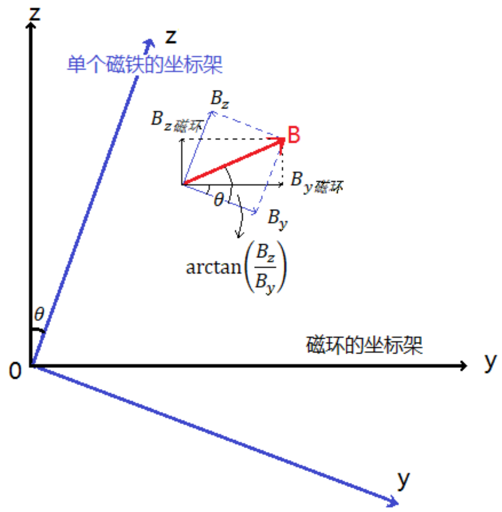
Graph 5

Assume that the magnet that intersects the positive direction of the Y-axis is number 1, and the number is arranged anti-clockwise 1~4n（n is Positive integer), $\varphi=(i-1)\frac{2\pi}{n}$, then at this coordinate point (x, y, z) corresponds to the coordinate frame of the original No. I magnet:$(x_i,y_i,z_i)^T=\begin{pmatrix}1&0&0\\0&cos\theta&-sin\theta\\0&sin\theta&cos\theta\end{pmatrix}(x,y-lcos\varphi,z-lsin\varphi)^T$

(That is, if we want to find the magnetic field of point (x, y, z) in space, we need to substitute $(x_i,y_i,z_i)^T$ into the formula obtained above), where θ is the rotation Angle of No. 1 magnet to the current magnet, and the rotation Angle corresponding to the i magnet:

$$\theta_i=\frac{\pi}{2}-\varphi+\frac{\pi}{2}(i-1)=\frac{\pi}{2}+(i-1)\frac{5\pi}{12}$$

After the coordinate frame is changed, the magnetic field under the coordinates of the magnetic ring is:



Graph 6

$$B_{x磁环}=B_x$$

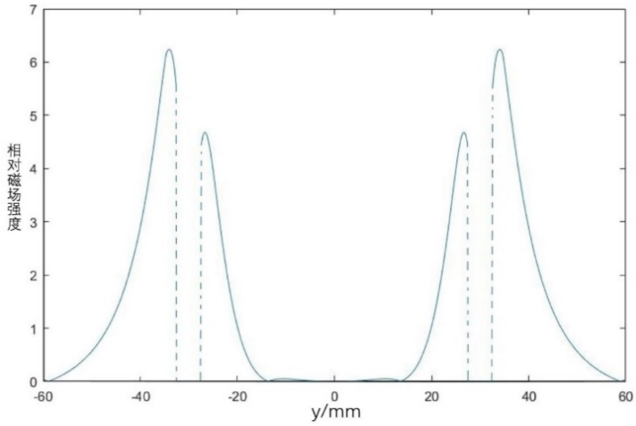
$$B_{y磁环}=\sqrt{B_y^2+B_z^2}\sin\left(\arctan\left(\frac{B_z}{B_y}\right)-\theta\right)$$

$$B_{z磁环}=\sqrt{B_y^2+B_z^2}\cos\left(\arctan\left(\frac{B_z}{B_y}\right)-\theta\right)$$

2 simulation

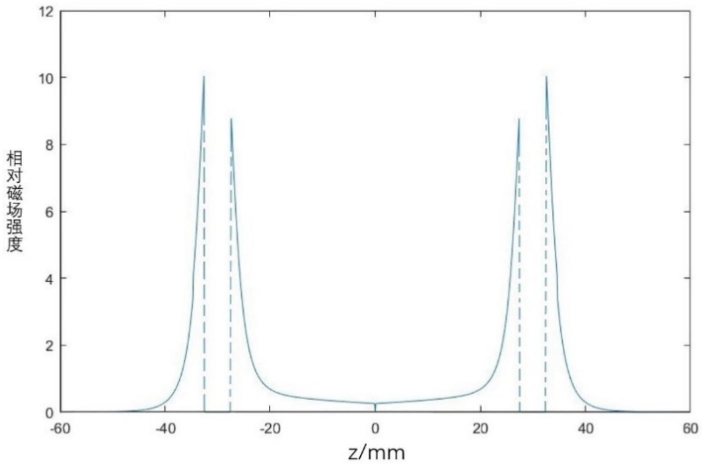
Let's find the numerical solution of the magnetic field at any point in space, again using 24 magnets as an example.

Then we can get: Relation diagram of magnetic field intensity B on y axis:



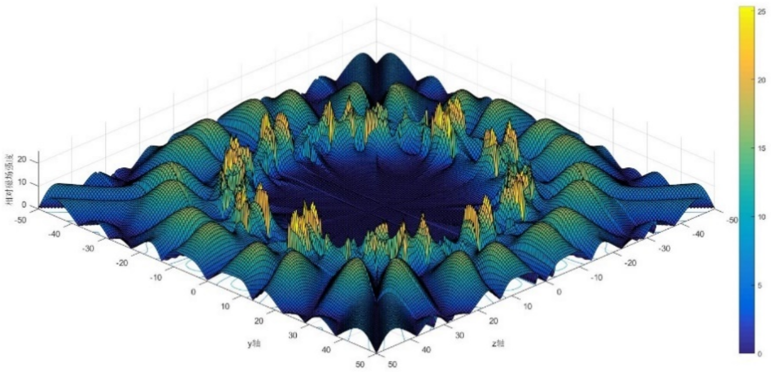
Graph 7

Relation diagram of magnetic field intensity B with respect to Z at Z axis:



Graph 8

When x=0, the magnetic field distribution in the YZ plane is:

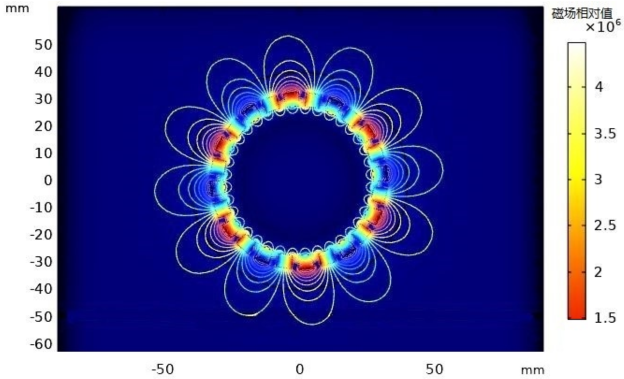


Graph 9

Note 1 All magnetic fields are relative.

Note 2: The magnetic fields in the figure do not include the magnet region (i.e. 27.5mm< R <32.4mm).

It is obvious that the magnetic field is well concentrated near the magnet.

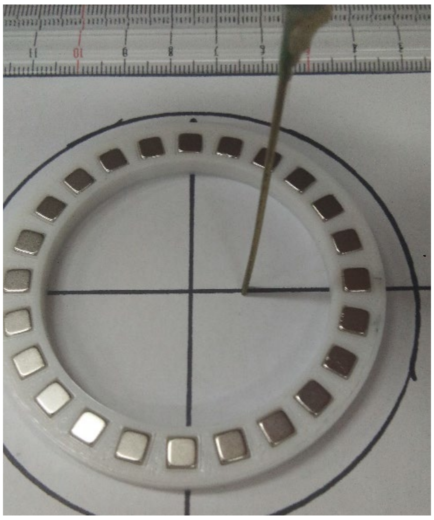


graph 10

Fig. 10 shows the distribution of magnetic induction lines and the concentration of magnetic flux distribution.

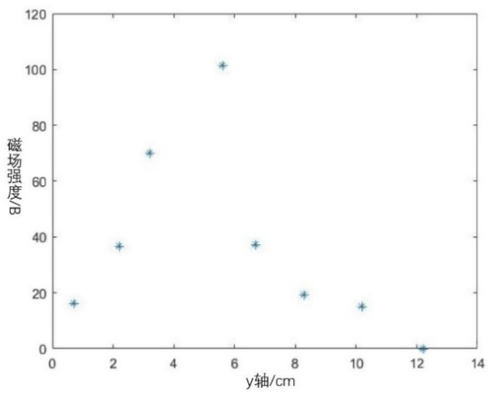
3. experiment

setup：24 5*5*5mm square magnet，circular frame by3D printing:

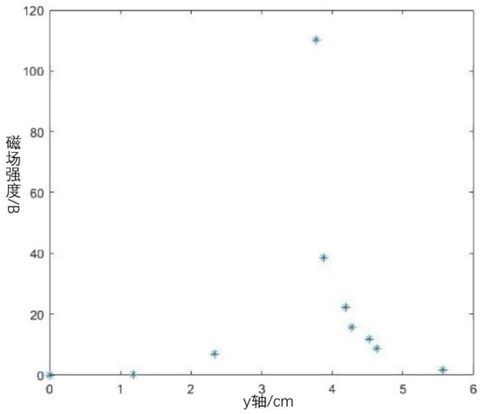


Graph 11

Measure magnetic field along the axis，plot as graph11：



Graph 12（y-B）



Graph 13（z-B）

It can be seen that, consistent with the theoretical curve, the magnetic field is concentrated near the magnet. When the distance from the magnetic ring is further and further away, the magnetic field decays rapidly. Inside the magnetic ring, the magnetic field is very small at the center of the magnetic ring and increases near the magnet.

4. conclusion

The magnetic field of the magnet can be calculated from Biosafar's law, and then the total magnetic field in space based on the special magnet arrangement can be calculated. Simulation and experiment show the magnetic field concentration of Helbeck array.

This kind of magnet array can be used in maglev train because of its good concentration of magnetic field.

When the magnetic ring has a rotating angular speed, put it on the conductor track, the conductor will generate inductive current, and then provide suspension force, and thrust, but its speed will be reduced, so just put a motor in the middle of the magnetic ring to keep it rotating, there can be continuous thrust。

5. reference

[1]. 海尔贝克阵列百度百科： https://baike.baidu.com/item/海尔贝克阵列/4092015?fr=aladdin

[2].”static_field_halbach_rotor”from COMSOL AC/DC

[3].胡友秋，程福臻，叶邦角编著，电磁学与电动力学，上【M】.科学出版社.2008