Venturi Apparatus (Poiseuille's and Bernoulli's Law)

Your group experimented last week with volumetric flow through tubes of different radii. It was found that the radii of the tube influenced the velocity of the liquid traveling through the tube. Today – you will add to this knowledge by examining the relationship between the PRESSURE and VELOCITY change in a tube of changing radius. This relationship is called the Bernoulli Equation.

From the last week we know that the VOLUMETRIC FLUID FLOW, Q, CAN BE DEFINED AS:

$$\text{FLUID FLOW} = Q = \frac{CHANGE\ IN\ VOLUME}{TIME\ OF\ CHANGE\ IN\ VOL}$$

We can also define this volumetric flow as

$$Q = VELOCITY\ OF\ FLOW\ (v) * CROSS - SECTIONAL\ AREA\ OF\ TUBE(A) = v * A$$

From the definitions above it is seen that **Q (the volumetric fluid flow), must remain constant if a tube changes from r_o (large radius in Figure 1) to r in the continuous tube in Figure 1. This simply states that the tubes themselves are not leaking therefore and that the liquid must continue to flow and cannot remain motionless in either tube.**

Figure 1: Continuous tube of varying radii

Physicists call this equality in the fluid flow value a 'continuity equation' since it is a statement of CONSERVATION OF ENERGY (remember from previous lab Etot = KE + PE) since one cannot lose fluid (no leaking) therefore the overall volumetric fluid flow must remain constant from one point to another as the fluid travels down the tube. Since we are working here with VOLUME – we can think of the conservation of energy in terms of ENERGY/VOLUME. Look at the image from hyperphysics that provides both the ENERGY/VOLUME equation AND a graphic similar to Figure 1.

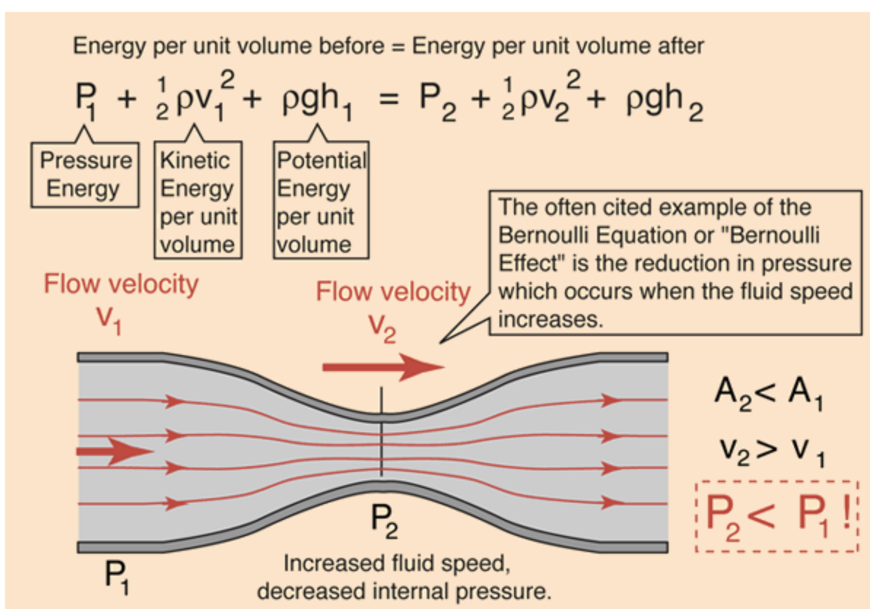

Figure 2: Hyperphysics image demonstrating conservation of Energy/Volume

We can write the Q equation for the values defined in figure 1 as:

$$A_o v_o = Av = Q \; as \; defined \; above$$

If this MUST be true – what happens to the velocity when the liquid moves from the larger radius area to the smaller radius area?
It is obvious IF the fluid must have the same FLUID FLOW that the velocity must INCREASE GREATLY in the smaller radius tube.
This seems to contradict what we saw in lab last week – but think carefully – in lab last week ALL the pails had approximately the SAME PRESSURE. So – for this equation above to be true PRESSURE MUST also change – but how?

Let's rewrite the conservation equation (see Figure 2):

$$\frac{Etotal}{Volume} = P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho_2 v_2^2 + \rho g h_2$$

But – if the tube is horizontal (as in the apparatus used in this lab) such that all liquid flows at the same height – then height does not change $(\rho g(h_1 - h_2) = 0)$ and we can simplify this equation to become:

$$\frac{Etotal}{Volume} = P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_2 v_2^2$$

Now – we can rearrange the equation so we write the change in pressure in terms of a change in velocity:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

since the fluid is the same through both tubes radii, $\rho_1 = \rho_2 = \rho$. Now – we have written above Bernoulli's Equation that is simply a restatement of conservation of energy in terms of volumetric fluid flow.

If we can measure the change in pressure and the change in velocity – we can demonstrate this Bernoulli Equation. Why is this equation important to biologist or earth scientists?

We can think again about blood flow in the body OR underground water reservoirs – fluids flows like magma through cracks in the Earth's crust to a pressure by the Bernoulli Equation. This gives us as scientist ONE more parameter – or variable – to test our results and use for experimentation. Bernoulli's equation allows us to look at pressure and velocity as the blood flows through a body's circulatory system.

Materials and Method:

In today's lab special equipment will be used that will allow 4 pressure sensors to measure the pressure changes as water passes through different radii openings. It is therefore essential that the water is free flowing without large air bubbles around the pressure sensor OR the pressure measured will not be of the flowing water in the apparatus. Look at the pressure sensor and Venturi apparatus in Figure 3; note that there are 4 pressure sensors – two located at wide areas of the apparatus and two others in the narrow radius sections. Therefore we expect the pressure to change as the water flows from the larger to smaller radius locations.

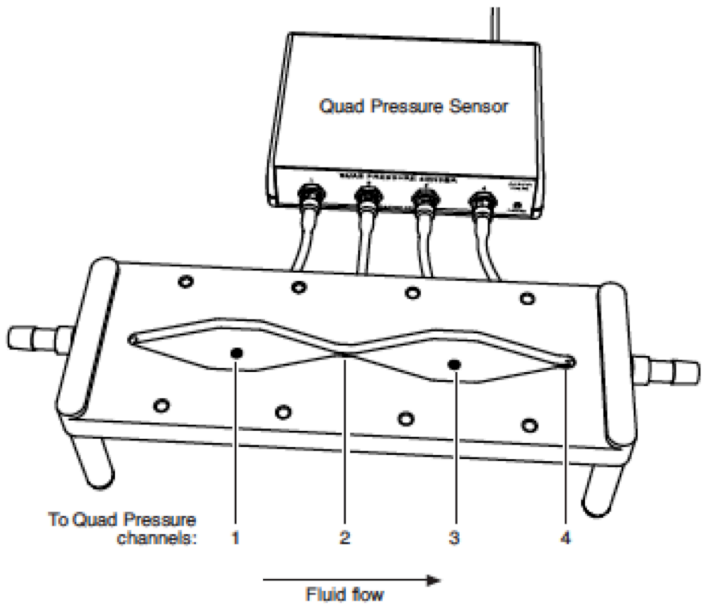

Figure 3: Venturi water flow apparatus and 4-port pressure sensor (provided by PASCO)

This apparatus and sensor system will be attached, via hoses, to a large graduated cylinder and water collector. The Quad Pressure Sensor will be attached to Pasco Capstone 850 interface and students will be able to monitor pressure and velocity of the liquid exiting the graduated cylinder with respect to time. Look at figures 4 for an image of the complete experimental layout.

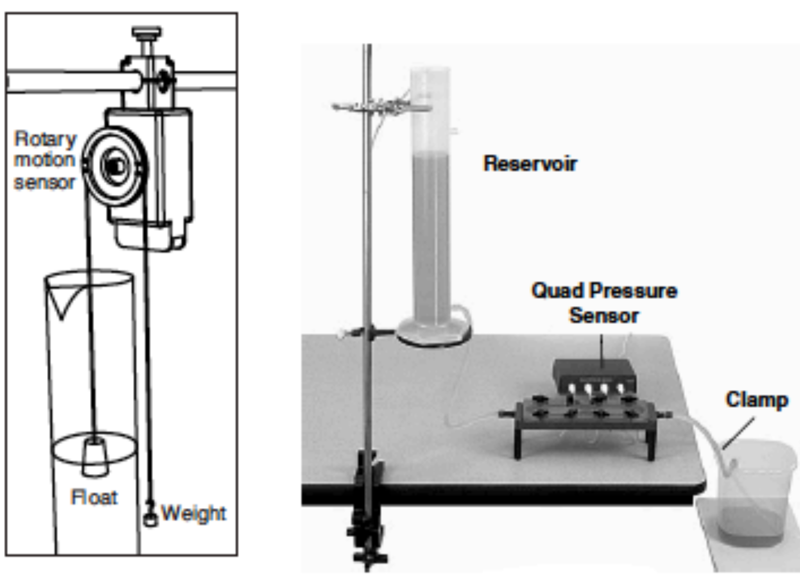

Figure 4: Table Top setup with inset detailing rotary motion sensor usage to capture velocity of water out of the reservoir

Capstone will capture data and graphs can be created of the parameters velocity (of the liquid in the reservoir (emptying from the graduated cylinder), and pressures at

4 locations in the Venturi Apparatus. How can these values be used to verify Bernoulli's Equation?

Experimental Procedure:

1. The experiment should be setup according to figure 4 when you arrive. If not- put water through the system and test for leaks before. Also – make sure no air bubbles are in the Venturi reservoir – close the clamp with water still in the Venturi apparatus and refill the reservoir for testing. (Please see instructor on how to tilt the apparatus to get rid of air bubbles if having any trouble. Tilting too much may allow water to pass into the Pressure sensors and therefore break the pressure sensors beyond repair.)
2. Line up the rotary motion sensor over the reservoir (graduated cylinder) and ensure that the mass is free hanging and the float (a golf ball with a hook) is not resting on the sides of the reservoir. Then clamp the rotary motion sensor above the reservoir (see Figure 4).
3. In Capstone enable the Linear Velocity measurement of the rotary motion sensor and set the Linear Scale value to Large Pulley Grove in Properties. (Choose – hardware tab – click on rotary motion sensor in the image – notice the word Properties at bottom – click and choose Large Pulley Groove as well as high precision (default is low).)
4. Create a graph of the velocity of the water and run a test to ensure that the rotary motion sensor is functioning correctly and the golf ball is following the water line as it drains. Ensure that NOT ALL water is emptied – ONLY DRAIN DOWN TO 300mL so that the Venturi apparatus is not drained to allow air into the system.
5. Using the value of the radius of the graduated cylinder = .031m calculate the cross sectional area of the cylinder. Now, calculate the fluid flow, Q, using the cross sectional area and the value measured for 'v' when testing equipment.
6. Look at this value – does it appear to make sense in terms its magnitude when compared to the flow rates calculated last week for different radii tubes? Ask your instructor if you are unsure.
7. Now – you are ready to start the experiment – make sure that the quad pressure sensor is connected to Capstone if not already done earlier. Refill the reservoir and record height of the water in the cylinder (in mL). Press Record in Capstone, open the clamp, and allow water to empty through the system until the cylinder is drained to 300mL. Clamp down the exit tube to stop the flow and stop recording data.
8. Record the exact value in mL of the new level of water in the reservoir.
9. View your data on graphs of pressure versus time and velocity vs. time. (Note – the 4 sensor plots should be on the same graph. You can right click on the vertical axis and choose the option "to add similar data" to the graph – choose ALL 4 **absolute pressure** sensor readings.)
10. Select a time interval from this graph = 2 seconds in which all of the pressure measurements are as constant as possible and relatively noise free.

11. Within the chosen 2-second time interval, determine the average and standard deviation of each pressure measurement: **P**1, **P**2, **P**3, and **P**4.
12. **Over the same 2-second interval**, determine the average velocity of the emptying of the cylinder from the velocity vs. time graph. Using this value and the cross-sectional area determined in step 5, calculate the flow rate, $\boldsymbol{Q} = \boldsymbol{v} * \boldsymbol{A}$.
13. If there were no friction or turbulence as the water flows from Area1 (wide area) to Area2 (small channel), the pressures in both wide sections (**P**1 and **P**3) would be equal. You will find that this is not the case and there is a small loss of pressure. We can find the average difference between P1 and P3 – this will be the averaged estimated LOSS in the system due to the frictional and turbulence losses. Keep this value as you will need it later to interpret your data results.
14. Use the measured flow rate from above, **Q**, and the fact that flow rates in different areas of tubing must equal – thus $A_o v_o = Av$ (as shown in Figure 1 – note that the large areas = 1.99×10^{-4} m^2 and the constricted areas = 0.452×10^{-4} m^2) to calculate the fluid velocity in the wide areas of the tube ($\boldsymbol{\upsilon}_0$), and the velocity in the venturi constriction channels ($\boldsymbol{\upsilon}$).
15. Which pressure sensors data reflects $\boldsymbol{\upsilon}_0$? Make sure to assign this value to the appropriate pressure sensor data. Do the same for the constricted velocities calculated. (Remember that Q is a constant since the water flows INTO and OUT of the apparatus.)
16. Use these values of $\boldsymbol{\upsilon}_0$ and $\boldsymbol{\upsilon}$ AND the Bernoulli equation to calculate the predicted pressure changes (**P1-P2, P2-P3, and P3-P4**).
17. Repeat using the experimentally measured pressures (**P1-P2, P2-P3, and P3-P4**). Note that the pressure is given in kPa – this is not the proper unit for the pressure if measuring in SI units. So convert the pressure values to Pa = N/m^2 by using the conversion 1kPa = 1,000 Pa or N/m^2. **Make sure to also multiply the uncertainties (stdev from graph data) by this conversion value to ensure the proper uncertainties units for comparison**.
18. Compare these calculated values (step 15) using the model Bernoulli's equation to the experimentally measured pressure differences (step16). To determine the uncertainties in the pressure differences one must COMBINE the uncertainties of each pressure stdev (σ) – to do this use the equation below as an example for the combined uncertainty for (P1-P2):

$$\sigma_{Pdifference} = \sqrt{\sigma_{P1}^2 + \sigma_{P2}^2}$$

19. Calculate the uncertainties for each difference, **P1-P2, P2-P3, and P3-P4**.

20. Using the loss estimate from step 12 – reassess the value of P2 and P4 (after converting to SI unit of Pascal) and again compare to the measured values to theory – if one adds the measured loss – how do the values from the model equation compare to the actual measured differences?

21. Using the mL measurements of the height of the water in the graduated cylinder – measure the actual height difference using a ruler or calipers. Convert this measurement to meters.

Remember last week when I stated that ALL buckets had to have the SAME water height so that at each tube drainage location the PRESSURE was constant?
After draining 30mL from the bucket – this water level was almost identical THUS P1-P2=0.
In this experiment the water drained a sizeable amount AND thus the pressure changed. BUT we only used 2-seconds of data. How much did the PRESSURE pushing the liquid through the Venturi Apparatus change as the water drained from the reservoir? We can calculate this value!

22. We have the velocity from the graph AND the fact we ONLY analyzed data over a 2-second interval. Therefore, if we multiply the velocity by the time we can determine the height change in the reservoir – record this value. (Note that this change in height MUST be smaller than the entire of change in height recorded for the experiment. If not, something is wrong with your velocity and the experiment needs to be rerun.)
23. Use the (potential energy)/(Volume) equation in Figure 2 ($\frac{1}{2}\rho g \Delta h = \Delta PE$) to determine the change in potential energy during the draining.
24. This change in Energy/Volume will appear as an energy loss in the Venturi apparatus for **ALL P1-P2, P2-P3, and P3-P4 values (using measured pressures)**. Looking at this value discuss if NOW the measured pressure differences with associated uncertainties are equal within precision to the calculated pressure differences. Discuss.

In your report be clear how using the Flow rate equation allowed for testing of the Bernoulli's equation.
Also discuss areas of the laboratory materials and methods that likely increased uncertainties and why you believe these steps were dominant sources of uncertainty in the experiment.

Finally, make sure that the data results indicate the type of uncertainty found – either making the measured value larger or smaller than predicted. For example, if you stated that air resistance impacted the experimentally measured value for the acceleration of gravity when a ball was falling, ensure the net measured acceleration is, in fact, smaller than that of acceleration of gravity predicted. If the ball was only falling through the air the resistance is a drag force in the opposite direction of the motion of the ball AND thus, would decrease the measured value of acceleration.