RC and RL Decay

Objectives:

1. Study charge/discharge in RC and RL circuits.
2. Understand the relationship between voltages across R and C (or L).
3. Understand sampling rate in measuring the signal of an exponential function.

I. Theory:

RC Circuit

In a circuit consisting of *R*, *C* and a power supply/switch combo (all in series as shown in Figure 1(a), the switch can be set to two positions: when connected at Position 1, the capacitor is charged by the power supply with a constant voltage (or amplitude) V_0; when set to Position 2, the power supply is taken out of the closed circuit and the capacitor discharges.

Let us consider the *charging* process first. When the switch is turned to Position 1, the current starts flowing into the circuit, which results in charges accumulating on the plates of capacitor. This also causes the voltage across the capacitor to increase (while the current decreases gradually) till the capacitor is fully charged to the voltage V_0. It should be noted that the circuit in this case is the same as Figure 26.22 in your textbook (which can be solved using Kirchhoff's loop rule as shown in Section 26.4). Adopting the textbook's solution (equation 26.13), we have the current in this circuit as a function of time,

$$i(t) = I_0 e^{-\frac{t}{\tau}}, \tag{1}$$

which indicates an exponential decay. The maximum current $I_0 = V_0 / R$ appears at $t = 0$ when the circuit is connected to the power supply, while the time constant $\tau = RC$ depends on the resistance and capacitance in the circuit. It can be seen that at time $t = \tau$, the current decreases to $I_0 / e \sim 0.37 I_0$, while at $t >> \tau$ it decays to zero (**NOTE:** $e \approx 2.718$ here is the math constant, not the charge of electron). The quantity RC has units of seconds, while R and C are in ohms and farads, respectively.

Applying Ohm's law to equation (1) allows the voltage across *R* to be easily calculated,

$$v_R(t) = V_0 e^{-\frac{t}{\tau}} . \tag{2}$$

So by applying Kirchhoff's loop rule in this simple series circuit ($V_0 - v_R(t) - v_C(t) = 0$), we can get the voltage across *C* as,

$$v_C(t) = V_0 (1 - e^{-\frac{t}{\tau}}) . \tag{3}$$

Equations (2) and (3) signify that $v_R(t)$ decays with time, while $v_C(t)$ gains the same amount in charging the capacitor.

Now let's consider the *discharge* process. When the switch is turned to Position 2, the power supply is switched out (or disconnected) from the circuit (similar to Figure 26.22 in your textbook). When this happens, the previously charged capacitor starts discharging through the closed circuit. The physics of this case is fully solved in Section 26.4 of your textbook, where the current (defined in equation 26.17) is exactly the same as equation (1), except it runs in the opposite direction. Thus, the voltage across *R* is

$$v_R(t) = -V_0 e^{-\frac{t}{\tau}}, \tag{4}$$

and across *C* is

$$v_C(t) = V_0 e^{-\frac{t}{\tau}},$$

where $t = 0$ in equations (4) and (5) is the moment when the power supply is switched out of the circuit. In othe words, $v_R(t)$ starts at $-V_0$, $v_C(t)$ at V_0, and both end up at zero as $t \to \infty$.

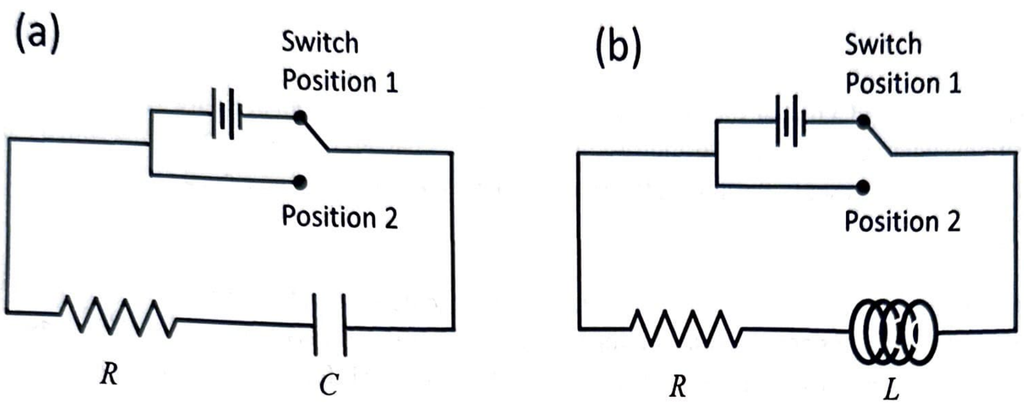

Figure 1. RC circuit (a), and RL circuit (b).

RL Circuit

The RL circuit (Figure 1b) is similar to the RC circuit (Figure 1a), except the capacitor *C* is replaced by an inductor *L* (it's also been fully solved in Section 30.4 of your textbook). First turning the switch to Position 1 at $t = 0$, the current in a RL circuit will start from zero, increase gradually with time, and reach the maximum $I_0 = V_0 / R$ at $t >> \tau$ (according to equation 30.14),

$$i(t) = I_0(1 - e^{-\frac{t}{\tau}}).$$

It can be seen that at $t = \tau$ (where the time constant $\tau = L/R$ depends on the resistance and the inductance), the current increases to $I_0(1 - 1/\mathrm{e}) \sim 0.63 I_0$. **NOTE:** in order to make L/R in the units of seconds, L and R have to be in henrys and ohms, respectively; be sure to keep everything in SI unit.

Applying Ohm's law to the resistor gives the voltage across as,

$$v_R(t) = V_0(1 - e^{-\frac{t}{\tau}}), \qquad (6)$$

and likewise applying Kirchhoff's law will give the voltage on the inductor as,

$$v_L(t) = V_0 e^{-\frac{t}{\tau}}. \qquad (7)$$

Thus it can be seen from equations (6) and (7) that at $t >> \tau$, the circuit reaches a steady state where nothing varies with time any more; in this case i and v_R maximize at I_0 and V_0, respectively, and v_L reduces to zero.

After the circuit reaches this steady state, set the switch to Position 2. Without the power supply, the current in the circuit starts decaying (according to equation 30.18 in your textbook),

$$i(t) = I_0 e^{-\frac{t}{\tau}}.$$

, voltages on R and L are, respectively,

$$v_R(t) = V_0 e^{-\frac{t}{\tau}} \tag{8}$$

$$v_L(t) = -V_0 e^{-\frac{t}{\tau}}, \tag{9}$$

with the time constants defined as $\tau = RC$ and $\tau = L/R$, respectively.

II. Experiment

A. RC Circuit

Step 1. Pick R and C with nominal values: R=3.9kΩ and C=2.2μF. Measure the actual values of both and calculate instrument uncertainties. Calculate the time constant τ and associated uncertainty.

Step 2. Make a RC circuit according to Figure 1(b). The power supply/switch combo is provided by the output ports on the Pasco interface when the signal generator is set to a "positive-square wave". The hardware/software setting procedure is similar to the previous lab (Induction and Magnetism).

Signal generator setting: waveform: positive-square wave; amplitude: 2V; offset: 2V; frequency: make the frequency correspond to a period ~10τ (**HINT:** solve for this).

Step 3. Measure the $v_R(t)$ and $v_C(t)$ at the same time. Connect voltage sensors between your circuit components and channels A and B on the Capstone interface. Plot all three voltage waveforms, $v_R(t)$, $v_C(t)$ and the output from the signal generator, in the same graph and compare. *Does the trend agree with the theory qualitatively?*

Step 4. Pick a decay portion from $v_R(t)$ and fit it with an exponential function. From this fit, find the time constant and uncertainty. *Does it agree with the calculated value?* Do the same to $v_C(t)$.

B. RL Circuit

Use R=3.9kΩ and the large coil to make the circuit. The coil is mainly an inductor ($L = 0.80 \pm 0.04H$), but with a resistance as well ($R_L = 63.5\Omega$). Make sure you use the total resistance $R + R_L$ when calculating the time constant. **You need to adjust the frequency of the square wave AND the sampling rate according to the time constant.** Follow the same steps as in Part A. Record $v_R(t)$ and $v_L(t)$, fit the decay, and find the time constant and uncertainty. *Answer the same questions in your discussion.*

III. Lab report:

III. Lab report: Standard lab report, including summary of theory (focus on relevant equations) and procedure, data table(s) (for time constant calculation), plot(s) with fit function, and discussion.