Pendulum

Introduction

This experiment is designed to study the motion of a pendulum. The pendulum consists of a rod and a mass attached to it. You will measure the period of the pendulum. Using the period of the pendulum, you will calculate the gravitational acceleration and test whether your measurements agree with the theoretical value.

Reference

Freedman & Young, University Physics, 12th Edition: Chapter 13, section 5, and 6.

Theory

A *simple pendulum* consists of a small mass attached to a massless string (or rod). For a small angle of oscillation, less than 10 degrees, the period of the pendulum is given as:

$$T = 2\pi\sqrt{\frac{d}{g}} \tag{1}$$

where d is the length of the string in meters and g is the gravitational acceleration in m/s^2.

A *physical pendulum* is an extended body which oscillates about a fixed point. The period of oscillation for a physical pendulum is given as:

$$T = 2\pi\sqrt{\frac{I}{m_0 gH}} \tag{2}$$

where m_0 is the mass of the extended object, H is the distance from the center of mass (CM) of the extended object to the pivot point, g is the gravitational acceleration and I is the moment of inertia of the extended object about the pivot point.

The moment of inertia of a rod rotating about its CM is given by:

$$I_{CM-rod} = \frac{1}{12}ml^2 \tag{3}$$

where l is the length of the rod and m is the mass of the rod. For an oscillating rod, the distance from the pivot point to the CM of the rod is h, therefore, the moment of inertia of the system can be written as:

$$I_{rod} = \frac{1}{12}ml^2 + mh^2 \quad (4)$$

The moment of inertia of the hanging mass M from the pivot point is:

$$I_{mass} = Md^2 \quad (5)$$

where d is the distance of the mass M to the pivot point. The total moment of inertia of the oscillator is the combination of the two moments of inertia:

$$I = I_{rod} + I_{mass} = \frac{1}{12}ml^2 + mh^2 + Md^2 \quad (6)$$

The CM of the extended object can be found using the following equation:

$$m_0H = mh + Md \quad (7)$$

Equation (2) can now be re-written as:

$$T = 2\pi\sqrt{\frac{I}{m_0gH}} = 2\pi\sqrt{\frac{\frac{1}{12}ml^2 + mh^2 + Md^2}{(mh + Md)g}} \quad (8)$$

Procedure

1. Remember in all data collecting today that we are assuming the period of the pendulum will not change as long as the initial angle is smaller than 10 degrees. This means that each time the pendulum is put into motion as long as the angle of oscillation is less than 10 degrees, the period should remain constant. This assumption holds true when damping is ignored, which is a good approximation during short time scales.

2. Most cell phones have a stop watch feature. If you do not have a stop watch feature on your phone, you can use an online stopwatch or other time piece to measure the time. Record the time it takes for the pendulum to complete one period. Repeat this process 10 times so that you have 10 measurements of a single period of the pendulum. Calculate the average period and the standard deviation of these 10 measurements.

3. Now, measure the time for 10 contiguous periods. Repeat this step 10 times. Calculate the average and standard deviation of the measurements. Now, divide the average and the STDEV by 10 and compare the value to individual period measurements calculated in Step 3.

4. Now you will collect data of the period of the pendulum using the computer. You will use the photo-gate sensor and the rotational motion sensor. The pendulum will oscillate between the U-sides of the photo-gate sensor, while the pendulum is attached to the rotational motion sensor. The rotational motion sensor will measure angular position as the pendulum swings. The photo-gate sensor will measure the period of oscillation directly as it passes through the U-sides of the sensor.

5. The photo-gate works by maintaining a signal, a red light, between the two U-sides of the sensor. Look at the sensor; notice two small holes on each of the U-sides of the apparatus. A signal is sent and received at these points. If you place the pendulum between these holes, the signal is 'blocked'. We will measure the pendulum's period using the 'blocking' of the signal by the pendulum. To examine a full period of the pendulum, we need to measure the time from the first pass through the photo-gate until the pendulum passes again the in same direction. Three passes constitute one period. This process is shown in Figure 1.

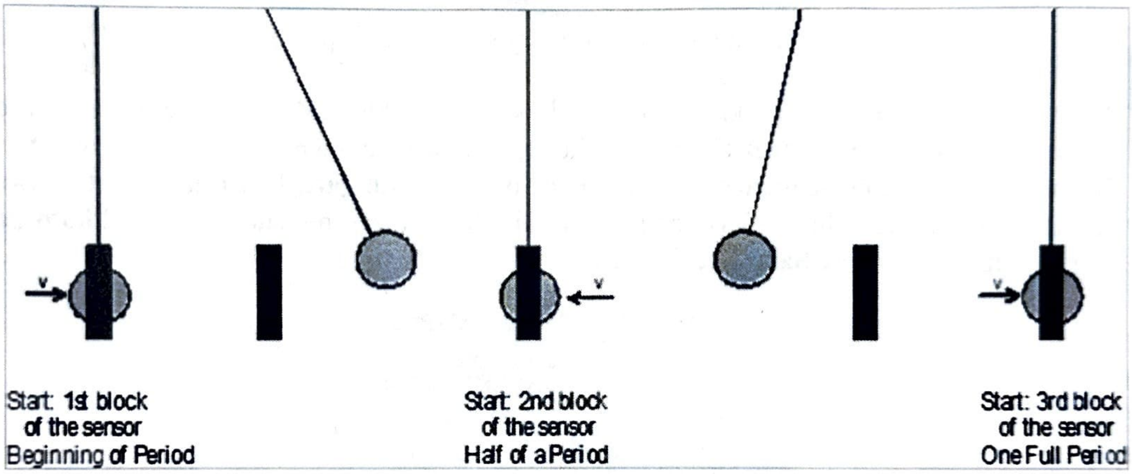

Figure 1: Photo-gate process.

6. The photo-gate sensor must be oriented perpendicular to the pendulum's path to function correctly. Be sure that the screw, which fastens the weight to the rod, is facing one of the sensor hole in order to avoid asymmetric blocking. Make sure the photo-gate sensor and the rotational motion sensors are plugged into the **PASCO** interface box. Find the **photo-gate** on the sensor list in the **Hardware** icon after clicking on port 1 on the left. Do the same for the rotational motion sensor but put the yellow and black leads into ports 3 and 4 respectively, see Figure 2.

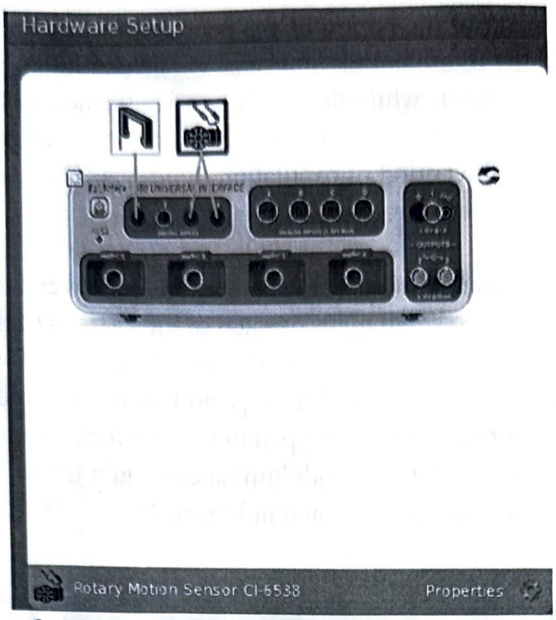

Figure 2: Photo-gate and rotary sensors in Capstone.

7. From the discussion above it is known that the sensor will be 'blocked' three times for one period. In **Capstone,** under the **Timer Setup** tab, choose **"Build your own Timer"**. Click on **"Next"** button. Under **"Timer Sequence Devices"**, click on the **photo-gate, Ch. 1** and select **blocked.** Do this three times. Click **"Next"** and rename the timer measurement name to **"Photo gate Period"** and then choose **Save.** See Figure 3.

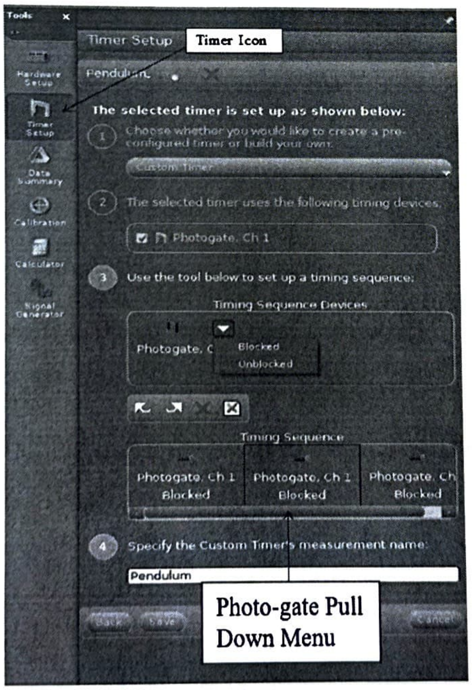

Figure 3: Setup photo-gate sensor.

8. The rotational motion sensor will collect data of the angular position of the pendulum. Inspecting the angular position versus time plot of the pendulum, we can measure the length of time for one

period. Click the Icon for the **Rotary Motion Sensor**. Then click on **Properties** on the bottom right. Make sure that under the drop-down box labelled "Linear Accessory" the option **Rack and Pinion** is selected.

9. Drag two graph icons into the workspace area assigning each sensor to one of the graphs. Set the y-axis in the upper graph to "**Angle (rad)** and in the lower graph to "**Photo gate Period**", from the selection list.

10. Make sure before you swing the pendulum you press the **Record** button. This will allow the rotational motion sensor to calibrate its 0°. Start acquiring data for at least 15 seconds. Look at the Photo Gate Period versus Time graph. If you start the swing with an angle larger than 10°, you will notice an initially strong downward trend. Let the motion settle until the period remains basically constant. This the portion of data you must use in your analysis.

11. Now, let us turn our attention to the photo-gate sensor graph. This graph allows us to examine the periods of oscillation over time. Calculate the mean and the standard deviation of this data set. To calculate the mean and the standard deviation go back to the **Photo Gate Period** graph and click on the Σ icon. A drop down window will appear where you can select Mean and Standard Deviation by checking them. Now, look at the graph legend box and you will see these values, see Figure 4.

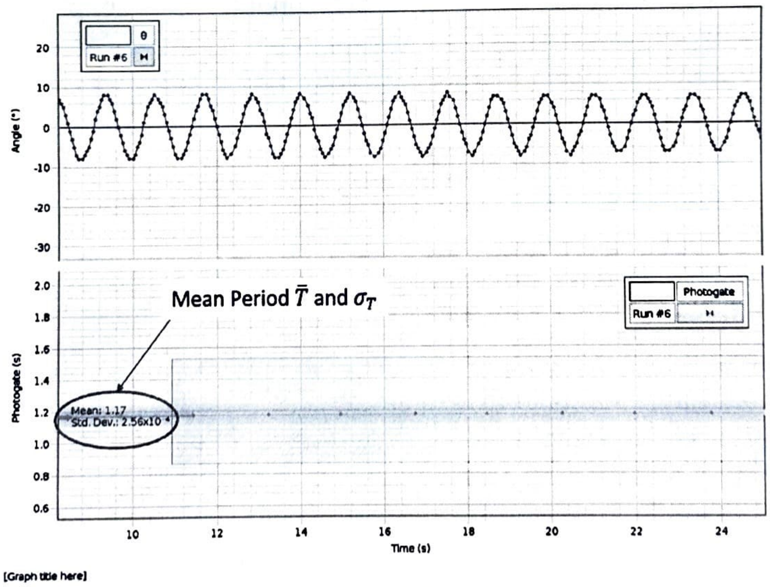

Figure 4: Plots of angular position vs time, and pendulum period vs time.

12. Calculate the CM of the rod/mass system, H, using Equation (7).

13. Using the stop watch data calculate the acceleration of gravity, with uncertainty, as if it were a simple pendulum, use Equation (1) and use d instead of l. To calculate the uncertainty in g, solve Equation (1) for g, and propagate the uncertainties in length and period, using the partial

derivatives in quadrate that we introduced last week. Does the value of g fall within 1 to 2 standard deviations of the accepted value?

14. Now calculate the acceleration of gravity using the Equation (8) for the physical pendulum. Use Equation (2) to find the uncertainty, and check to see if the value of g fall within 1 to 2 standard deviations of the accepted value? Make sure to write down your formula for the propagated uncertainty for g in your notebook.

Repeat steps 13 and 14 using the average value for the period and its uncertainty measured using the photogate.

Make the following table in Excel and fill in all the data. Use Equations (1) and (8) to calculate your experimental value of g, using the period of oscillations you measured.

Method	Period T and g (m/s^2) using Simple Pendulum Data		Period T and g (m/s^2) using Physical Pendulum (mass and rod)	
	T	g	T	g
Stop Watch (10 times)				
Photo-gate				

Which timing method gave you the best result? Which formula (simple or physical pendulum) gave you the best result? Explain.