Standing Waves (Resonance) on a String

Learning Goals:

1. The main goal of this lab is to observe the resonant frequencies of a string which is fixed on both ends and to verify the theoretical relationship between the number of antinodes, the mass per unit length of the string, the tension, the length of the string, and the fundamental frequency of vibration.
2. Determine resonant frequencies given the fundamental frequency of the string.
3. Determine the relationship between the resonant frequencies and the number of antinodes of a string of length L with hanging mass m.
4. State how mass (tension) is related to the number of antinodes at a constant input frequency.

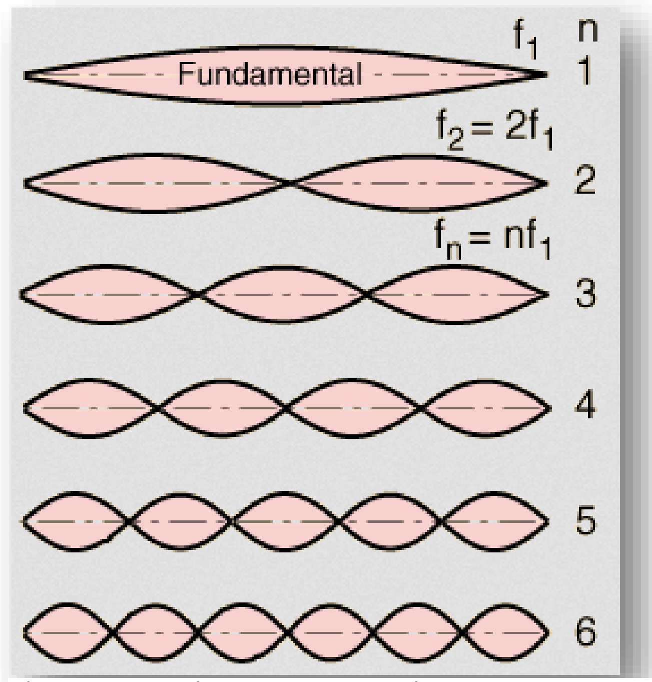

Figure 1: Standing waves on a string

Materials:

Capstone, Excel, signal generator, string, mass, pulley

References:

Giancoli, Physics 7th Edition: Chapter 11, sections: 6, 7, 13; Chapter 12, section 4

Introduction:

This experiment studies standing waves on a string using the apparatus shown in Figure 4. The frequency of the oscillator (see at far end of the image) can be adjusted in Capstone to meet the necessary conditions for standing waves. At a constant frequency input, the string length can be adjusted by sliding the clamp that holds the oscillator or the pulley. Finally, the change in tension can be explored by changing the mass hanging over the pulley.

How can we describe this motion theoretically to compare to experimental results? We know if we shake a string (fastened at one end) the resulting wave on the string depends upon how quickly or slowly we shake. At times the string appears to have almost no wave traveling up or down the string and at other times the wave is visible and appears to have points on the string that appear motionless.

Thus, at certain frequencies waves have small amplitudes and at other specific frequencies the amplitude increases. What is going on when this happens? The frequency is matching with the length of the string such that the wave exhibits maximal amplitude. Why this happens is the theoretical topic to discuss.

If we look at the shape of the waves as we shake the sting – we see that it appears to be sinusoidal. How can we determine the fundamental frequency? We know that the period of oscillation is related to the frequency of oscillation, namely $T = 1/f$.

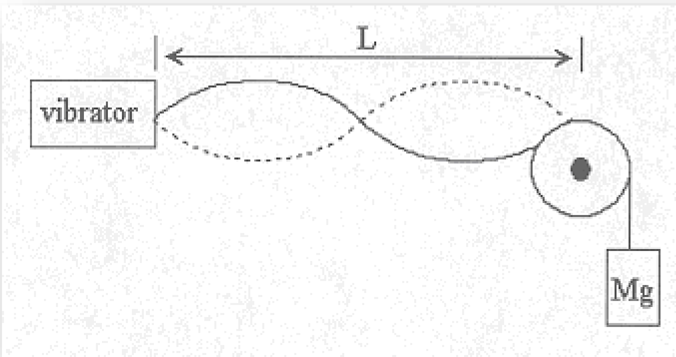

Figure 2: Sketch of experimental setup

In the sketch to the left we see that the wavelength for this sine wave is L – the length between the two end nodes when the amplitude is at its maximum. This makes intuitive sense because if the wavelength was longer – the end node at the pulley would force the wave to zero earlier than its natural zero point. If the wavelength was shorter, the wave would begin to rise and then again be forced to zero at the pulley.

We know when determining the wavelength of any signal we must find the point where the periodic signal begins to repeat itself, thus defining the wavelength as the distance of the periodic wave form.

In figure 3, notice the arrows showing the direction of the wave's progression with respect to distance

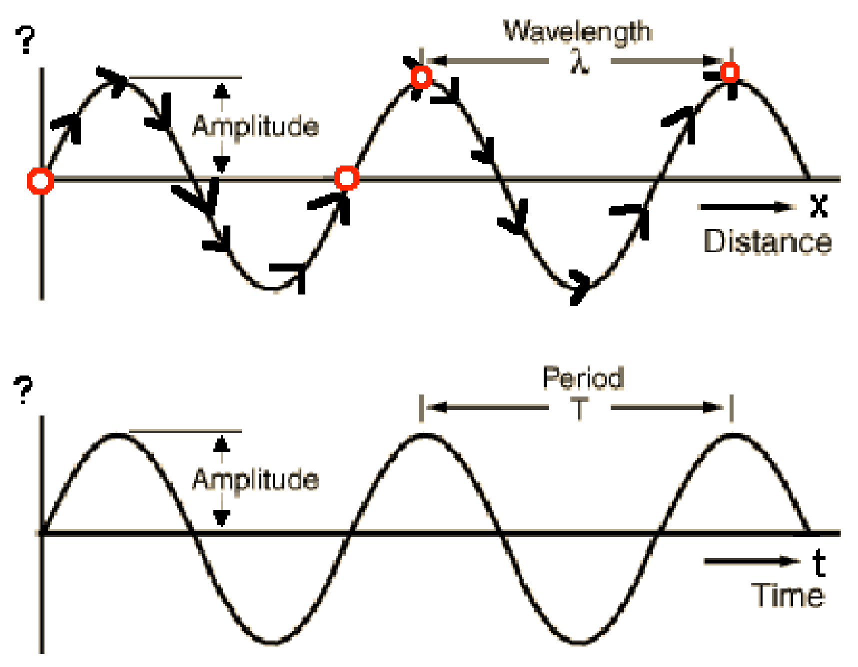

Figure 3: wavelength and period measurements

and time. These graphs show how the wave travels down the string.

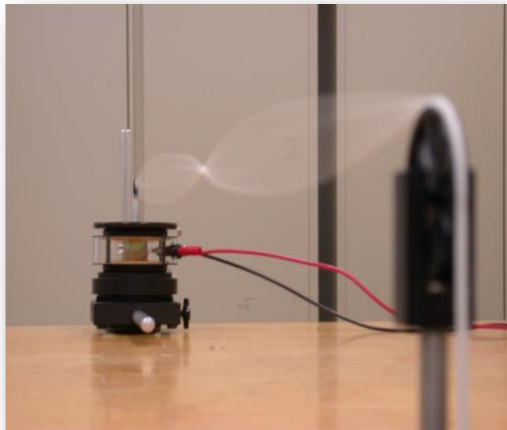

Figure 4: Experimental Setup

When sketching arrows on a periodic signal such as the sine wave shown here, one wavelength (in length) or one period (in time) is found when the arrows cross the starting position and travel in the same direction. Notice that one can determine the wavelength by either starting at a zero crossing and finding the next zero crossing with arrows in same direction – thus restarting the signal OR by looking at the change in distance from one peak to the next peak. These are the two most convenient points used typically although theoretically any starting point can be used as long as one stops when the starting point is found again with the arrow in the same direction.
The same is true for the period. Notice in the 2nd graph in the image the x-axis is time rather than distance thus the time interval now measures the period of the oscillation rather than the wavelength. It is important to read axes when trying to determine attributes of a signal and not confuse the period and wavelength measurements.

Now that we can find the wavelength and period of a periodic signal such as the sine wave we need to apply this to the problem at hand. In figure 1 on the first page one sees that many periods or ½ a period can form on a string fixed at two ends. We also see the word 'fundamental' by the ½ period shown in the image. This is the wavelength (or frequency) that will allow for ½ a period to form on the string. So, think about the standing wave – the wave travels DOWN the string and then reflects and RETURNS. We see this from the +maxima traveling down and the –maxima reflecting back. Our eyes see this as standing still because the motion is very quick (higher frequency). It is important for you to realize that it is moving even though your eyes see the wave as appearing to be standing.

For propagation of these waves on a string fixed at both ends (as opposed to a hollow tube with one end closed), standing waves occur when half the wavelength is equal the length, L, of the string, or when two half wavelengths equals the length, or three half wavelengths, etc. If we want to create an equation that will properly represent this, we need to first write the equation for ½ wavelength.

$$\frac{1}{2}\lambda = L$$

We also know that we can use multiples of this equation to solve for shorter wavelengths. So, we can have (2) half wavelengths or $2 * \left(\frac{1}{2}\right)\lambda = L = \lambda$ and if one has 3 half wavelengths then $3 * \left(\frac{1}{2}\right)\lambda = L =$

$\frac{3}{2}\lambda$. If we continue in this way, we see that the wavelength increases in increments of ½ wavelengths. Thus we can write the general equation as:

$$\frac{n}{2}\lambda = L \text{ and solving for wavelength we find: } \lambda = \frac{2L}{n}$$

for n = 1, 2, 3, ...

How can we relate this to a frequency? Since the Capstone software only allows users to change the frequency of the oscillation, we need a way to convert the wavelength into frequency to conduct the experiment examining the relationship between the frequency and the number of antinodes (or maxima) for each standing wave found.

If we think about the units of both the frequency and the wavelength we will find that these values determine the wave speed. Since frequency = Hertz = 1/seconds and wavelength = meters if we multiply these two values together we find speed:

$$f * \lambda = \frac{1}{s} * m = \frac{m}{s} = v$$

If we can determine the speed of the wave – we can determine the frequency that we need to use the Capstone software to collect data of changing frequency. But how can we do this? Well, it turns out if we think about the velocity in terms of the force of tension on the string and the linear mass density (how much the string weighs on Earth per unit length), we can find the velocity. It should make sense that how fast the wave travels should depend upon the force pulling on the end of the string (over the pulley) and the mass of the string itself as the thicker and less stretchy the string the more difficult it will be to create a wave of any amplitude on it.
Let's look at dimensional analysis and see if we can reason the velocity equation from the units of these two values that we theorize must influence the velocity.

$$Force = F_{tension\ on\ string} = mg = kg * \frac{m}{s^2}$$

$$\mu = \frac{mass}{length} = \frac{kg}{m}$$

$$velocity = \frac{m}{s}$$

Inspection above gives the theorized equation from your text, namely:

$$v = \sqrt{\frac{F_{Tension}}{\mu}} = \sqrt{\frac{kg * {}^{m}/_{s^2}}{{}^{kg}/_{m}}} = \sqrt{\frac{{}^{m}/_{s^2}}{{}^{1}/_{m}}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$

Now, we have an equation for the velocity and we can plug this into the equation above to determine frequency as well as write explicitly the equation for any wavelength previously determined:

$$f * \left[\frac{2L}{n}\right] = \sqrt{\frac{F_{Tension}}{\mu}} = \sqrt{\frac{mg}{\mu}}$$

and if we solve for the number of antinodes (the integer 'n' in the equation we find:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}}$$

From this equation we can find the relationship between the number of antinodes (n) and the frequency (*f*). If we hang a 200g mass over the pulley and have the distance from the pulley to the oscillator = 1m we can determine the mass density of the string. This has been done for you and the mass density of the string with a 200g mass attached to the end is:

$$\mu = 0.00401 \pm 0.00004\ \frac{kg}{m}$$

Think about why this linear mass density needs the mass on the end of the string determined to provide a precise measurement? (Hint: Is the string stretchy? How could this impact mass if a heavier mass is attached to the end of the string?)

Proper experimental technique requires that ONLY 1 variable in a predictive equation be changed at a time to determine how this variable affects another. In this case – we would want to conduct three experiments examining how the number of antinodes is affected by changes in the frequency applied to, the length of, and the tension on a string in resonance (standing wave). To do this one would write each equation isolating the variable and setting all other values as constants – this means that during experimentation ONLY the ONE value should change to see how this value affects the antinodes written as 'n'.

To examine how the frequency affects the number of antinodes – arrange the equation as shown:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} = f * \left(2L * \sqrt{\frac{\mu}{mg}}\right) = n = constant\ * f$$

$$constant = \left(2L * \sqrt{\frac{\mu}{mg}}\right)$$

To examine how the length affects the number of antinodes – arrange the equation as shown:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} = L * \left(2f * \sqrt{\frac{\mu}{mg}}\right) = n = constant\ * L$$

$$constant = \left(2f * \sqrt{\frac{\mu}{mg}}\right)$$

To examine how the tension on the string affects the number of antinodes – arrange the equation as shown:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} = \frac{\left(2fL\sqrt{\mu}\right)}{\sqrt{mg}} = \frac{constant}{\sqrt{mg}} \qquad \text{with} \qquad constant = \left(2fL\sqrt{\mu}\right)$$

Now, with the equation above – written to isolate each of the three variables predict the shape of the following plots:

- Antinodes vs. frequency
- Antinodes vs. length
- Antinodes vs. force of tension

Discuss in your group and sketch the 3 plots to turn in with your group lab report. It is not important if after conducting the experiment you find your sketches are not correct – so please do not change sketches – you only need to state what was wrong with your group's thinking at first that was realized after the data was plotted. Make sure your instructor sees the 3 sketches before you start your experiment.

Because of time restrictions we can only conduct two experiments today. Your group will examine the frequency vs. antinodes and the tension vs. antinode. Write down your hypotheses to be turned in with your report.

Experiment:

Experimental procedure and Data Collection:

1. Setup the distance between the pulley and the oscillator to 1m. Use a meter stick to take this measurement and discuss HOW your group determined the end point for the pulley node. Make sure to detail HOW your group determined the end points of this measured distance **L** in your group report. For the first part of the experiment, hand a mass of 200 g on the string.

2. Connect the two wires (black and red) from the Pasco oscillator to Output 1 in the 850 Interface.

3. Open Capstone. Click on Signal Generator. Click on 850 Output 1. The waveform should be set to "Sine". Start at a frequency of 25 Hz and increase it in increments of 1-10 Hz looking for the resonant frequencies on the string that has 4 antinodes.

4. Once the string is close to resonance (you observe a stable standing wave), change the stepping increment to 0.1 Hz and fine-tune your resonant frequency. You may need to go back and forth a few times to determine the frequency at which the LARGEST displacement occurs- -that is

your resonant frequency for the 4 antinode oscillation, i.e. $n = 4$. Record this resonant frequencies and number of antinodes in a table in Excel.

5. Examine how the frequency affects the # of antinodes. Keeping the length and the mass constant, change the frequency required to produce antinodes $n=1, 2, 3, 5, 6, 7$, and 8. Record these frequencies and antinodes in Excel.

6. Now, return the oscillator to the frequency found in Step 4 (for 4 antinodes) to examine how the tension affects the number of antinodes. Keeping the frequency and the length constant, change the mass (and therefore Tension) on the string until you find as many standing waves as possible between 0.05 kg – 0.80 kg. Note that your group should be able to find 5 data points.

Analysis:

1. Create graphs of the **antinode vs. frequency** and **antinode vs. tension** in Excel.

2. Using the trendline feature in Excel, choose the appropriate fit for the two plots and fit your group's data. Make sure the fit equations show on the two graphs.

3. Predict the value of the constant.

4. Now, for all linear data – compare the slope value from Excel to the constant value shown in the theory section. How does your data compare to the predicted constant value calculated? Discuss.

5. If the data is not linear – compare the calculated constant value for the proper equation shown on page 5 to the fit constant from the Excel trendline feature. How does your data compare to the predicted constant value calculated? Discuss.

6. The value of μ is only known to 2.5% precision. Is this uncertainty sufficient to state that your Excel fit constants equal the calculated constant value for these experiments? Explain.

7. Discuss in your group the impact of the pulley friction. How does your group predict this friction, if present, would impact data results? Explain reasoning.

8. Discuss the stretchy string – if the μ changes during the experiment with changing masses would its value tend to increase or decrease? Explain.