Physics 161

# Rotational Motion and Angular Momentum

## Introduction

You will be exploring various aspects of rotational motion in this lab. In Part I, you will use Video Point to examine a two-dimensional circular motion of an object. You will investigate the relationships among acceleration, velocity and position of the object. In Part II, you will observe the conservation of angular momentum by drop an object onto a rotating disk. You will measure the initial and final angular velocities and calculate the initial and final angular momenta and their uncertainties to test if angular momentum is conserved during this collision.

## Reference

Young and Freedman, University Physics, 13$^{th}$ Edition: Chapter 10, sections 1-5.

## Theory

**Part I**: If an object is moving around a circle at any given instant, the distance of the object to the origin must be constant. Therefore:

$$r = \sqrt{x^2 + y^2} \tag{1}$$

where *x* and *y* are the coordinates of the center of the object and *r* is the radius of the circular path. Uniform circular motion can be described as the motion of an object in a circle at a constant speed. The speed at any given instant can also be calculated as:

$$v = \sqrt{v_x^2 + v_y^2} \tag{2}$$

where $v_x$is the velocity in the $x$ direction, $v_y$ is the velocity in the $y$ direction, and $v$ is the tangential velocity of the object. The angular velocity is expressed as:

$$\omega = \frac{v}{r} \tag{3}$$

The acceleration of the object is called centripetal acceleration which points towards the center of the circle. This acceleration can be calculated using the following equation:

$$a = \frac{v^2}{r} = r\omega^2 \tag{4}$$

You will use these equations to analyze the video of an object moving around a circle.

**Part II**: Angular motion is similar to translational motion in many ways. For example, instead of linear velocity, $v$, we have angular velocity, $\omega$, of a rotating body. Similarly, there are equivalent angular quantities for acceleration, momentum, force and mass. In the case of a rotating mass, we use the quantity moment of inertia, $I$, to describe how readily that body can undergo angular acceleration. The moment of inertia, $I = \sum m_i r_i^2$, depends upon the mass of the object and how the mass is distributed about the axis of rotation.

In the absence of any external torques, such as friction, on an object, angular momentum is conserved, i.e. initial and final angular momenta are equal. Conservation of angular momentum means that if the moment of inertia, $I$, of a rotating object is changed, the angular velocity, $\omega$, will change by some factor so that the total angular momentum, $L$ is conserved:

$$L_i = L_f \tag{1}$$

$$L = I\omega \tag{2}$$

$$I_i \omega_i = I_f \omega_f \tag{3}$$

The moment of inertia for a disk of uniform density rotating about its center is:

$$I = \frac{1}{2} M R^2 \tag{4}$$

# Procedure

### Part I: Object moving around a circle.

You should open the UCM.avi file into the **Capstone** program. This is a video of a turntable with two dimes placed on top of it. As the turntable rotates at constant angular speed, the dimes will travel in circular paths.

1. Open **Capstone**. Click on the down-arrow in the Video Analysis icon in Figure. 1 below and open the file **UCM.mov**.

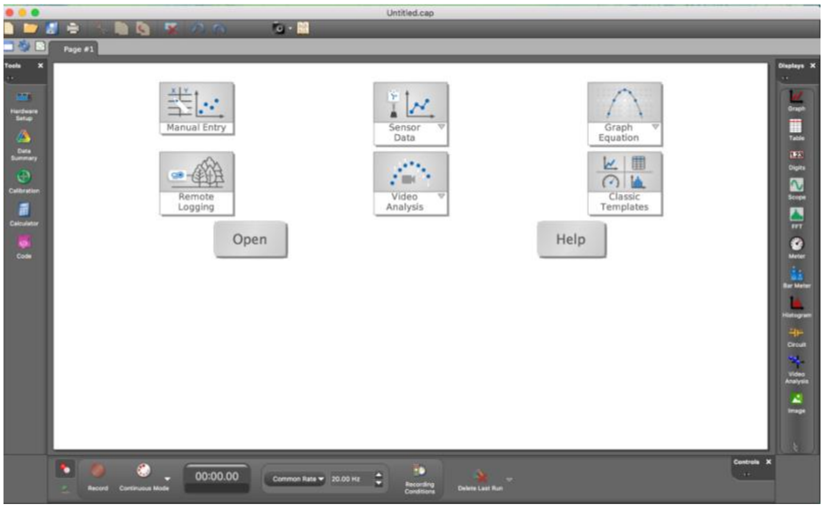
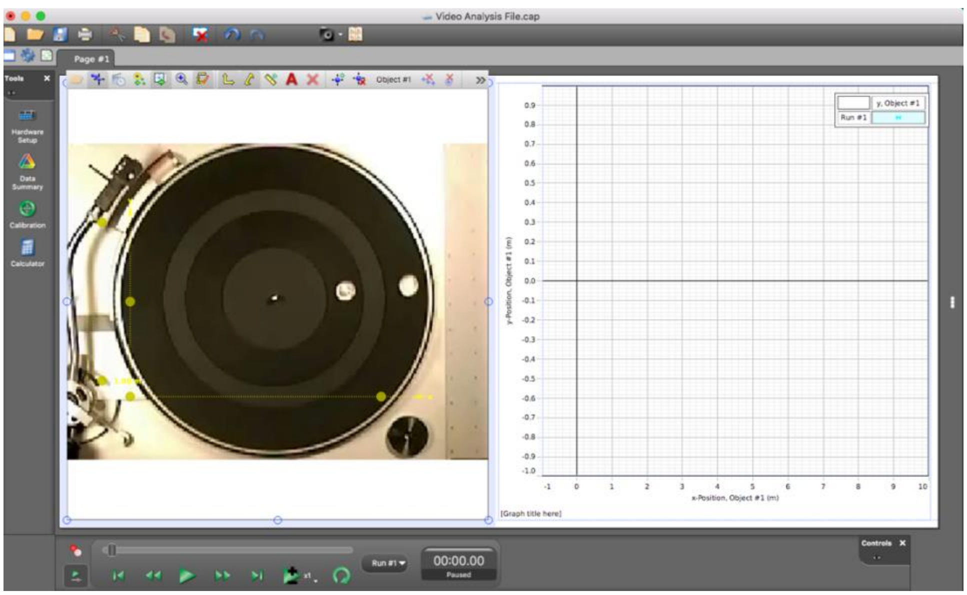



Figure 1: Open file UCM.mov.

2. You should see the turntable as in Figure 2.




3. To calibrate the image use the yellow calipers to fit to the diameter of the turntable. Set the calipers to 30.5 *cm*, see Figure. 3a. In the setup pane, set the origin of your coordinates at the center of the turntable. The point you mark should be the tip of the shaft that comes out of the middle of the turntable, see **Error! Reference source not found.**b.

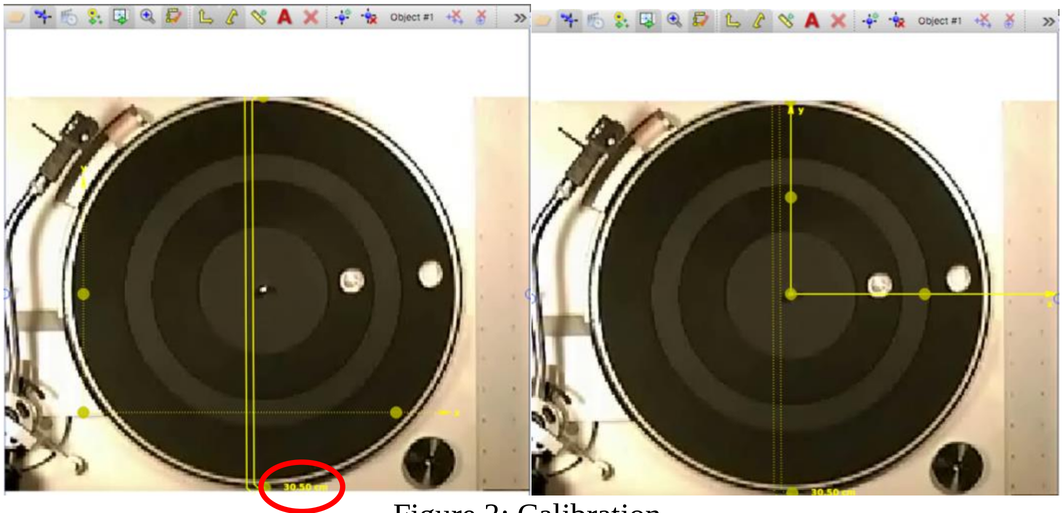
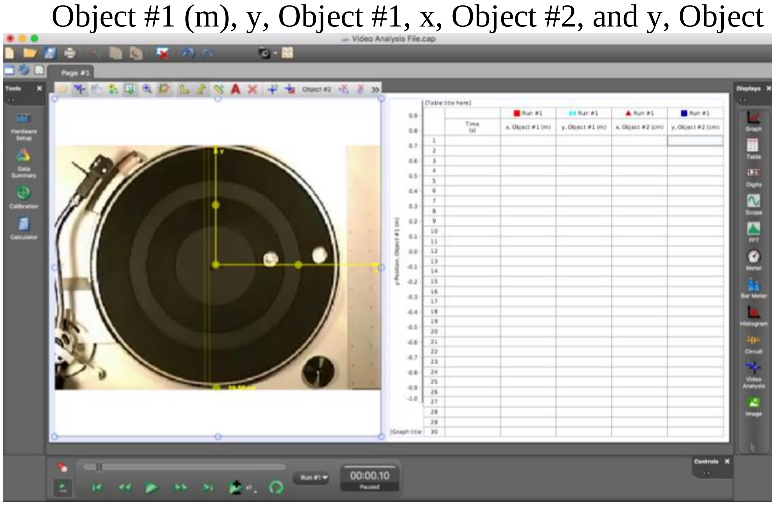
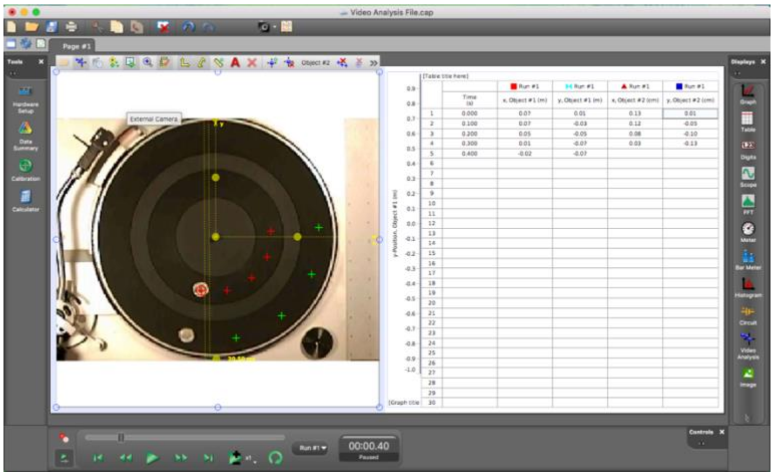



Figure 3: Calibration.

4. Create a new object from the menu above the movie, it should now say Object #2 to the right of the delete object button. You will be able to toggle between the two objects by clicking on that name. Next make a table, insert three empty columns to the right, and select time, x, Object #1 (m), y, Object #1, x, Object #2, and y, Object #2.



5. Select Object #1, place your cursor over the center of mass of the dime closest to the origin. Mark the center of this dime in each frame by clicking the mouse. The video will automatically advance to the next frame. Continue in this fashion, clicking for a data point in each frame until you reach the end of the video. A dialog box will pop up and ask if you want to take another data set, click on “Yes.” Repeat the above steps for the second dime by starting from the beginning and selecting Object #2. Don’t worry if your first data set appears to have disappeared it will reappear as you go around.



6. Copy, or export data to a file , and paste or open it in Excel.

7. Your spreadsheet should have separate columns for $t$, $x_1$, $y_1$, $x_2$, and $y_2$. The time in seconds is in the first column. The second and third columns include the $x$ $y$ positions for the first dime, and the $x$ $y$ positions of the second dime should be listed in the fourth and fifth columns.

8. In a new column in Excel, calculate the radius of the circular path for the first dime using equation 1. Repeat this for the second dime.

9. Now calculate the *x* and *y* components of the velocity for both dimes just as you did in the Projectile Motion lab. Calculate the speeds, $v_1$ and $v_2$, for each time step using equation 2. Is $v_1$ constant? Is $v_2$ constant? Is $v_1$ equal to $v_2$? For the closest dime to the center of rotation, produce plots of *x*-position, *y*-position and $r$ versus time, place all these in one graph. Now, for the same dime, plot $v_x$, $v_y$, and $v$ versus time, place all these in one graph.

10. Finally for each time step use the linear speed $v$ and radius $r$ to calculate the angular speed $\omega$ at each time point for both dimes. From the time series calculate the average and standard deviation for the angular speed for each dime. Add the uncertainties in quadrate as you did for the momentum conservation lab and use the total uncertainty to assess whether the two dimes move with the same angular speed. If not, what could account for this difference?

11. Calculate the centripetal acceleration using equation 4 for both dimes.

**Part II**: Conservation of Angular Momentum

1. Measure and record the radius and mass of the disk and calculate *I*.

2. Open Pasco Capstone. Choose **Create Experiment**. Select **Hardware Setup** and set a **Rotary Motion Sensor**, see Figure 2. Set the **Rotary Motion Sensor** to 20 *Hz* at the bottom near Recording Conditions

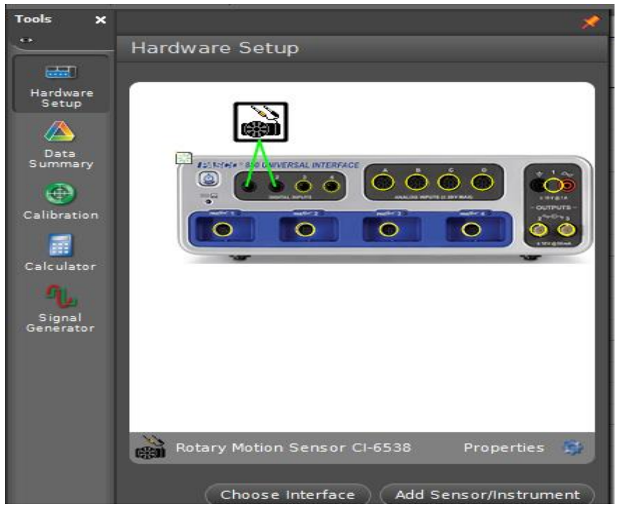



Figure 2: Rotary motion sensor.

3. Click on Graph under the Display Tab to the right and set a plot area. Set the $y$ axis of the graph to Angular velocity $\omega$ in radians/s.

4. Now with the disk attached to the rotary motion sensor, give the system a spin in the clockwise direction. Press **Record** to begin collecting data.

5. Carefully drop a second disk on top of the spinning disk. Make sure that there is very little excess motion when the disk lands. It may take a few practice trials to achieve minimal disturbance of motion. Wait a couple of seconds and press **Stop**.

6. You should get a graph resembling the one shown in Figure 3.

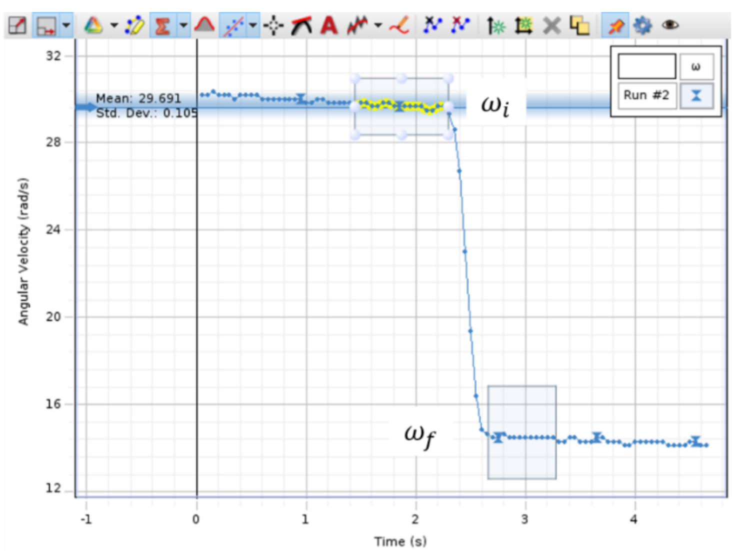



Figure 3: Angular velocity vs time.

7. As you did in the linear momentum lab export at least five points just before and after the sudden change to excel.

8. Calculate the initial and final angular momenta from the angular velocity measurements by multiplying by their respective moments of inertia. Calculate the mean and uncertainty in these values from the data. Based on these values is angular momentum conserved?