RC circuit response to sinusoidal input voltage

Objective: Learn how a resistor and capacitor in series react to a sinusoidal voltage source.

Introduction:

In a RC circuit with a sinusoidal voltage source (shown in Fig.1), the voltage and current on

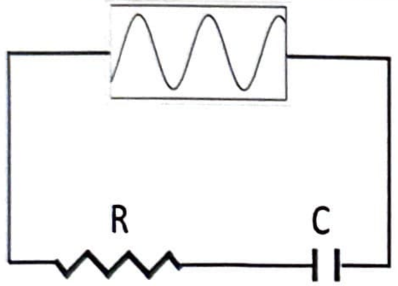

Figure 1. Circuit diagram

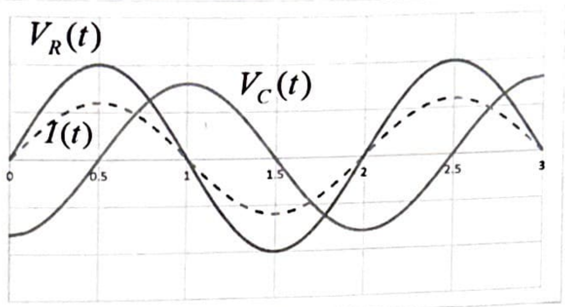

Figure 2. Voltages and current.

each component (R or C) are also sine functions.

The source voltage from the signal generator can be written in a sine function of time.

$$V_0(t) = A_0 \sin(2\pi ft + \phi_0) \tag{1}$$

Where A_0, f, and ϕ_0 denote the amplitude , frequency , and phase, respectively.

The voltage across the resistor is a sine function of the same frequency f, but with a different amplitude A_R and phase ϕ_R.

$$V_R(t) = A_R \sin(2\pi ft + \phi_R) \tag{2}$$

The current through the resistor is also a sine function with the same frequency and phase as the voltage. Its amplitude I_R relates to the voltage amplitude A_R by Ohm's law.

$$A_R = I_R R \tag{3}$$

On the other hand, the voltage on a capacitor is always lagged behind the current by 90 degrees while the amplitudes also obey the Ohm's law with the reactance X_C in place of resistance.

$$V_C(t) = A_C \sin(2\pi ft + \phi_C) \tag{4}$$

$$A_C = I_C X_C \tag{5}$$

$$X_C = 1/(2\pi f C) \qquad (6)$$

The current is the same for all components (R, C, and the signal generator) in the circuit since they are in series. This means that $V_C(t)$ is 90degrees behind $V_R(t)$), ie.

$$\phi_R - \phi_C = 90^\circ \qquad (7)$$

Also, the amplitude of the voltage function is proportional to the resistance (or reactance):

$$A_R / A_C = R / X_C \qquad (8)$$

Fig 2. Shows $V_R(t)$, $V_C(t)$, and the current in the circuit.

According to Kirchhoff's loop law, $V_0(t) = V_R(t) + V_C(t)$, which can be formulated into phasor addition (same as vector addition) shown in Fig. 3. The three amplitudes satisfy

$$A_0 = \sqrt{A_R^2 + A_C^2} \qquad (9)$$

And the phases are related by

$$\phi_0 - \phi_C = \tan^{-1}(A_R / A_C) \qquad (10)$$

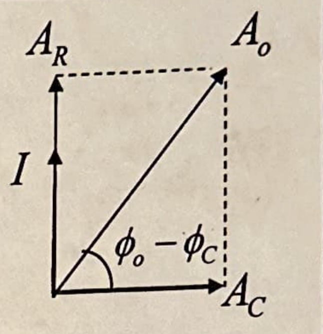

Figure 3. Phasor diagram.

Experiment:

1. Choose a capacitor with nominal value $2.2\mu F$ and a resistor with nominal value 3900Ω. Measure the actual values with a multimeter and calculate the associated uncertainties
2. Assemble a RC circuit as shown in Fig.1 using the voltage source from the Pasco interface. Set the signal generator in the Pasco Capstone to Sine Wave, frequency to 25Hz, and amplitude to 2V. Attach the voltage sensors to resistor and capacitor.
3. Pick a suitable sampling rate and plot voltages across the resistor, capacitor, and the source (output from Pasco).
4. Measure the amplitude (maximum value) of each function. Verify equations (7) through (10).
5. Now, change the frequency by at least doubling its value. Repeat steps 3 and 4 for this new frequency.

Discussion:

Do measured values satisfy the equations in steps 4 and 5?

When the frequency is changed, what happens to the voltage maxima in the graph? Is this expected? Why?