Physics 161

Moment of Inertia

Introduction

In this experiment you will study the relationship between force and angular motion by observing the relationship between torque, moment of inertia and angular acceleration for two masses on a rotating rod. By changing the geometry of this system you will change the response of the system and gain insight into how Newton's Laws describe angular motion.

Reference

Young and Freedman, University Physics, 13th Edition: Chapter 3, section 4; Chapter 9, sections 1-4.

Theory

The moment of inertia measures the distribution of mass in a body and how difficult that body is to accelerate angularly with respect to a center of rotation. In this experiment, a falling mass will accelerate a rotating object in a horizontal plane. You will find the moment of inertia of a rod with two masses attached to it.

The basic equation for rotational motion is:

$$\sum \tau = I\alpha \qquad (1)$$

where α is angular acceleration in units of rad/s^2, τ is applied torque in $N\ m$, and finally I is the moment of inertia in units of $kg\ m^2$.

The system's moment of inertia can be calculated as the sum of the moments of inertia for each component of the system. Our system is comprised of a rod and two attached masses. We will assume the two masses to be point masses. The moment of inertia for each mass is:

$$I_{mass} = mr^2 \qquad (2)$$

where r is the distance from the point mass to the axis of rotation located at the center of the rod. Because the masses can be moved along the rod, r will be adjusted to change their moment of inertia. The moment of the inertia of the rod with mass M and a length L is:

$$I_{rod} = \frac{1}{12} M_{rod} L^2 \tag{3}$$

The moment of inertia for a rod with length L and two masses on each end at a distance r is simply the sum of the components as defined by Equations (2) and (3):

$$I_{theoretical} = 2mr^2 + \frac{1}{12} M_{rod} L^2 \tag{4}$$

The first term in Equation (4) is multiplied by two because there are two point masses. Given the moment of inertia of the entire system, I, and the torque, τ , applied by the mass M hanging from the pulley, angular acceleration, α can be found. The rotational motion is described by:

$$\tau_{net} = I\alpha \tag{5}$$

Torque τ is a rotational "force" that causes rotation with an angular acceleration, α, to an object with a moment of inertia, I. The rotating object will be attached to a pulley placed at the center of the rotary motion sensor which has a string wrapped around it. The string has a mass tied to one end and is laid over an additional pulley which allows the falling motion of the mass to be converted to a torque on the rotary motion sensor. For the purposes of this lab, the applied torque will be due to a mass accelerated by gravity acting on the pulley of the rotary motion sensor. You might expect the torque to be given by the weight of the hanging mass, $m_{hang}g$, times the moment arm, R_{pulley}, $\tau \approx m_{hang} g R_{pulley}$. However, the torque actually arises from the tension in the string, not the weight of the hanging mass. The analysis using Newton's laws, see Appendix, shows that the torque is:

$$\tau = m_{hang} g R_{pulley} - m_{hang} R_{pulley}^2 \alpha \tag{6}$$

where m_{hang} is the mass hanging on the pulley. The experimental moment of inertia can be found using the following equation:

$$I_{experiment} = \tau / \alpha \tag{7}$$

By adjusting the position of the masses on the rod, we can observe how an increased moment of inertia will result in a decreased angular acceleration for the same torque. It is the same type of relationship as the one you observed for Newton's Second Law.

Procedure

Moment of Inertia of a Rod with Two Masses Attached

1. Weigh the rod and the point masses and record these values.

2. Set up the apparatus as shown in Figure 1, making sure to connect the rotary motion sensor to the interface box.

Figure 1: Experiment setup. Rotary motion sensor.

3. Setup the pulley to give positive values for the angular position. This means that the rotary motion sensor will turn in a counterclockwise direction as the mass on the pulley drops. The motion sensor may have an indication of which direction is positive taped to it.

4. Adjust the measurement rate to 10 Hz on the **Rotary Motion Sensor**.

5. Adjust the masses so they are as far from the axis of rotation as possible without falling off. Measure and record this distance, r_0. Hang a mass of 50 grams from the pulley, press **Record**, and release the hanging mass.

6. Press **Stop** just before the hanging mass reaches the table. You will repeat this for three trials with different positions for the masses and create a table with the following format:

Run	$r\ (m)$	$R_{pulley}\ (m)$	$m_{hang}\ (kg)$	$I_{masses} = 2mr^2$	$I_{Theory} = 2mr^2 + I_{rod}$	α	τ	$I_{experimental} = \frac{\tau}{\alpha}$	$\sigma_{I_{experimental}}$
$r = r_0$			0.05						
$r = 0.5r_0$			0.05						
$r = 0$			0.05	0					

For the case $r = 0$, remove the masses completely from the rod.

7. Create plots in Excel for the following:
   a. angular position, θ, in radians versus time,
   b. angular velocity, ω, in radians/s versus time, and
   c. angular acceleration, α, in radians/s^2 versus time.

8. Using Equations (6) and (7) and your time series for alpha, calculate the mean and standard deviation for $I_{experimental}$ and put those values in your table.

IMPORTANT: r is the distance between the center of the point mass and the axis of rotation of of the rod, while R_{pulley} is the radius of the pulley on the rotary motion sensor around which the string is wrapped.

For Your Lab Report:

Include a sample calculation of the torques

$$\tau = m_{hang} g R_{pulley} - m_{hang} R_{pulley}^2 \alpha$$

Use the uncertainty you calculated for $I_{experimental}$ determine how many $\sigma' s$ $I_{theoretical}$ and $I_{experimental}$ differ by. Remark on these results in your notebook and make sure to discuss any sources of discrepancy that may not be accounted for by the statistical uncertainty.

Are your plots of angular acceleration consistent with the assumption that angular acceleration is constant?

Appendix

The rotary motion sensor has a radius, R_{pulley}, which is the distance from the axis of rotation at which the force, T, acts. T is the tension in the string attached to the pulley and is due in part to the weight hanging on the string, Mg, where M is the mass of the hanging mass and g is the acceleration due to gravity. Figure 2 below shows a rough sketch of the set up.

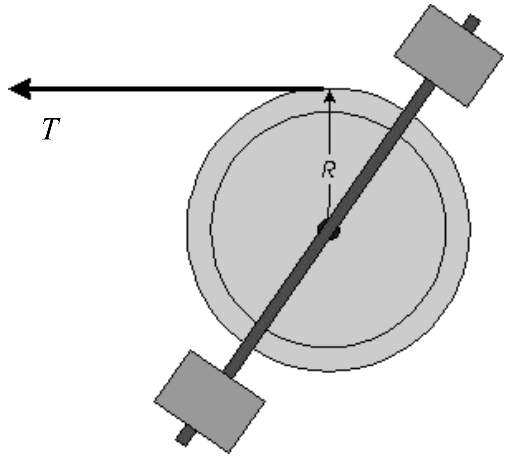

Figure 2: Experiment setup sketch.

The application of Newton's Second Law to the hanging mass results:

$$\sum F_y = m_{hang}a \tag{a.1}$$

$$\sum F_y = m_{hang}g - T = m_{hang}a \tag{a.2}$$

The downward direction is taken to be positive. We can write the linear acceleration, a, as:

$$a = R_{pulley}\alpha \tag{a.3}$$

Where α and R_{pulley} are the angular acceleration and the radius of the rotary motion sensor, respectively. Continuing to solve the equation gives us:

$$m_{hang}g - T = m_{hang}R_{pulley}\alpha \tag{a.4}$$

$$T = m_{hang}g - m_{hang}R_{pulley}\alpha \tag{a.5}$$

T is the tension in the string. It is a force, not a torque. To find the torque acting on the pulley, we must multiply by the distance from the axis of rotation at which the force acts:

$$\tau = R_{pulley}T \tag{a.6}$$

$$\tau = m_{hang} g R_{pulley} - m_{hang} R_{pulley}^2 \alpha \qquad \text{(a.7)}$$

For the set up of this experiment, take the following example where $m_{hang} = 0.06\ kg$, $R_{pulley} \approx 0.025\ m$ and $\alpha \approx 2.58\ rad/sec^2$. Substituting these values results in:

$$m_{hang} g R_{pulley} = 14.7 \times 10^{-3} Nm$$

$$m_{hang} R_{pulley}^2 \alpha = 0.0969 \times 10^{-3} Nm$$

From this example, we see that $m_{hang} g R_{pulley} \gg m_{hang} R_{pulley}^2 \alpha$ so we can approximate $\tau = m_{hang} g R_{pulley}$.