Physics 161

Static Equilibrium and Introduction to Error Propagation

Introduction

You will observe static equilibrium for a meter stick suspended horizontally with masses suspended from it. You will then propagate uncertainties in the measured masses, forces and distances to determine the overall uncertainty in the sums of the forces and torques to determine whether your measurements accurately reflect static equilibrium.

Reference

Young and Freedman, University Physics, 12th Edition: Chapter 11, section 3

Theory

Part I: When forces act on an object, this object has translational and rotational motions due to unbalanced forces. Static equilibrium occurs when the net force and the net torque are both equal to zero. We will examine a special case where forces are only acting in the vertical direction and can therefore be simply summed without breaking them into components:

$$F_{net} = F_1 + F_2 + F_3 + \cdots = 0 \tag{1}$$

Torques may be calculated about an axis of your choosing:

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \cdots = 0 \tag{2}$$

where torque is specified by the equation:

$$\tau = Fd \tag{3}$$

where *d* is the lever arm or moment arm for the force. The lever arm is the perpendicular distance from the line of force to the axis about which you are calculating the torque.

Normally, up is "+" and down is "-" for forces. For torques, by convention, clockwise is defined as "-" and counterclockwise as "+". For this experiment, be consistent with the force and torques signs. Make sure you understand what a "+" or "-" value of the force or torque means directionally.

Procedure

Static Equilibrium

1. Weigh the meter stick, including the metal hangers.

2. Attach the force sensor cords to the Interface box as you have done in previous labs. Set up the hardware in Pasco by adding two force sensors for channels A and B.

3. For today's lab you do not need to graph the force sensors over time. Instead, drag the **Digits** icon from the "Displays" menu, on the right hand side, to the main page area. This will give you a digital readout of the force sensor data for any given time. Repeat this step to create two **Digit** displays, one for each force sensor on the meter stick. In the **"Select Measurement"** icon choose "**Force Channel A (N)**" and "**Force Channel B (N)**" in the two Digit displays.

4. Remove the meter stick from the force sensor hooks, click on **Record** button, and tare each force sensor without any added weight to establish zero. Next, measure a mass on the scale. Then hang this known mass on each force sensor to verify that it is reading correctly. If you need to increase the precision of the display, hover the mouse over the **"increase digits"** icon shown in the Figure 1. This will increase the number of significant figures.

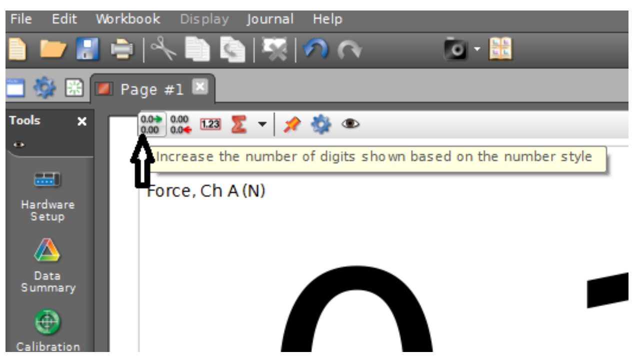

Figure 1: Digital display toolbar. Increase number of digits.

5. While maintaining the Record button on, set up the meter stick and *force sensors* as shown in Figure 2. The meter stick will be suspended from a beam via the two force sensors. These sensors are used to determine the upward vertical forces at these positions. The force sensors must be attached vertically.

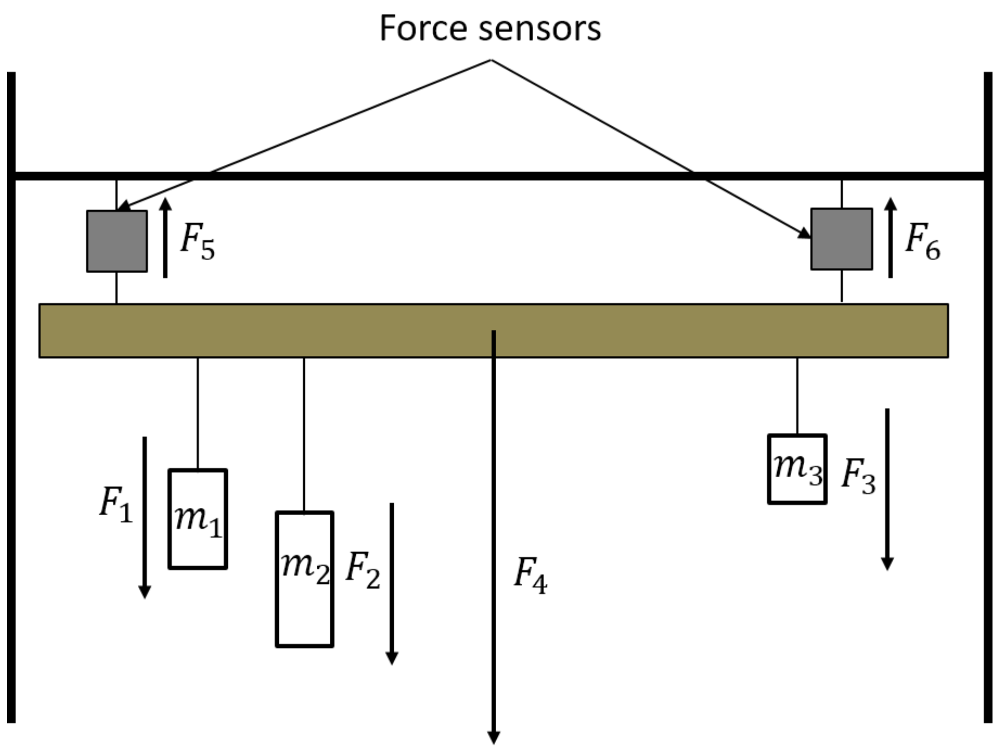

Figure 2: Diagram of the torque experiment setup.

6. Attach three masses 100g, 20g and 50g, to the meter stick using metal hangers.

7. Balance your system by moving the three weights and watching the Digits displays. All forces must be vertical; therefore, make certain the meter-stick is level and be sure the force sensors are pulling straight upward. Adjust the location of the masses by moving the metal hangers so that the force sensors read almost identical values.

8. Record the position of each mass on the meter-stick.

9. Create the following data sheet to record the results. Calculate the sum of the masses responsible for the positive forces and the sum of those responsible for the negative forces. Forces 5 and 6 are readings from the force sensors. Check to see if the sums are equal.

Mass(g)	Force #	Force (N)	Position (m)
$m_1 = 100$	F1	$(m_1 + m_h) \times g =$	
$m_2 = 20$	F2	$(m_2 + m_h) \times g =$	
$m_3 = 50$	F3	$(m_3 + m_h) \times g =$	
$m_4 =$	F4	$m_4 \times g =$	
	F5		
	F6		

10. Using the zero position (x = 0 m) of the meter stick as the axis of rotation, and counterclockwise torques as positive, determine the sum of the torques acting in both directions. Create a new data sheet, and record these values. Imagine the lever arm is now located at the axis point in the middle of the meter stick (x = 0.50 m), recalculate these new torques.

Torque Calculation Table				
	Lever Arm (m) Axis at x=0m	Torque (N-m) Axis at x = 0m	Lever Arm (m) Axis at x=0.5m	Torque (N-m) Axis at x=0.5m
F1				
F2				
F3				
F4				
F5				
F6				
Σ of Positive Torques				
Σ of Negative Torques				
Sum of all Torques				

Error propagation.

We have already seen that when you are comparing multiple measurements that you need to combine the uncertainties from those measurements. If, as we usually assume, the fluctuations are uncorrelated, the uncertainties should be combined in quadrate. So far we have calculated uncertainties for any quantity we wish to compare by taking multiple measurements of that quantity and calculating a standard deviation. For instance, when we calculated the linear momentum and its uncertainty we simply multiplied the mass times the velocity for a series of measurements. In doing so we are assuming that our measurement of mass was perfect. To be more accurate, we should include some uncertainty in our mass measurement as well. Unlike combining uncertainties for added (or subtracted) quantities, multiplication requires a more sophisticated process of combination. Luckily any functional dependence, not just multiplication, can be addressed with the quadrature formula. The uncertainty in any quantity $f(x, y, z)$, given the uncertainties in x, y, and z is:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma_z^2} \qquad (4)$$

This formula is valid as long as the variables x, y, and z are independent and uncorrelated.

For instance, the uncertainty in torque $\tau = m\,g\,r$ due to one of the masses on the meter stick would be:

$$\sigma_\tau = \sqrt{\left(\frac{\partial\tau}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial\tau}{\partial g}\right)^2 \sigma_g^2 + \left(\frac{\partial\tau}{\partial r}\right)^2 \sigma_r^2} = \sqrt{(gr)^2\sigma_m^2 + (mr)^2\sigma_g^2 + (mg)^2\sigma_r^2} \qquad (5)$$

In your notebook make estimates for the uncertainties for mass, gravity, distance r, and the force meter. These can be based on the accuracy of your measuring device, say 0.5% for a mass scale, or your ability to measure the position on a meter stick, which certainly cannot be much smaller than a half of a mm.

Once you have the uncertainties for each individual force and torque then you will need to find the combined uncertainty for the sums of these quantities. Using the formula above, where, $\sigma_F = \sqrt{\sigma_{F_1}^2 + \sigma_{F_2}^2 + \sigma_{F_3}^2 + \cdots}$. You will find that the propagated uncertainty for the sum is the same quadrate formula we have been using for combing sums in previous labs.

Calculate the uncertainties for the sums of the forces and torques for both parts 10 and 11 and use these to determine whether the sums of the torques and forces are significantly different from zero and discuss these results in your notebook.