RC Decay

Learning Goals:

1. The main goal of this lab is to study the discharge behavior of a capacitor.

2. You will learn how to determine the time constant of an RC circuit.

Materials:

Capacitor; Resistor; Digital Multimeter; DC power Supply; Cables; Timer; Excel.

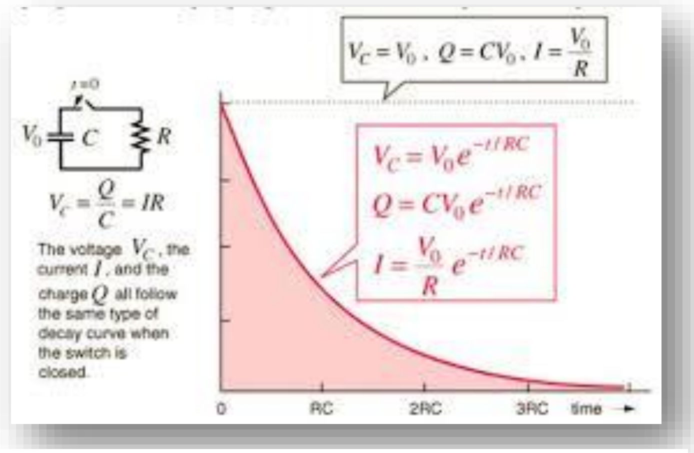

Figure 1: RC decay

References:

Giancoli, Physics 7th Edition: Chapter 19, Section 6

Introduction:

A capacitor is a device for storing charge and energy. It consists of two conductors insulated from each other. A typical capacitor is called a parallel-plate capacitor. When the two plates are connected to the wires of a charging device such as a DC power supply or a battery, positive charge is transferred to the plate connected to the positive terminal of the charging device and negative charge of equal magnitude is transferred to the other plate. This flow of charge continues until the potential between the conductors equals the applied potential difference. The amount of charge (Q) depends on the geometry of the capacitor and is directly proportional to the potential difference (V). The proportionality constant is called the capacitance (C).

$$Q = CV \qquad (1)$$

The gap between the plates is usually filled with a dielectric material to increase the capacitance. Charge is measured in Coulombs (C) and voltage in Volts (V), so capacitance has the units Coulombs (C)/Volt (V) which is called a Farad (F).

This experiment will investigate the discharge of a capacitor through a resistance. The decrease of charge on the capacitor is exponential as a function of time. This is an important type of behavior in nature since it occurs whenever the instantaneous change in a quantity is only dependent on the amount present at the time. Population and bacterial growth, money in savings account, and energy use may follow such a law.

Written in electrical symbols the charge, as a function of time, on a discharging capacitor is given by:

$$Q = Q_0 e^{-\frac{t}{RC}} \qquad (2)$$

where Q is the instantaneous charge at time t, Q_o is the initial charge at the beginning of the time interval t, R is the resistance in the circuit, C is the capacitance of the capacitor used, and e is the base of the natural logarithm, e = 2.71828...

Since it is easier to measure the voltage across a capacitor than the charge, we can combine equations (1) and (2) to give

$$V = V_0 e^{\frac{-t}{RC}} \qquad (3)$$

The product RC is called the time constant of the circuit and is represented by the Greek letter tau (τ). It has units of ohms-farads, (which equals seconds), and is the time required for the voltage on the capacitor V to fall from V_o to V_o/e, as can be seen by substituting time t = RC into equation 3 above. The value 1/e ~ 0.368, so at t = RC, V = 0,368V_o. Since this contains an awkward fraction, ($t_{½}$) or half-life is often determined experimentally instead of the RC time constant. ($t_{½}$) is defined as the time necessary for V to decrease to ½ of its initial value. When the voltage has decayed to ½ of the original voltage, we have V = ½V_o which can be solved to give $\frac{V}{V_0} = \frac{1}{2}$ Replacing V with the right side of equation 3, gives

$$\frac{1}{2} = e^{\frac{-t_{0.5}}{RC}}$$

Taking the natural log of both sides, we have:

$$ln\frac{1}{2} = \frac{t_{0.5}}{RC}$$

Rearranging this yields:

$$t_{0.5} = 0.693RC \qquad (4)$$

One way to analyze the data can be understood by taking the natural log of each side of equation 3.

$$lnV = \ln\left[V_0 e^{\frac{-t}{RC}}\right] = lnV_0 + \ln[e^{\frac{-t}{RC}}]$$

$$lnV = lnV_0 - \frac{t}{RC} \tag{5}$$

So if you plot lnV vs. t, the resulting graph should be linear with a slope of -1/RC.

Experiment:

In this experiment the exponential discharge or decay is slow enough that measurements can be made with a clock or watch. The following circuit will be used to measure the discharge of the capacitor (Figure 2).

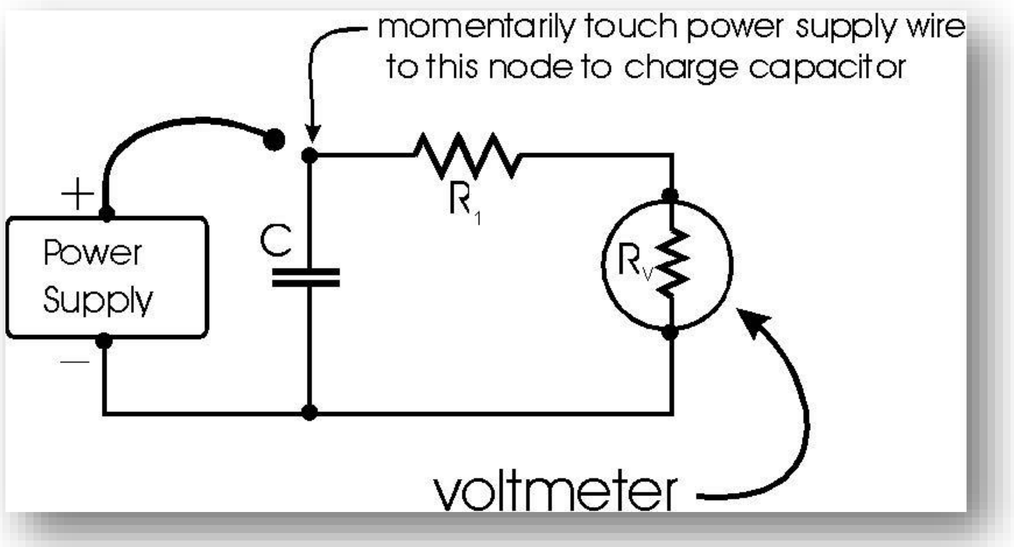

Figure 2: Circuit used to measure slow discharge of a capacitor. R1 = 10 MΩ, C = 1.0μF and the voltmeter is represented by its internal resistance of Rv = 10 MΩ

In this circuit the elements are R1, R_v and C:

Table 1: Values of circuit elements

R1	10 MΩ
Rv	10 MΩ
C	1.0 μF

The voltmeter in this circuit is connected in series with the other components. This is an unusual use of the voltmeter as they are normally connected in parallel to other components. Figure 2 shows the voltmeter in terms of its internal resistance which is 10 MΩ. There are obviously many more components inside the voltmeter to allow it to function as a voltmeter, but these are not shown here so we can focus on the effect the voltmeter has on the flow of current through the circuit. Momentarily touching the positive wire from the power supply to the indicated point charges the capacitor. When the power supply is then disconnected, the capacitor will discharge through the resistor and voltmeter. (Note that it is important that the power supply wire be physically disconnected after charging in order for this experiment to function properly.)

The size of the current however, is extremely small and would require a very sensitive meter to record. However, the large resistance of a multimeter used in the voltmeter mode results in a measureable voltage drop with even a small current flow. Using the voltmeter in series thus results in a reading that is proportional to the voltage on the capacitor. It should be noted that if the power supply is set to 10 V, the initial reading of the voltmeter should be 5.0 V since the internal resistance of the voltmeter (10 MΩ) is in series with a 10 MΩ resistor.

Experimental Procedure and Data Taking:

1. Verify that all circuit components are connected in series and the lead from the positive outlet of the power supply is not connected to the circuit.

2. Charge the capacitor. Use a stopwatch and, working in a team, read the voltage from the voltmeter every 2 seconds and record the values in your notebook. (One person watches the time and calls out intervals, one reads the voltmeter out loud and the third records the voltage.) Graph the voltage vs. time.

Analysis:

1. Predict the time constant $\tau = RC$ from the measured values of the circuit elements R and C. Note that R is the total circuit resistance. R_v is given. R1 and C need to be measured. Record the value in your notebook.

2. Create a graph of the measured voltage values. Plot time on the x-axis and voltage on the y-axis.

3. Pick the half-time $t_{1/2}$ off your graph to calculate the time constant. Use equation 4.

4. In your graph insert an excel trendline and display the equation. Determine RC from the trendline equation.

5. Create a new graph. Plot $\ln V$ vs. time. Is the resulting graph linear? Refer to equation 5 and calculate a value for RC. Perform a linear regression on the data to determine the uncertainty in RC.

6. You have now determined the time constant RC with different methods. Compare these time constants. Which method is the most accurate one?

7. Replot the data using a logarithmic axis for voltage and add a linear trendline. This should produce a straight line plot.

8. Change either R or C to get and RC value that is different by more than 30%. Calculate the predicted value of the time constant.

9. Take data and use two different methods to calculate the experimental value of RC.

10. Calculate the uncertainty in the predicted value of the time constant. Assume the uncertainty of R_v to be 1%. Show all steps in the uncertainty calculation. (This is to be done in your notebook)

11. Turn in the Excel spreadsheet by uploading it to the spreadsheet drop box in the RC Decay content folder on Blackboard and submit a group report by the end of this class.

References:

Adapted from a previous handout.