Physics 161

Conservation of Linear Momentum

Introduction

In this lab, you will be observing the conservation of momentum under two different conditions. In **Part I**, you will observe the elastic collision of two carts with magnets that repel one another, and test the principle of conservation of momentum and energy. In **Part II**, you will observe the perfectly inelastic collision of two carts with Velcro ends that attach on contact, and test conservation of momentum.

Reference

Young and Freedman, University Physics, 13th Edition: Chapter 8 sections: 1-4.

Theory

The conservation of linear momentum, $p = mv$, is an important concept. When linear momentum is conserved within a closed system, the initial and final total linear momenta are equal. A closed system is defined when there are no outside forces acting on the system. For this lab, we will assume that there are no significant outside forces, such as friction, present during the collisions. This will not literally be the case but since the collision is short in duration, it is reasonable to make this assumption because the friction is small and the duration over which it acts is short. Therefore, $\sum p_i = \sum p_f$. We will examine the motion of two carts in one dimension so the initial and final momentum equations are, respectively:

$$p_i = m_1 v_{1i} + m_2 v_{2i} \quad \text{and} \quad p_f = m_1 v_{1f} + m_2 v_{2f} \tag{1}$$

The conservation of momentum yields:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{2}$$

When two objects collide we can examine the collision in terms of the impulse. Impulse is defined as the product of the net Force applied during the collision and the time interval of the impact:

$$Impulse = F\Delta t \tag{3}$$

Newton's second law can be rewritten in terms of linear momentum as:

$$F = ma = m\frac{\Delta v}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t} \tag{4}$$

Therefore, it can be said that the net force applied to an object of constant mass equals the time rate of change in momentum, $\Delta p/\Delta t$.

The total impulse may also be described as the change in momentum for objects without mass changes:

$$Net\ Impulse = \Delta p \tag{5}$$

Thus, we can write that:

$$F_{net}\Delta t = \Delta p = m(v_f - v_i) \tag{6}$$

Part I: Elastic collisions.

An elastic collision is one in which kinetic energy and linear momentum are conserved, $KE_f = KE_i$ and $p_i = p_f$:

$$KE_i = \frac{1}{2}(m_1 v_{1i}^2 + m_2 v_{2i}^2) \tag{7}$$

$$KE_f = \frac{1}{2}\left(m_1 v_{1f}^2 + m_2 v_{2f}^2\right) \tag{8}$$

If there are no external forces acting on the objects in a collision, such as friction, the momentum is conserved. Kinetic energy must be conserved in order to define the collision as elastic. Kinetic energy is often dissipated through vibrations of each object upon collision. To minimize the loss of kinetic energy, we use similar poles of magnets, which repel one another, to prevent physical contact between the two carts during the collision.

Part II: Inelastic collisions.

Collisions where $KE_f < KE_i$ are referred to as inelastic collisions and the energy dissipated is $\Delta KE = KE_i - KE_f$. A collision where the two objects stick together is referred to as a perfectly inelastic collision because the maximum possible energy is dissipated, typically in the form of heat. However, linear momentum is still conserved. You will observe this in the case of two carts which become attached upon collision by the Velcro fabric on their ends.

Procedure

Both experiments will be conducted using the track with a motion sensor at each end.

Part I: Elastic collisions.

1. Attach the two motion sensors to the **PASCO 850 Universal** interface. Double-click on each motion sensor icon and set the trigger rate to 25 Hz. Set the motion sensor to the short-range designation ( ).

2. Figure 1 shows velocity versus time data for a collision where m_2 is initially at rest. To create this graph with both velocities plotted on the same graph you need to set the y-axis first to the velocity of one of the carts, then click on "**Add new y-axis**" button ( ) from the graph toolbar and add the velocity of the second cart.

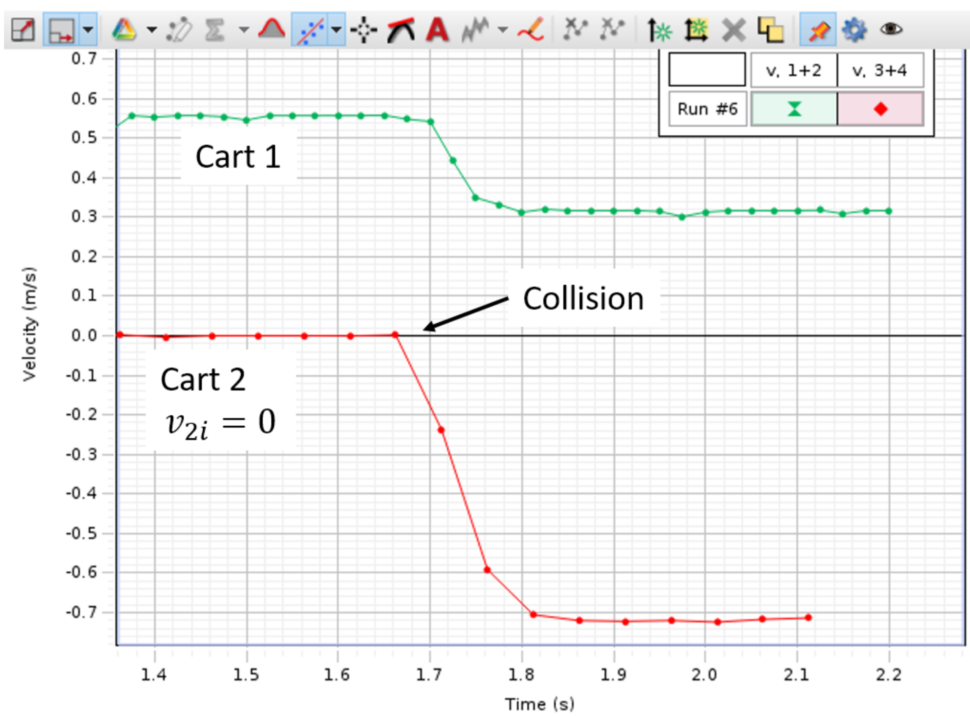

Figure 1: Velocity vs time of the two carts measured throughout the collision.

Since the motion sensor for m_2 is on the opposite end of the track and oriented in the opposite direction, <u>you will need to reverse the sign of v_2 in order to get the appropriate</u>

velocity relative to the system. This can be done in **Capstone** but it is easier to reverse the sign manually by multiplying v_2 results by -1 when you record them into Excel.

There is actually some friction acting on the carts, as shown by the slight downward slope of the otherwise horizontal lines, see Figure 1. It is important to determine the velocity from the graph immediately before and after the collision. This way, the time over which friction has acted is minimal and therefore its effect on the conservation of the linear momentum will be minimal.

3. Orient the carts with the magnets toward each other. Since the magnets are strong you should keep your watch and any USB memory stick away from them.

4. Create a table in Excel like the one below for your data on elastic collisions. In all cases m_1 is the incident cart and m_2 is stationary. Remember that the sign of the velocity for m_2 has to be reversed.

m_1	m_2	v_{1i}	v_{2i}	v_{1f}	v_{2f}	P_i	P_f	$\sigma_{\Delta P}$	KE_i	KE_f	$\sigma_{\Delta KE}$
Mass of cart1	**Mass of cart2**										
Mass of cart1	**Mass of cart2 +0.5 kg**										

5. Measure the masses of the carts and record them into the table. In the second trial add 0.5 Kg to cart 2.

6. Press **Record**, push one cart gently towards the other, record the collision, let them separate and press **Stop**. Run the trials listed in the table and graph them using **Capstone**. The magnets produce a collision that is approximately elastic when the carts are repelled solely by the magnetic repulsive force. Therefore, your results will be most accurate if you avoid accelerating the carts so hard that they physically hit each other.

7. Transfer at least five data points for the initial and final velocities for both carts to excel, we will use them to measure the momenta and the estimates for the uncertainty in momenta.

8. Determine if momentum is conserved. Multiply the data points for velocity by their respective masses, and calculate the mean and uncertainty for each of the four data sets. Find the total initial and final momenta by summing the momenta of the carts. Now find the difference between the total initial and final momentum. It will not be exactly zero, but that does not necessarily mean momentum has not been conserved. We must compare this difference to the uncertainty in the difference. To calculate this uncertainty we will sum the uncertainties of the individual momenta. We may try just summing them to begin with $\sigma_{\Delta P} = \sigma_{P_{1i}} + \sigma_{P_{2i}} + \sigma_{P_{1f}} + \sigma_{P_{2f}}$. This formula is only correct if the individual uncertainties are correlated. In general we assume the opposite, that the uncertainties are completely uncorrelated. In which case adding the uncertainties from different measurements would be

done as though they were orthogonal vectors, $\sigma_{\Delta P} = \sqrt{\sigma_{P_{1i}}^2 + \sigma_{P_{2i}}^2 + \sigma_{P_{1f}}^2 + \sigma_{P_{2f}}^2}$. Now divide the difference between the initial and final momenta by the total uncertainties. How many sigmas is the difference in each case? In general, we say that something is in agreement if it is within 2 sigmas. If your results differ by more than 2 sigmas for either uncertainty measurement think of a few reasons why the statistical fluctuations do not explain the discrepancy. Make sure to document these results in your notebook.

9. Using your velocity data, calculate the initial and final kinetic energies and their uncertainties in the same way you did the momentum. And again, test energy conservation based on the total uncertainty, this time only use the quadrate formula. Make sure to document these results in your notebook.

Part II: **Perfectly Inelastic Collisions**

10. Orient the two carts with their Velcro ends towards each other so that the carts will stick together in a collision.

11. Create a table to record the data for Part II. Repeat the Steps 5 through 9 from Part I. For uncertainties only use the quadrate formula, and treat the two masses as one after the collision. Label each table accordingly, elastic collision and inelastic collision.

m_1	m_2	v_{1i}	v_{2i}	v_f	P_i	P_f	$\sigma_{\Delta P}$	KE_i	KE_f	$\sigma_{\Delta KE}$
Mass of cart1	**Mass of cart2**									
Mass of cart1	**Mass of cart2 +0.5 kg**									