Physics 161

Vectors & Newton's Laws I

Introduction

In this laboratory you will explore a few aspects of Newton's Laws using a force table. By establishing equilibrium between three forces on the force table, you will gain a better understanding of vectors and vector addition.

Reference

Young and Freedman, University Physics, 13th Edition: Chapter 1, section 1.7-1.9, Chapter 4 sections 4.1, 4.3, 4.5

Theory

Forces are vector quantities and in order to be added, they must be broken into their separate components. ***In this experiment, you will work in two dimensions and resolve the x and y components of each force relative to an assigned axis.***

There are two major methods for adding vectors to find the sum, or resultant vector. A graphical approach involves drawing the vectors by hand on a graph, where length is proportional to the magnitude of the vector and the angle of the vector describes its orientation or direction in space. In using the "tail to tip" or parallelogram method to add vectors, it is important that the orientation/angle of each vector be maintained and also that the magnitude is carefully measured using a ruler to ensure that the vectors are the appropriate relative lengths. The length of the resultant vector can be converted back to magnitude and the angle measured as is to determine the orientation of the vector.

Graphical methods have limited accuracy and therefore an analytical method is usually preferable. Analytical vector addition is outlined in your textbook. For this experiment, the forces that are balanced are the forces of tension in the three strings. The magnitude of each tension force vector is equal to the magnitudes of the corresponding weights on the ends of the strings (mg). Remember that the force of tension is the same everywhere in the string; we assume the string to be massless. According to Newton's Second Law, the vector sum of the three tension forces is $\sum \vec{F} = m\vec{a}$, and if there is no acceleration, this sum must be equal to 0. The magnitudes of tension forces, F_1, F_2, and F_3 are equal to $m_1 g$, $m_2 g$, and $m_3 g$, respectively.

Figure 1: Force table.

Forces are said to be in equilibrium when the acceleration of the object is zero. Therefore, when three masses balance with the knot where they are joined over the center point of the force table, the net tension force on the knot for both x- and y-components is 0 N (see Figure 1). To write Newton's Second Law $\sum \vec{F} = m\vec{a}$ in component form, we need to define our x and y axes. Assume that our x-axis points toward mark 0^o on the force table, and our y-axis points toward 90^o. As always, angles are measured from the positive side of x-axis, counter-clockwise. But, we can equally use negative angles, i.e. $\sin(-25^o) = \sin(335^o)$.

Our vector equation, $\sum \vec{F} = m\vec{a}$, consists of two scalar equations because we are dealing with a two-dimensional space:

$$\sum F_y = m_1 g \sin\theta_1 + m_2 g \sin\theta_2 + m_3 g \sin\theta_3 = 0 \tag{1}$$

$$\sum F_x = m_1 g \cos\theta_1 + m_2 g \cos\theta_2 + m_3 g \cos\theta_3 = 0 \tag{2}$$

For the purposes of this experiment, two known masses, m_1 and m_2, will be assigned to specific angles; a third mass m_3 will then be adjusted and placed at such an angle as to establish equilibrium between the three tension forces. The third tension force $\vec{F}_3$, with a magnitude of $m_3 g$, will not be equal to the resultant vector of the first two tension forces, $\vec{F}_1$ and $\vec{F}_2$. However, it will have the same magnitude as the resultant vector of $\vec{F}_1$ and $\vec{F}_2$, but will point in the opposite direction.

Gravity is canceled in our scalar equations for the components of the forces, so we only need to work with the mass values. After cancelling g, our Equations (1) and (2) are reduced to:

$$\sum R_y = m_1 \sin\theta_1 + m_2 \sin\theta_2 = -m_3 \sin\theta_3 \tag{3}$$

$$\sum R_x = m_1 \cos\theta_1 + m_2 \cos\theta_2 = -m_3 \cos\theta_3 \tag{4}$$

Angles θ_1, θ_2 and θ_3 are read on the force table scale. The magnitude and direction of resultant vector, R can be found in the standard way:

$$\sqrt{R_x^2 + R_y^2} = R \tag{5}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \tag{6}$$

Procedure

You will vary the masses and the angles on two strings of the system, each time finding the vector that results in equilibrium of the system.

1. Set one of the strings to 20° and the other to -25°, i.e. to 335°, by sliding the bases of the pulleys to where the center notch at the base lines up with the desired angle.

2. On each of the first two strings, place a 50g mass on the plastic hooks. The mass of each hook is about 5g, please ***be sure to add this to the hanging mass to get the correct total mass*** of each vector.

3. With the third string, adjust the angle of the pulley and the mass on the hook until the ***knot (which joins all three strings) is above the center of the hole in the force table***. This may take some practice, but if you are observant of how changing the angle and the mass affect the location of the knot, you will become more efficient. ***Pay close attention*** to all of the strings to be sure that they are still on the grooves of the pulleys as they tend to slip out and get caught in the axle of the pulley; this will distort the forces on the knot and therefore the accuracy of your results.

4. Enter your results for the mass and angle required to balance the forces in Table 1 (columns 8 and 10) below, as well as in a spreadsheet in Excel (to be turned in with your Results).

5. Repeat steps 1 through 4 using angles of -20° and 100° and masses of 50 g and 100 g.

6. For both trials, calculate and enter the x and y components of the resultant vector, R_x and R_y by using the sum of components from the first two masses. **In Excel, angles must be in radians**:

Use the function 'radians()' to convert degrees into radians, and the function 'degrees()' when necessary to convert back into degrees.

7. Use the Pythagorean Theorem to calculate the overall magnitude of each of resultant vector, R, from the x and y components. Remember, the resultant, R, should have the same magnitude as the third mass you found experimentally.

8. Calculate the angles of the resultant vector using Equation (6) (*see the appendix of your text if you need a review of trigonometry rules).* The theoretical angle of the vector, which counters the resultant, can be found by adding 180 degrees to the angle of the resultant vector. Enter this theoretical angle in column 9 of Table 1. Compare this to the experimental value (column 10) you obtained in Steps 1 through 4.

9. Estimate the accuracy of your angle and mass measurements (i.e. uncertainties regarding columns 8 and 10). Are the differences between your theoretical and measured values for the mass and angle (columns 7 and 9) within the accuracy of your measurements (e.g. N_σ)?

Table 1: Resultant vector based on the masses and the angles.

1	*2*	*3*	*4*	**5**	**6**	*7*	*8*	*9*	*10*
				$\boldsymbol{R_y}$	$\boldsymbol{R_x}$	$\boldsymbol{R}$			
$m_1\ (g)$	$\theta_1\ (°)$	$m_2\ (g)$	$\theta_2\ (°)$	$m_1 \sin\theta_1 + m_2 \sin\theta_2$ (g)	$m_1 \cos\theta_1 + m_2 \cos\theta_2$ (g)	$\sqrt{R_x^2 + R_y^2}$ (g)	$m_3(g)$	θ_T (°)	θ_m (°)
55	20	55	-25						
55	-20	105	100						