

## Frequency Components of Non-Sinusoidal Waves

### 1 Purpose

(A) The frequency components of a square wave are obtained by capturing the waveform and using the FFT algorithm. They are compared with the frequency component of a sine wave of the same frequency. (B) Two sinusoidal waves with slightly different frequencies are added, and the frequency components observed, and compared with those obtained by passing the signal through a diode.

### 2 Introduction

It is possible to add up sine waves of different amplitude, frequency and phase to obtain almost any wave or pulse shape. This procedure is called “**Fourier synthesis**”; the reverse, extracting the frequency components which make up an arbitrary wave shape, is “**Fourier analysis**”.<sup>1</sup>

What we will do in this experiment is to capture some waveforms with the digital oscilloscope, and use the “Fourier Analysis” feature of the spreadsheet Excel to study the frequency components.

### 3 Pure Sine Wave

Connect the waveform generator directly to the oscilloscope: Set the frequency to about

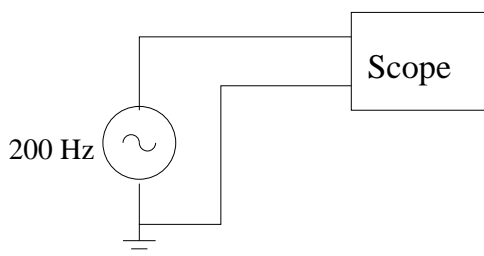


Figure 1: Setup for single waveform.

200Hz, and set the waveform to sinusoidal. The amplitude should be about 1 volt. Set the time scale so that the scope shows at least 5 cycles. Read this waveform out with

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<sup>1</sup>To be precise, when the wave form,  $f(t)$  is periodic, it may be written as an (infinite) Fourier series. When the wave form is aperiodic, i.e. a single “pulse”,  $f(t)$  is expressed as an integral, the Fourier transform, instead of a series. And when  $f(t)$  is only a discrete set of  $N$  “samples”, as we would get from an oscilloscope capture, it is expressed as a finite series called the “discrete Fourier transform” or DFT. A special algorithm to perform the DFT rapidly is the “fast Fourier transform” or FFT.

the Waveform Data Capture program. Copy the data to the clipboard and paste it into Excel.

## 4 Square Wave

Now simply switch the waveform generator waveform to square wave, changing nothing else. Repeat the above capture procedure. This time leave at least 3 columns free between the first data set and where you paste in this one.

## 5 Finding the Frequency Components

Select **Data, Data Analysis, Fourier Analysis**. The instructions in this and the following section assume that the times are put in column A, and the signal voltages in column B. As Input Range, enter B1:B2048, if your voltage data is in column B. (We can't use all 2500 points from the scope because the algorithm requires that the number of points be a power of 2.) Select Output Range and enter C1:C2048.

What you will see next is a set of two numbers for each cell. This is because the frequency decomposition algorithm (the FFT) produces them. They can be put into the form of a magnitude and a phase. Excel's algorithm produces 2 components, the real and imaginary parts of a complex number. In this experiment, we are only interested in the magnitude, so in the adjacent column, (D), enter =IMABS(C1) and copy this to the rest of the column.

## 6 Frequencies

The FFT algorithm produces frequency components over a finite set of frequencies. The lowest one is 0, corresponding to a constant level. The highest one is:

$$f_{max} = \frac{1}{2\Delta t}$$

where  $\Delta t$  is the time interval between samples. In addition, the second half of the output column is associated with negative frequencies. So in this data set, there are only 1024 positive frequencies. We are only interested in the lower part of the frequency spectrum, so in the next column, compute the frequency for each component for the first 200 numbers in the magnitude column. To do this, enter 0 in cell E1. In cell E2 enter =E1 + 1./((A3 - A2)\*2048.) . Then copy cell E2 to cells E3 to E200. This will calculate the correct frequency for the corresponding component in column D.

## 7 Continuing

Next, repeat the above procedure for the square wave data. You don't have to construct the frequency column, because it is the same as for the sine wave. Make a graph of the FFT magnitude vs. frequency, for the first 200 points. If the sine-wave frequency is  $f_0$ ,

do you see peaks in the square-wave spectrum at every integer multiple of  $f_0$ ? If not, for what “harmonics” do the peaks occur?

## 8 Making Beats

Now take 2 waveform generators, and add their signals as shown in Figure 2 below. Make the generator amplitudes each about 5 volts.

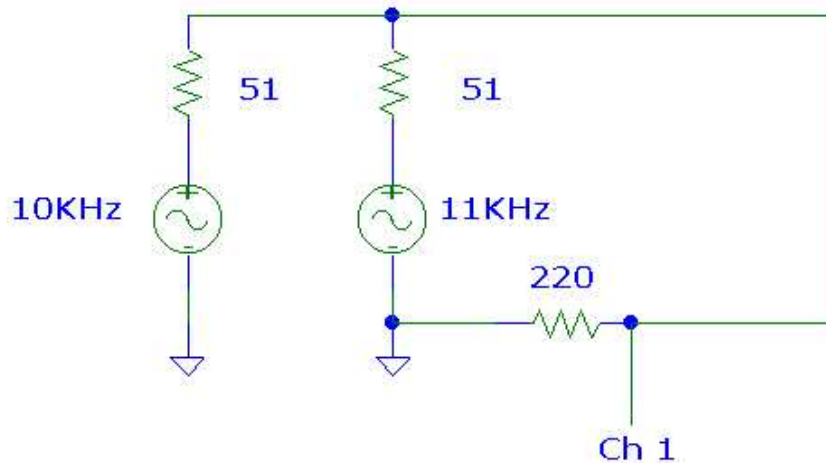


Figure 2: Setup for addition of two sinusoidal signals.

On the scope, you can see the “interference” between the two signals. Set the time scale so that you can see at least 5 beats. An example of this is shown in Figure 3.

Now repeat the capture and FFT analysis you did above. Is there a peak in the frequency spectrum corresponding to the “beat” frequency?

## 9 Effect of a non-linear element

A non-linear element (one for which the output current is not proportional to the voltage drop) has interesting effects on the frequency response. The simplest non-linear circuit element is a diode. Insert a diode in your circuit as shown in Figure 4. A diode conducts in only one direction, and in the forward direction, the response is non-linear.

Repeat the capture and FFT analysis. Make a plot comparing the frequency spectrum with and without the diode. What additional frequencies does the diode produce?

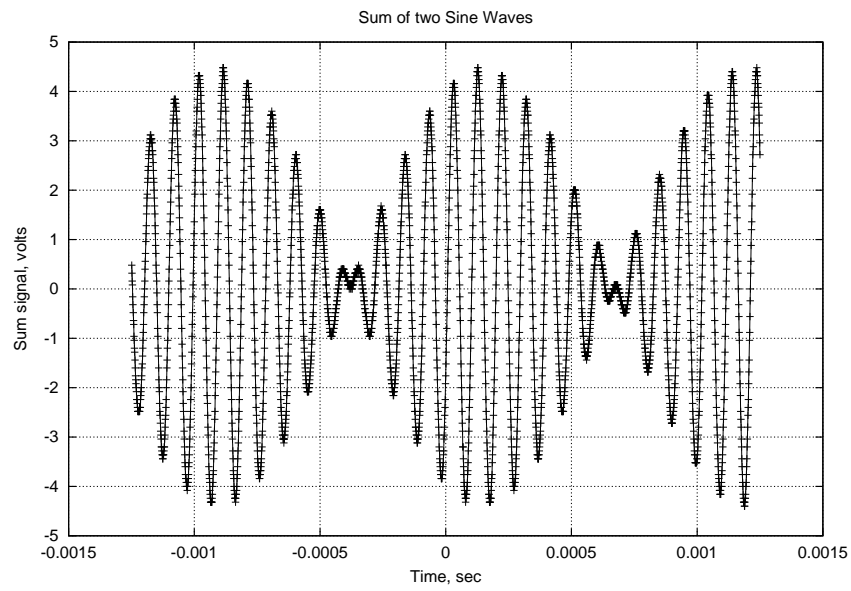


Figure 3: Sum signal vs. time

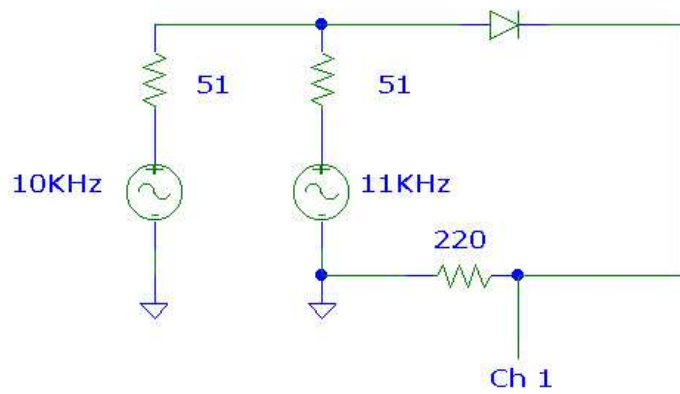


Figure 4: Signal addition with a diode included.

## 10 Frequency Components of A Rectified Beat Signal

Why should a rectified sine wave contain harmonics? We may write it as:

$$V(t) = +\sqrt{\sin^2 \omega t} + \sin \omega t$$

This is shown in Figure 5.

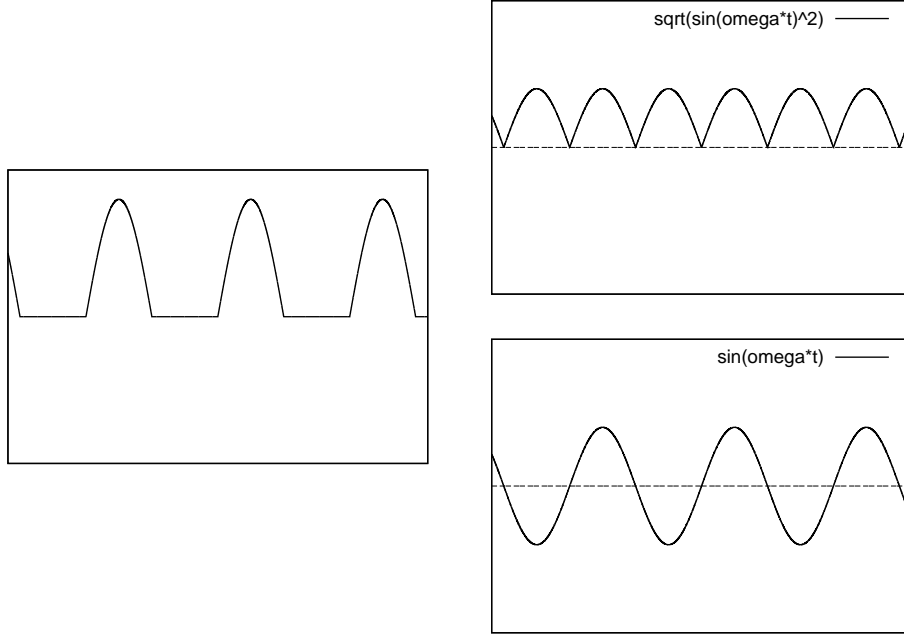


Figure 5: Left: rectified signal; Right:  $\sqrt{\sin^2 \omega t}$ ;  $\sin \omega t$

For two sine waves,

$$V(t) = \sqrt{(\sin \omega_1 t + \sin \omega_2 t)^2} + \sin \omega_1 t + \sin \omega_2 t$$

Expanding the square:

$$(\sin \omega_1 t + \sin \omega_2 t)^2 = \sin^2 \omega_1 t + \sin^2 \omega_2 t + 2(\sin \omega_1 t \sin \omega_2 t)$$

Trigometric identities can be used to show that

$\sin^2 \omega_1 t = (1 - \cos 2\omega_1 t)$  and  $\sin^2 \omega_2 t = (1 - \cos 2\omega_2 t)$  and that  $2(\sin \omega_1 t \sin \omega_2 t) = \cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t$

So the rectified sine wave contains **at least**

$\omega_1$ ,  $\omega_2$ ,  $\omega_1 - \omega_2$ ,  $\omega_1 + \omega_2$ ,  $2\omega_1$ , and  $2\omega_2$ . The  $\sqrt{\quad}$  in the equation generates additional harmonics.