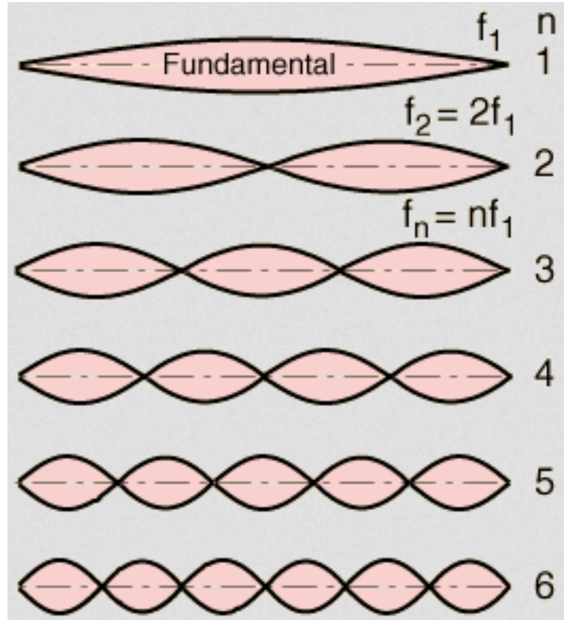
George Mason University
Standing Waves (Resonance) on a String
Group Lab Report



Learning Goals:

After completion of this laboratory students will be able to

- **Determine resonant frequencies given the fundamental frequency of the string.**
- **Determine the relationship between the resonant frequencies and the number of antinodes of a string of length L with hanging mass m.**
- **State how the Mass (Tension) is related to the number of antinodes at a constant input frequency.**

Reference

Giancoli, Physics 7th Edition: Chapter 11, sections: 6, 7, 13; Chapter 12, section 4

Lab Overview

In this lab, you will observe the resonant frequencies of a string fixed at both ends (one end attached to a harmonic oscillator and the other end hanging over a pulley weighted by a mass.) With the data you collect, you will demonstrate the relationship between the number of antinodes (or maxima) on the string (as shown in the figure above) and the input frequency. You will also show the relationship between the string length and number of antinodes as well as the dependence of the mass (or tension) on the string's antinodes. Remember for each experimental procedure ALL other parameters must remain constant. Thus, 3 experiments must be conducted. Because of time constraints, groups will explore 1-2 of the procedures and share results so all students can analyze results collected in class for each group's report.

Background Theory

This experiment studies standing waves on a string using the apparatus shown in Figure 3. The frequency of the
Oscillator (see at far end of image) can be adjusted in Capstone to meet the necessary conditions for standing waves. At a constant frequency input, the string length can be adjusted by sliding the clamp that holds the oscillator or the pulley. Finally, the change in tension can be explored by changing the mass hanging over the pulley.

How can we describe this motion theoretically to compare to experimental results? We know if we shake a string (fastened at one end) the resulting wave on the string depends upon how quickly or slowly we shake. At times the string appears to have almost no wave traveling up or down the string and at other times the wave is visible and appears to have points on the string where that appear motionless. Thus, at certain frequencies waves have small amplitudes – at other specific frequencies the amplitude increases and the maximal amplitude wave becomes defined as seen in figure 1. What is going on when this happens? The frequency is matching with the length of string such that the wave exhibits maximal amplitude. Why this happens is the theoretical topic to discuss.

If we look at the shape of the waves of the string as we shake it – we see that it appears to be sinusoidal. We know from last week that this means that how stretchy the string is will dictate how high the maximal amplitudes can be.
We also know that there is a fundamental frequency that the string will oscillate in just the same way as the spring/mass system. So – how can we determine this fundamental frequency? Think about the frequencies inverse AND the shape of a sine wave. Last week we saw that the period of oscillation is related to the frequency of oscillation, namely T = 1/*f*.

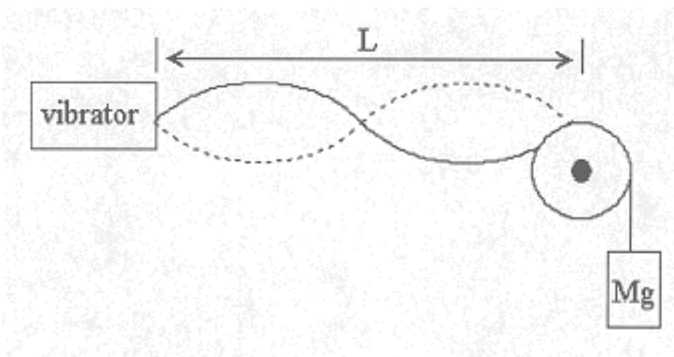


Figure 1: Sketch of experimental setup

We see the sketch to the left that the wavelength for this sine wave is L – the length between the two end nodes (1 by the oscillator/vibrator and the other by the pulley/mass) when the amplitude is at its maximum. This makes intuitive sense because if the wavelength was longer – the end node at the pulley would force the wave to zero earlier than its natural zero point. If the wavelength was shorter, the wave would begin to rise and then again be forced to zero at the pulley.

We know when determining the wavelength of any signal we must find the point where the periodic signal begins to repeat itself, thus defining the wavelength as the distance the periodic wave form is unique.

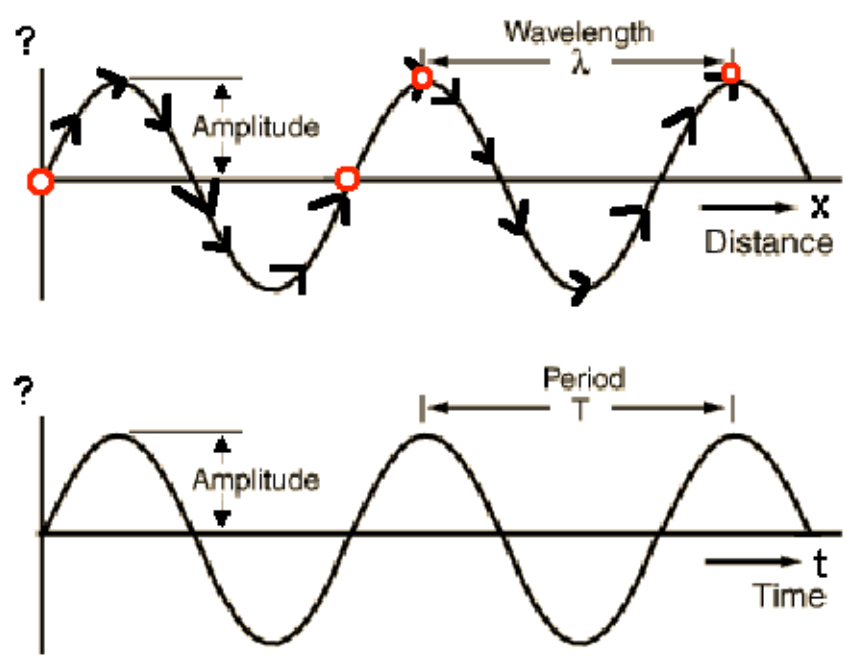


Figure 2: wavelength and period measurements

Look at the image to your left and notice the arrows showing the direction of the wave progression with respect to distance and time. These graphs show how the wave travels down the string.
When sketching arrows on a periodic signal such as the sine wave shown here, 1 wavelength (in length) or 1 period (in time) is found when the arrows cross the starting position and travel in the

same direction. Notice that one can determine the wavelength by either starting at a zero crossing (and finding the next zero crossing with arrows in same direction – thus restarting the signal OR by looking at the change in distance from 1 peak to the next peak. These are the two most convenient points used typically although theoretically any starting point can be used as long as one stops when the starting point is found again with the arrow in the same direction.
The same is true for the period. Notice in the 2nd graph in the image the x-axis is time rather than distance – thus the time interval now measures the period of the oscillation rather than the wavelength. It is important to read axes when trying to determine attributes of a signal and not confuse the period and wavelength measurements.

Now that we can find the wavelength and period of a periodic signal such as the sine wave we need to apply this to the problem at hand. In the image on the first page one sees that many periods or ½ a period can form on a string fixed at two ends. We also see the word 'fundamental' by the ½ period shown in the image. This is the wavelength (or frequency) that will allow for ½ a period to form on the string. So, think about the standing wave – the wave travels DOWN the string and then reflects and RETURNS. We see this from the +maxima traveling down and the –maxima reflecting back. Our eyes see this as standing still because the motion is very quick (higher frequency). If one uses a strobe – which is a light source the flickers at a frequency set by the user – one can see the wave actually move down and reflect back on the string. It is important for you to realize that it is moving even though I eyes see the wave as appearing to be standing.

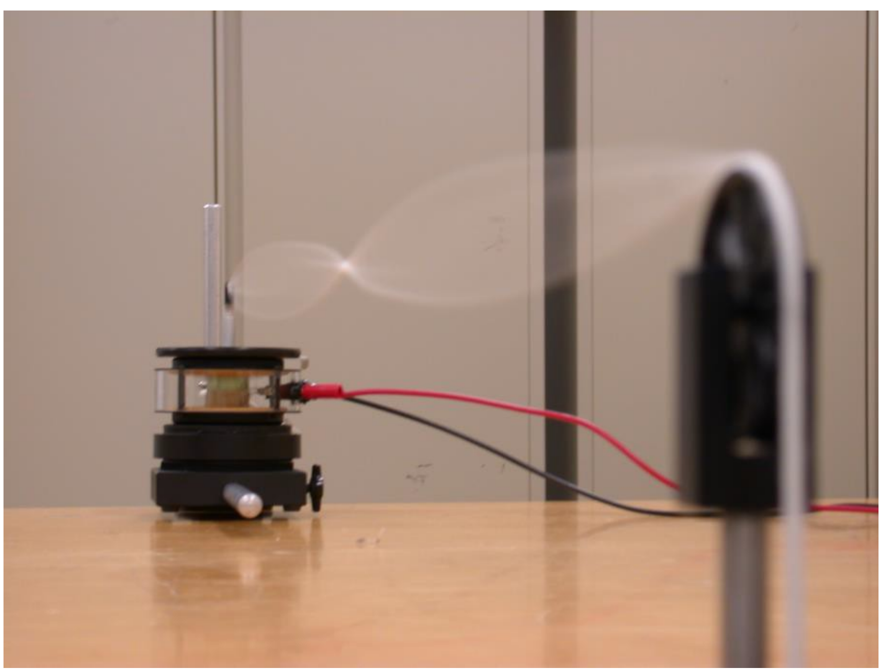


Figure 3: Experimental Setup with Pasco equipment

For propagation of these waves on a string fixed at both ends (as opposed to a hollow tube with one end closed), standing waves occur when half the wavelength is equal the length, *L*, of the string, or when two half wavelengths equals the length, or three half wavelengths, etc. If we want to create an equation that will properly represent this we need to first write the equation for ½ wavelength.

$$\frac{1}{2}\lambda = L$$

Now, we also know that we can use multiples of this equation to solve for shorter wavelengths.

So, we can have (2) half wavelengths or $2 * \left(\frac{1}{2}\right)\lambda = L = \lambda$ and if one has 3 half wavelengths then $3 * \left(\frac{1}{2}\right)\lambda = L = \frac{3}{2}\lambda$. If we continue in this way we see that the wavelength increases in increments of ½ wavelengths. Thus we can write the general equation as:

$$\frac{n}{2}\lambda = L \text{ and solving for wavelength we find: } \lambda = \frac{2L}{n}$$

for n = 1, 2, 3, …

Now, how can we relate this to a frequency. Since the Capstone software only allows users to change the frequency of the oscillation we need a way to convert the wavelength into frequency to conduct the experiment examining the relationship between the frequency and the number of antinodes (or maxima) for each standing wave found.

If we think about the units of both the frequency and the wavelength we will find that these values determine the wave speed. Since frequency = Hertz = 1/seconds and wavelength = meters if we multiply these two values together we find speed:

$$f * \lambda = \frac{1}{s} * m = \frac{m}{s} = v$$

Ok, so if we can determine the speed of the wave – we can determine the frequency that we need to use the Capstone software to collect data of changing frequency. But how can we do this? Well, it turns out if we think about the velocity in terms of the force of tension on the string and the linear mass density (how much the string weighs on Earth per unit length) we can find the velocity. It should make sense that how fast the wave travels should depend upon the force pulling on the end of the string (over the pulley) and the mass of the string itself as the thicker and less stretchy the string the more difficult it will be to create a wave of any amplitude on it.

Let's look at dimensional analysis and see if we can reason the velocity equation from the units of these two values that we theorize must influence the velocity.

$$Force = F_{tension\ on\ string} = mg = kg * \frac{m}{s^2}$$

$$\mu = \frac{mass}{length} = \frac{kg}{m}$$

$$velocity = \frac{m}{s}$$

Inspection above gives the theorized equation from your text, namely:

$$v = \sqrt{\frac{F_{Tension}}{\mu}} = \sqrt{\frac{kg * {}^{m}/_{s^2}}{{}^{kg}/_{m}}} = \sqrt{\frac{{}^{m}/_{s^2}}{{}^{1}/_{m}}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$

Now, we have an equation for the velocity and we can plug this into the equation above to determine frequency as well as write explicitly the equation for any wavelength previously determined:

$$f * \left[\frac{2L}{n}\right] = \sqrt{\frac{F_{Tension}}{\mu}} = \sqrt{\frac{mg}{\mu}} \quad \text{Equation 1}$$

and if we solve for the number of antinodes (the integer 'n' in the equation we find:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} \text{ Equation 2}$$

from this equation we can find the relationship between the # antinodes (n) and the frequency (*f*). If we hang a 200g mass over the pulley and have the distance from the pulley to the oscillator = 1m we can determine the mass density of the string. This has been done for you and the mass density of the string with a 200g mass attached to the end is:

$$\mu = 0.00401 \pm 0.00004 \ \frac{kg}{m}.$$

Think about why this linear mass density needs the mass on the end of the string determined to provide an precise measurement? Hint: Is the string stretchy? How could this impact mass if a heavier mass is attached to the end of the string?)

Proper experimental technique requires that ONLY 1 variable in a predictive equation be changed at a time to determine how this variable affects another. In this case – we would want to conduct three experiments examining how the number of antinodes is affected by changes in the frequency applied to, the length of, and the tension on a string in resonance (standing wave). To do this one would write each equation isolating the variable and setting all other values as constants – this means that during experimentation ONLY the ONE value should change to see how this value affects the antinodes written as '*n*'.

To examine how the frequency affects the number of antinodes – arrange the equation as shown:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} = f * \left(2L * \sqrt{\frac{\mu}{mg}}\right) = n = constant \ * f$$

$$constant = \left(2L * \sqrt{\frac{\mu}{mg}}\right)$$

To examine how the length affects the number of antinodes – arrange the equation as shown:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} = L * \left(2f * \sqrt{\frac{\mu}{mg}}\right) = n = constant \ * L$$

$$constant = \left(2f * \sqrt{\frac{\mu}{mg}}\right)$$

To examine how the tension on the string affects the number of antinodes – arrange the equation as shown:

$$n = 2Lf * \sqrt{\frac{\mu}{mg}} = \frac{\left(2fL\sqrt{u}\right)}{\sqrt{mg}} = \frac{constant}{\sqrt{mg}} \qquad \text{with} \qquad constant = \left(2fL\sqrt{u}\right)$$

Now, with the equation above – written to isolate each of the three variables predict the shape of the following plots:

Antinodes vs. frequency
Antinodes vs. length
Antinodes vs. force of tension

Discuss in your group and sketch the 3 plots to turn in which your group lab report. It is not important if after conducting the experiment you find your sketches are not correct – so please do not change sketches – you only need state what was wrong with your group's thinking at first that was realized after the data was plotted. Make sure your instructor sees the 3 sketches before you start your experiment.

Because of time restrictions we can only conduct two experiments today. Discuss why it has been chosen that your group examine the frequency vs. antinodes and the tension vs. antinode and have chosen not to conduct the length vs. antinode. Write down your hypotheses to be turned in with your report.

Experimental Procedure: (Note that a full procedure section is not required as steps below provide questions that will allow the reader to understand how your group setup the equipment for measurements.)

1. Setup the distance between the pulley and the oscillator to 0.60m. Use a meter stick to take this measurement and discuss HOW your group determined the end point for the pulley node. Make sure to detail HOW your group determined the end points of this measured distance **L** in your group report.
2. Connect the two wires (black and red) from the Pasco oscillator to Output 1 in the 850 Interface.
3. Open Capstone. Click on Signal Generator. Click on 850 Output1. The waveform should be set to Sine. Start at a frequency of 25 Hz and increase it in increments of 1-10 Hz looking for the resonant frequencies on the string that has 4 antinodes.
4. Once the string is close to resonance (you observe a stable standing wave), change the stepping increment to 0.1 Hz and fine-tune your resonant frequency. You may need to go back and forth a few times to determine the frequency at which the LARGEST displacement occurs- -that is your resonant frequency for the 4 antinode oscillation, i.e. n = 4. Record this resonant frequencies and number of antinodes in a table in Excel.
5. To examine how the frequency affects the # of antinodes follow steps below.
    a. Keeping the length and the mass constant, change the frequency required to produce antinodes n=1,2,3,5,6,7,and 8. Record these frequencies and antinodes in Excel.
6. Now, return the oscillator to the frequency found in Step 4 to examine how the tension affects the number of antinodes.
    a. Keeping the frequency and the length constant, change the mass (and therefore Tension) on the string until you find as many standing waves as possible

between 0.05kg – 0.80kg. Note that your group should be able to find 5 data points.

7. Once all data is collected – write you findings on the board to share with other groups.
8. Create graphs of the antinode vs. frequency and antinode vs. tension in Excel. On each graph plot your data and the average class data. Calculate the standard deviation of the class data for both data sets.
9. These stdev values can be added as uncertainty to the plotted average values. Add these uncertainties as error bars to the plots and discuss if your group's data is within uncertainties of the average class data. Why might your data be inconsistent with the class data? Discuss.
10. Using the trendline feature in Excel, choose the appropriate fit for the two plots and fit your group's data. Make sure the fit equations show on the two graphs.
11. Now, for all linear data – compare the slope value from Excel to the constant value shown in the theory section (note each constant value differs depending upon which variable is examined – see page 5.) How does your data compare to the predicted constant value calculated? Discuss.
12. If the data is not linear – compare the calculated constant value for the proper equation shown on page 5 to the fit constant from the Excel trendline feature. How does your data compare to the predicted constant value calculated? Discuss.
13. The value of μ is only known to 2.5% precision. Is this uncertainty sufficient to state that your Excel fit constants equal the calculated constant value for these experiments? Explain.
14. Discuss in your group the impact of the pulley friction. How does your group predict this friction, if present, would impact data results? Explain reasoning.
15. Discuss the stretchy string – if the μ changes during the experiment with changing masses would its value tend to increase or decrease? Explain.
16. Using this explanation hypothesize what would happen to the data if this change in μ was significant. What this change seen in your group's data? Discuss.