The first step in determining the missing values is to compute the total circuit impedance. One of the rules for series circuits states that the total resistance is equal to the sum of the individual resistances. This basic rule is still true, but it must be bent a bit to fit this circuit. It is true that the ohmic values still add, but the inductive and resistive parts of the circuit are out of phase with each other by 90°. Vector addition must be used to determine the total impedance. The formula for determining impedance in a series circuit containing resistance and inductive reactance is $Z = \sqrt{R^2 + X_L^{\ 2}}$.

$$Z = \sqrt{24^2 + 32^2}$$

$$Z = \sqrt{276 + 1{,}024}$$

$$Z = \sqrt{1{,}600}$$

$$Z = 40\ \Omega$$

The total circuit current can now be computed using Ohm's law.

$$I_T = \frac{E_T}{Z}$$

$$I_T = \frac{120}{40}$$

$$I_T = 3 \text{ amps}$$

The current is the same at any point in a series circuit. The values of I_R and I_L are, therefore, the same as I_T.

Now that the amount of current flowing through the resistor is known, the voltage drop across the resistor can be determined using Ohm's law.

$$E_R = I_R \times R$$

$$E_R = 3 \times 24$$

$$E_R = 72 \text{ volts}$$

The true power, or watts, can be computed using any of the power formulas. In this example the true power will be computed using $E \times I$.

$$P = E_R \times I_R$$

$$P = 72 \times 3$$

$$P = 216 \text{ watts}$$

The voltage drop across the inductor can be determined using Ohm's law and reactive values.

$$E_L = I_L \times X_L$$

$$E_L = 3 \times 32$$

$$E_L = 96 \text{ volts}$$

Note that the resistor has a voltage drop of 72 volts and the inductor has a voltage drop of 96 volts. One of the rules for series circuit states that the total or applied voltage is equal to the sum of the voltage drop in the circuit. The circuit has 120 volts applied. Therefore, the sum of 72 and 96 should equal 120. In order for these two values to equal the applied voltage, vector addition must be employed. In a series circuit, the current is the same through all parts of the circuit. Since the voltage drop across the resistor is in phase with the current and the voltage drop across the inductor leads the current by 90°, the voltage drops across the resistor and inductor are 90° out of phase with each other. Total voltage can be determined using the formula

$$E_T = \sqrt{{E_R}^2 + {E_L}^2}$$

$$E_T = \sqrt{72^2 + 96^2}$$

$$E_T = \sqrt{14{,}400}$$

$$E_T = 120 \text{ volts}$$

The reactive VARs can be computed in a manner similar to determining the value for watts or volt amps, except that reactive values are used.

$$\text{VARs}_L = E_L \times I_L$$

$$\text{VARs}_L = 96 \times 3$$

$$\text{VARs}_L = 288$$

The value of inductance of the inductor can be computed using the formula

$$L = \frac{X_L}{2\pi f}$$

$$L = \frac{32}{377}$$

$$L = 0.0849 \text{ henry}$$

The apparent power or volt amps can be computed using formulas similar to those for determining watts or VARs, except that total circuit values are used in the formula

$$VA = E_T + I_T$$

$$VA = 120 \times 3$$

$$VA = 360$$

The circuit power factor can be computed using the formula

$$PF = \frac{P}{VA}$$

$$PF = \frac{216}{360}$$

$$PF = 0.6 \text{ or } 60\%$$

The power factor is the cosine of angle theta. In this circuit, the decimal power factor is 0.6. The cosine of angle theta is 0.6. To determine angle theta, find the angle that corresponds to a cosine of 0.6. Most scientific calculators contain trigonometric functions. To find what angle corresponds to one of the sin, cos, or tan functions, it is generally necessary to use the invert key, the arc key, or one of the keys marked $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$.

$$\cos \varnothing = 0.6$$

$$\varnothing = 53.13°$$

The current and voltage are 53.13° out of phase with each other in this circuit.

The circuit, with all completed values, is shown in Figure 11-4.

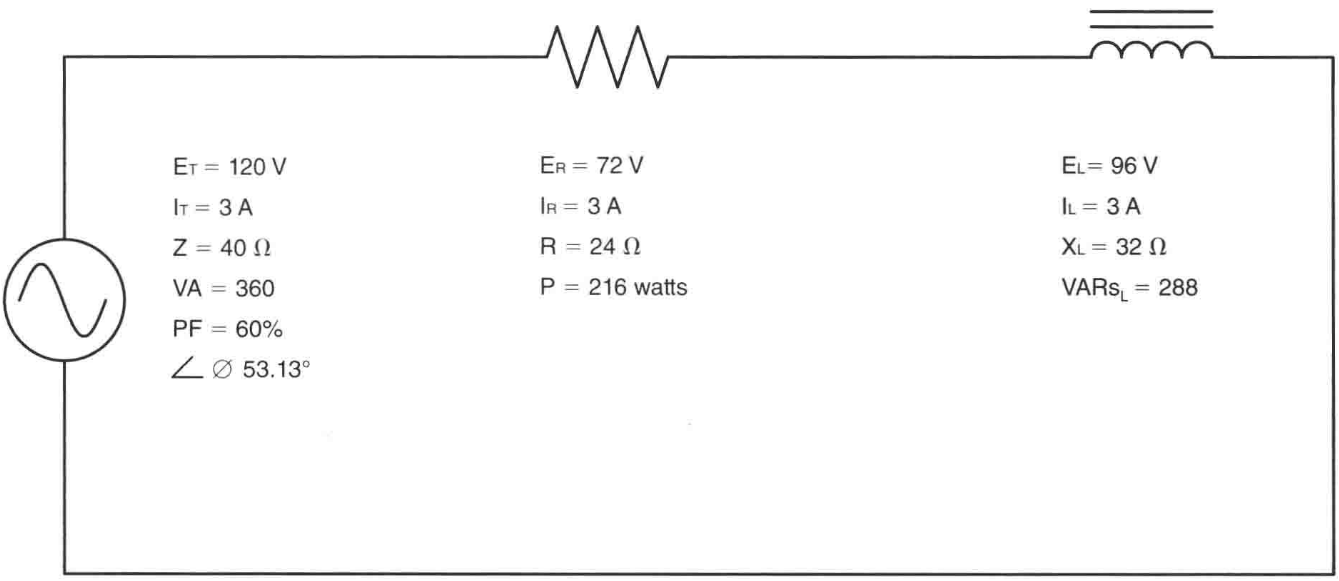


Figure 11-4 RL series circuit with all the missing values.

LABORATORY EXERCISE

Name ____ Date __________

Materials Required

1 0.5 kVA control transformer with two windings rated at 240 volts and one winding rated at 120 volts

1 100 ohm resistor

1 250 ohm resistor

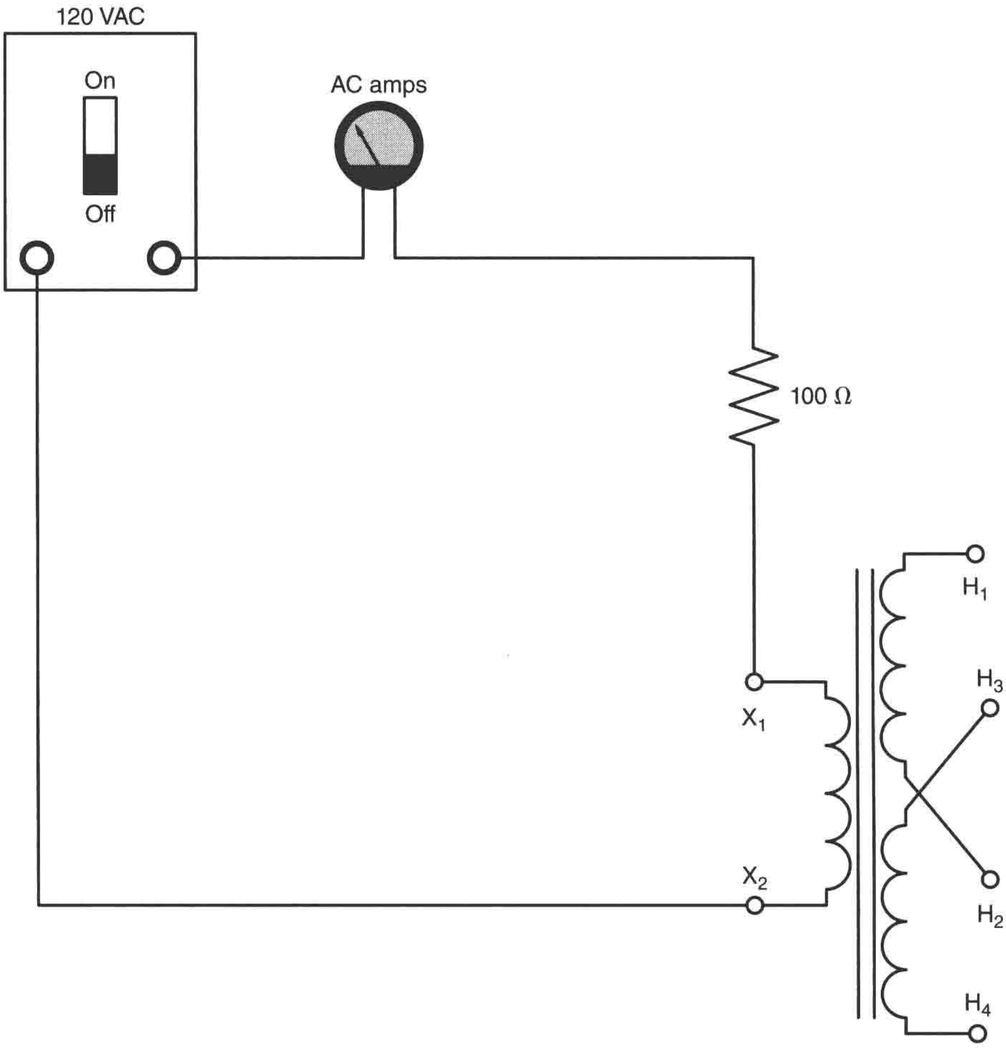


Figure 11-5 Connecting an RL series circuit.

AC ammeter (In-line or Clamp-on may be used. If a clamp-on type is employed, the use of a 10:1 scale divider is recommended)

Connecting wires

AC voltmeter

1 120 Volt AC power supply

Formulas for resistive-inductive series circuits shown in Figure 11-6

1. Connect the circuit shown in Figure 11-5. NOTE: During this experiment the H terminals of the transformer will provide a step-up in voltage. Be careful not to come in contact with these terminals.
2. Turn on the power and measure the current flow in the circuit with an AC ammeter.

   I = _______________ amps
3. Measure the voltage drop across the 100 ohm resistor an AC voltmeter.

   E_R = _______________ volts
4. Compute the true power in the circuit using the resistive values of voltage and current.

   $P = E_R \times I_R$

   P = _______________ watts

5. Measure the voltage drop across winding X_1 and X_2 of the transformer with an AC voltmeter. **Turn off the power.**

   E_L = _______________ volts

6. Compute the inductive reactance of the inductor (X_1 to X_2) using Ohm's law.

   $X_L = \frac{E_L}{I}$

   X_L = _______________ Ω

7. Compute the inductive VARs in the circuit using the following formula: $VARs_L = E_L \times I_L$.

   $VARs_L$ = _______________

8. Compute the apparent power (VA) using the total circuit values. $VA = E_T \times I_T$

   VA = _______________

9. Compute the circuit power factor using the following formula: $PF = \frac{P}{VA}$

   PF = _______________ %

10. Using the decimal value of the power factor, determine the phase angle difference between the voltage and current in this circuit. $\text{Cos } \angle\varnothing = PF$

    $\angle\varnothing$ = _______________ °

11. Replace the 100 ohm resistor with a 250 ohm resistor.

12. Turn on the power and measure the current in the circuit using an AC ammeter.

    I = __________ A

13. Measure the voltage drop across the 250 ohm resistor with an AC voltmeter.

    E_R = _______________ volts

14. Compute the true power in the circuit using Ohm's law.

    P = _______________ watts

15. Measure the voltage drop across the inductor (winding X_1 and X_2 of the transformer) with an AC voltmeter. **Turn off the power.**

    E_L = _______________ volts

16. Compute the inductive reactance of the inductor using Ohm's law.

    X_L = _______________ Ω

17. Compute the reactive VARs for the inductor using Ohm's law.

    $VARs_L$ = ___________________

18. Compute the inductance of the inductor.

    L = _______________ henry

19. Compute the apparent power of the circuit using Ohm's law.

    VA = _______________

20. Compute the power factor of the circuit.

    PF = _______________ %

21. Determine the value of angle theta.

    $\angle\varnothing$ = _______________ °

22. In an RL series circuit the voltage drop across the resistor and inductor are out of phase with each other. Add the voltage drop across the resistor in step #3 and the voltage drop across the inductor in step #5.

E = __________ volts

Is the sum of the two voltages greater than the voltage source?

____

23. Because the voltage across the resistor and the voltage across the indictor are out of phase with each other, vector addition must be used. If the components were a true 90° out of phase with each other, the vector sum should be equal to the voltage applied to the circuit. However, a transformer winding is not a true indictor or choke, and the winding does contain some amount of resistance which will prevent the voltage across the transformer winding from being 90° out of phase with the voltage across the resistor. Calculate the vector sum of the two voltage values using the following formula:

$$E_T = \sqrt{E_R^2 + E_L^2}$$

E_T = __________ volts

24. Disconnect the circuit and return the components to their proper place.

Review Questions

Refer to the formulas shown in Figure 11-6 to answer some of the following questions.

1. What is the phase angle difference between current and voltage in a pure resistive circuit?

____

2. What is the phase angle difference between current and voltage in a pure inductive circuit?

____

3. An inductor and resistor are connected in series. The resistor has a resistance of 26 Ω and the inductor has an inductive reactance of 16 Ω. What is the impedance of the circuit?

____

4. An RL series circuit is connected to a 120 volt, 60 Hz line. The inductor has a voltage drop of 54 volts. What is the voltage drop across the resistor?

____

5. An inductor is using 1.6 kVARs and has an inductive reactance of 14 Ω. How much current is flowing through the inductor?

____

6. An RL series circuit is connected to a 208 volt, 60 Hz line. The resistor has a power dissipation of 46 watts and the inductor is operating at 38 VARs. How much current is flowing in the circuit?

____

7. An RL series circuit is connected to a 480 volt, 60 Hz line. The apparent power of the circuit is 82 kVA. What is the impedance of the circuit?

____

8. An RL series circuit has a power factor of 82%. Determine angle theta.

____

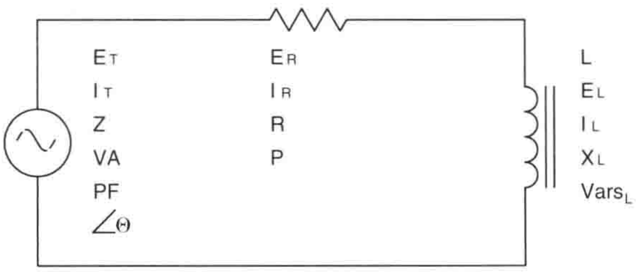
9. An RL series circuit is connected to a 240 volt, 60 Hz line. The inductor has a current of 8 amperes flowing through it. The X_L of the inductor is 12 Ω. The resistor has a resistance of 9 Ω. How much current is flowing through the resistor?

____

10. An RL series circuit has an apparent power of 650 VA and a true power of 375 watts. What is the reactive power in the circuit?

____

Resistive-Inductive Series Circuits



Note: To find values for the reisistor, use the formulas in the pure resistive section.

Note: To find values for the inductor, used the formulas in the pure inductive section.

$E_T = \sqrt{E_R^2 + E_L^2}$	$Z = \sqrt{R^2 + X_L^2}$	$VA = E_T \times I_T$	$I_T = I_R = I_L$
$E_T = I_T \times Z$	$Z = \frac{E_T}{I_T}$	$VA = I_T^2 \times Z$	$I_T = \frac{E_T}{Z}$
$E_T = \frac{VA}{I_T}$	$Z = \frac{VA}{I_T^2}$	$VA = \frac{E_T^2}{Z}$	$I_T = \frac{VA}{E_T}$
$E_T = \frac{E_R}{PF}$	$Z = \frac{R}{PF}$	$VA = \sqrt{P^2 + VARs_L^2}$	$VA = \frac{P}{PF}$
$PF = \frac{R}{Z}$	$P = E_R \times I_R$	$Z = \frac{E_T^2}{VA}$	$I_R = I_T = I_L$
$PF = \frac{P}{VA}$	$P = \sqrt{VA^2 - VARs_L^2}$	$E_R = I_R \times R$	$I_R = \frac{E_R}{R}$
$PF = \frac{E_R}{E_T}$	$P = \frac{E_R^2}{R}$	$E_R = \sqrt{P \times R}$	$I_R = \frac{P}{E_R}$
$PF = \text{Cos}\angle\Theta$	$P = I_R^2 \times R$	$E_R = \frac{P}{I_R}$	$I_R = \sqrt{\frac{P}{R}}$
$R = \sqrt{Z^2 - X_L^2}$	$P = VA \times PF$	$E_R = \sqrt{E_T^2 - E_L^2}$	$X_L = \sqrt{Z^2 - R^2}$
$R = \frac{E_R}{I_R}$	$E_L = I_L \times X_L$	$E_R = E_T \times PF$	$X_L = \frac{E_L}{I_L}$
$R = \frac{E_R^2}{P}$	$E_L = \sqrt{E_T^2 - E_R^2}$	$I_L = I_R = I_T$	
$R = \frac{P}{I_R^2}$	$E_L = \sqrt{VARs_L \times X_L}$	$I_L = \frac{E_L}{X_L}$	$X_L = \frac{E_L^2}{VARs_L}$
$R = Z \times PF$	$E_L = \frac{VARs_L}{I_L}$	$I_L = \frac{VARs_L}{E_L}$	$X_L = \frac{VARs_L}{I_L^2}$
$VARs_L = \sqrt{VA^2 - P^2}$	$VARs_L = E_L \times I_L$	$I_L = \sqrt{\frac{VARs_L}{X_L}}$	$X_L = 2\pi f L$
$VARs_L = I_L^2 \times X_L$	$VARs_L = \frac{E_L^2}{X_L}$	$L = \frac{X_L}{2\pi f}$	

Figure 11-6 Formulas for RL series circuits.

Unit 12 RL Parallel Circuits

Objectives

After studying this unit, you should be able to

- Discuss the voltage and current relationship in an RL parallel circuit.
- Determine the phase angle of current in an RL parallel circuit.
- Determine the power factor in an RL parallel circuit.
- Discuss the differences between apparent power, true power, and reactive power.
- Find the impedance in an RL parallel circuit.

In any parallel circuit, the voltage must be the same across all branches. Since the current is in phase with the voltage in a pure resistive circuit, and the current lags the voltage by 90° in a pure inductive circuit, the current flow through the inductive branch will be out of phase with the current through the resistive branch. The total current will be out of phase with the applied voltage by some amount between 0° and 90° depending on the relative values of resistance and inductance.

Determining electrical values in an RL parallel circuit is very similar to determining values in a series circuit with a few exceptions. Probably the greatest difference is calculating the value of impedance when the values of R and X_L are known. Recall that vector addition can be used with the ohmic values of an RL series circuit to determine the impedance.

$$Z = \sqrt{R^2 + X_L{}^2}$$

The same basic concept is true for an RL parallel circuit, except that the reciprocal value of R and X_L must be used.

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}}$$

Example: A resistor and inductor are connected in parallel. The resistor has a resistance of 50 Ω and the inductor has an inductive reactance of 60 Ω. Find the impedance of the circuit.

Solution:

$$Z = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{60}\right)^2}}$$

$$Z = \frac{1}{\sqrt{(0.02)^2 + (0.01667)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0002779}}$$

$$Z = \frac{1}{\sqrt{0.0006779}}$$

$$Z = \frac{1}{0.02604}$$

$$Z = 38.4\ \Omega$$

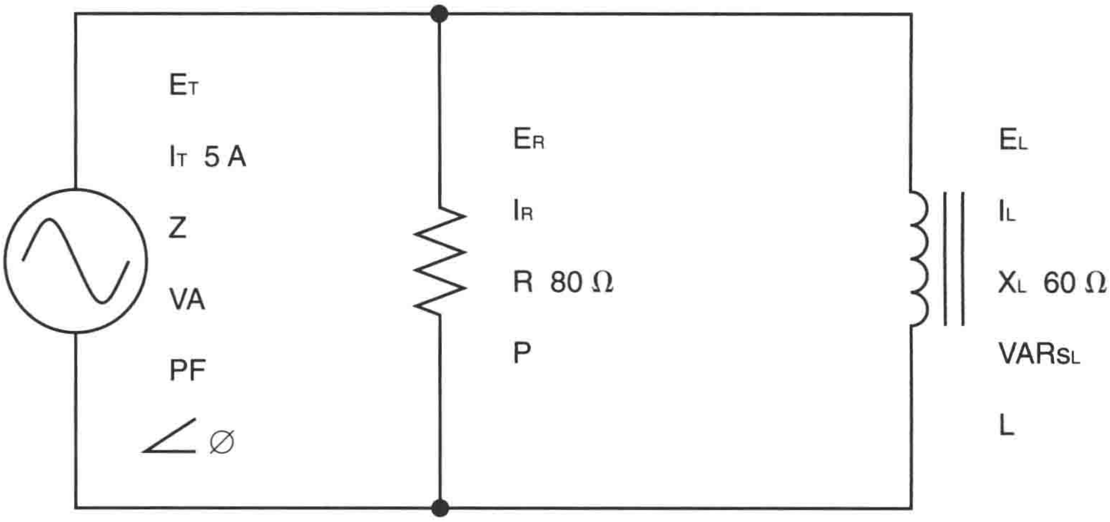


Figure 12-1 RL parallel circuit.

Example Circuit

An RL parallel circuit is connected to a 60 Hz line (see Figure 12-1). The resistor has a resistance of 80 Ω and the inductor has an inductive reactance of 60 Ω. The circuit has total current flow of 5 amps. The following values will be computed:

E_T - Total circuit voltage

Z - Circuit impedance

VA - Apparent power

I_R - Current flow through the resistor

P - True power

I_L - Current flow through the inductor

$VARs_L$ - Reactive power

L - Inductance of the inductor

PF - Power factor

∠∅ - Angle theta

The first step in determining the missing values for this problem is to determine the circuit impedance using the values of R and X_L.

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{80}\right)^2 + \left(\frac{1}{60}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.00015625 + 0.00027778}}$$

$$Z = \frac{1}{0.0208333}$$

$$Z = 48\ \Omega$$

Now that the value of impedance is known, the total voltage can be determined using Ohm's law.

$$E_T = I_T \times Z$$

$$E_T = 5 \times 48$$

$$E_T = 240 \text{ volts}$$

In a parallel circuit, the voltage is the same across all branches. Therefore, the voltage drops across the resistor and inductor are also 240 volts.

The apparent power can be computed using the following formula:

$$VA = E_T \times I_T$$

$$VA = 240 \times 5$$

$$VA = 1{,}200$$

The current flowing through the resistor can be computed using Ohm's law.

$$I_R = \frac{E_R}{R}$$

$$I_R = \frac{240}{80}$$

$$I_R = 3 \text{ amps}$$

The true power in the circuit can be computed using resistive values.

$$P = E_R \times I_R$$

$$P = 240 \times 3$$

$$P = 720 \text{ watts}$$

The current flow through the inductor can be computed using Ohm's law.

$$I_L = \frac{E_L}{X_L}$$

$$I_L = \frac{240}{60}$$

$$I_L = 4 \text{ amps}$$

The inductive VARs can be computed using the inductive values and Ohm's law.

$$VARs_L = E_L \times I_L$$

$$VARs_L = 240 \times 4$$

$$VARs_L = 960$$

The inductance of the inductor can be computed using the following formula:

$$L = \frac{X_L}{2\pi f}$$

$$L = \frac{60}{377}$$

$$L = 0.159 \text{ henry}$$

The power factor can be determined by comparing the true power and apparent power.

$$PF = \frac{P}{VA}$$

$$PF = \frac{720}{1{,}200}$$

$$PF = 0.6 \text{ or } 60\%$$

The cosine of angle theta is equal to the decimal power factor value.

$$\cos \angle\varnothing = PF$$

$$\cos \angle\varnothing = 0.6$$

$$\angle\varnothing = 53.13°$$

The circuit with all the missing values is shown in Figure 12-2.

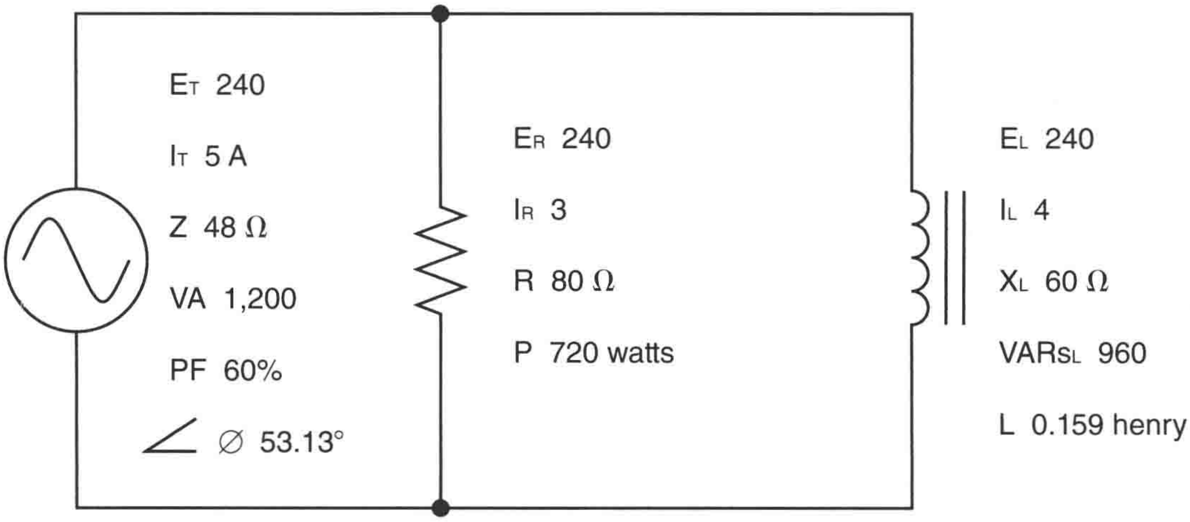


Figure 12-2 RL parallel circuit with all the missing values.

LABORATORY EXERCISE

Name ____ Date __________

Materials Required

Formulas for resistive–inductive parallel circuits shown in Figure 12-6

1 0.5 kVA control transformer with two windings rated at 240 volts and one winding rated at 120 volts

1 100 ohm resistor

1 150 ohm resistor

AC ammeter (In-line or clamp-on types may be used. If a clamp-on type is employed, the use of a 10:1 scale divider is recommended.)

Connecting wires

AC voltmeter

1 120 volt AC power supply

1. Connect the circuit shown in Figure 12-3.
2. Turn on the power supply and measure the total circuit current with an AC ammeter.

   I_T ______________ amps
3. **Turn off the power supply.**
4. Connect an AC ammeter in series with the 100 ohm resistor as shown in Figure 12-4.
5. Turn on the power supply and measure the current flowing through the lamp.

   I_R ______________ amps
6. **Turn off the power supply.**
7. Connect the AC ammeter in series with the inductor as shown in Figure 12-5.
8. Turn on the power supply and measure the current flow through the transformer winding.

   I_L ______________ amps
9. **Turn off the power supply**
10. Calculate the true power in the circuit using resistive value of voltage and current.
    $P = E_R \times I_R$

    P = ______________ watts
11. Calculate the value of inductive reactance using inductive values of voltage and current.
    $$X_L = \frac{E_L}{I_L}$$

    X_L ______________ Ω

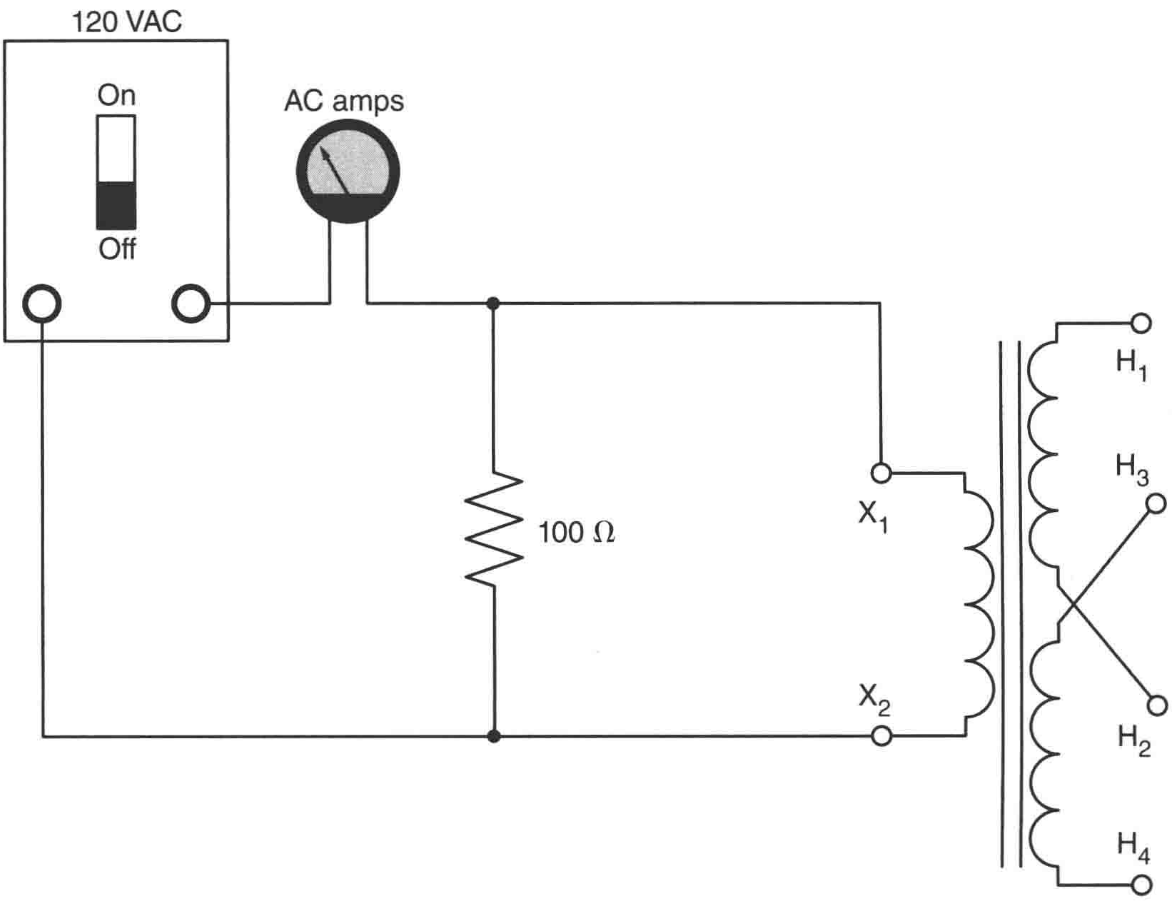


Figure 12-3 The ammeter is connected to measure the total circuit current.

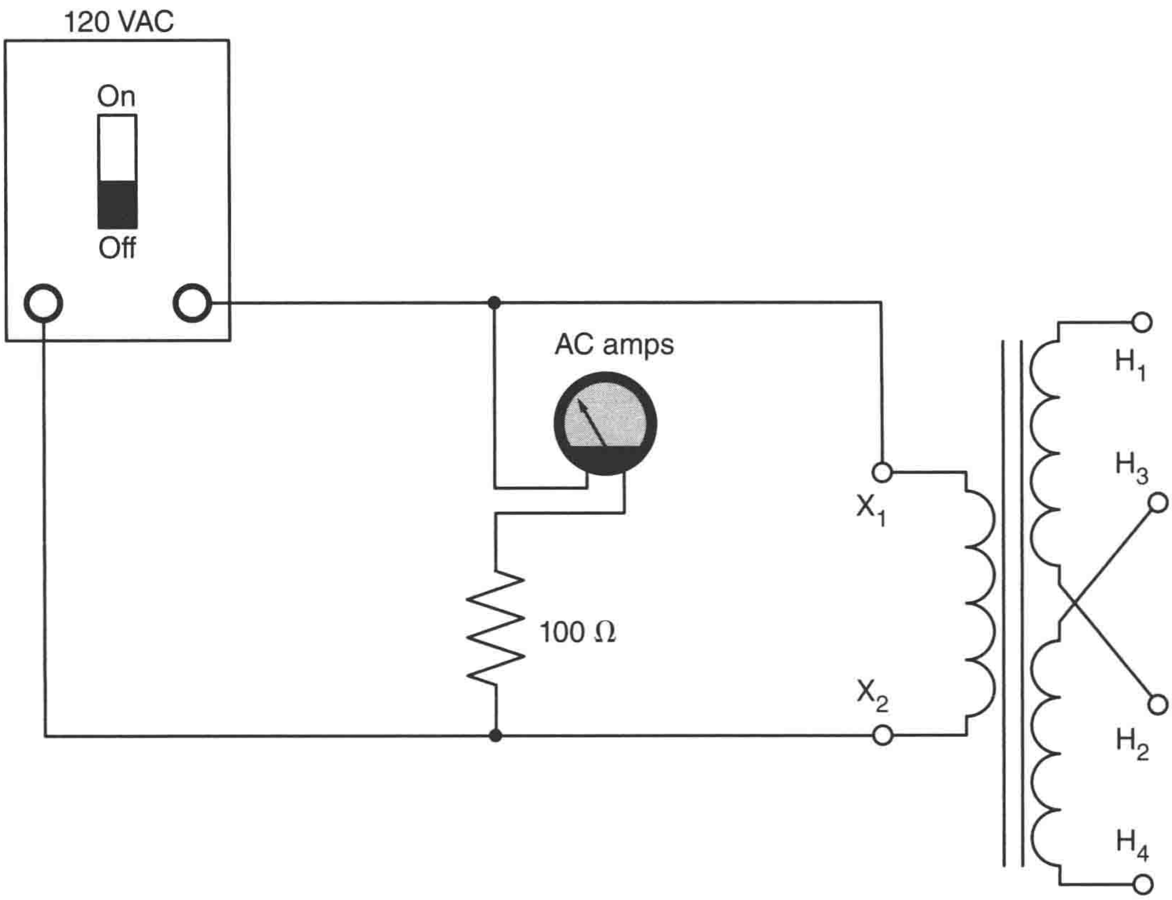


Figure 12-4 Measuring current through the resistive load.

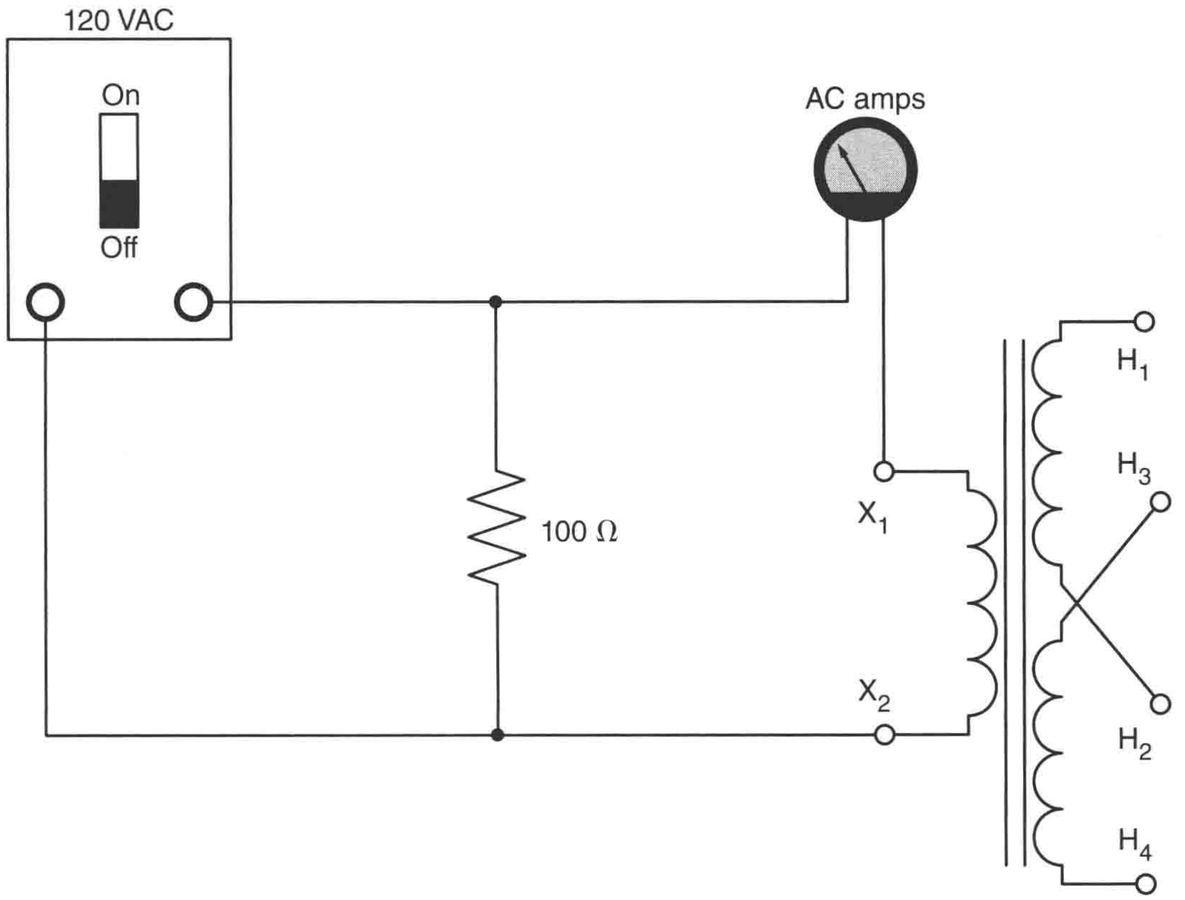


Figure 12-5 Measuring current through the inductive load.

12. Calculate the reactive power using inductive values of voltage and current.

    $VARs_L = E_L \times I_L$

    $VARs_L$ ______________

13. Compute the apparent power in the circuit using the total values.

    $VA = E_T \times I_T$.

    VA ______________

14. Compute the power factor using the following formula: $PF = \frac{P}{VA}$

    PF ______________ %

15. Compute the angle theta using this formula: $\cos \angle\varnothing = PF$

    $\angle\varnothing$ ______________ °

16. Add the current flow through the resistor and current flow through the inductor using the following formula:

    $I_T = I_R + I_L$

    I_T = __________ A

    Is the sum of the currents greater than the total current flow through the circuit?

    ____

17. In an RL parallel circuit the current flow through the resistor is out of phase with the current flow through the inductor. In a perfect circuit, the two currents would be 90° out of phase with each other. However, the transformer winding used as an inductor is not a perfect inductor and the current flow will not be at a 90° angle. Vector addition

must be employed to determine the total current value. Calculate the total circuit current using the following formula:

$I_T = \sqrt{I_R^2 + I_L^2}$

$I_T =$ _________ A

18. Replace the 100 ohm resistor with a 150 ohm resistor in the RL parallel circuit.
19. Reconnect the AC ammeter to measure the total circuit current. Turn on the power supply and measure the total current flow in the circuit.

    I_T ______________ amps
20. **Turn off the power supply** and reconnect the ammeter to measure the current flow through the 150 ohm resistor.
21. Turn on the power supply and measure the resistive current.

    I_R ______________ amps
22. **Turn off the power supply** and reconnect the ammeter to measure the current flow through the transformer winding.
23. Turn on the power supply and measure the inductive current.

    I_L ______________ amps
24. **Turn off the power supply.**
25. Compute the total impedance of the circuit using Ohm's law.

    Z ______________ Ω
26. Compute the true power in the circuit using Ohm's law.

    P ______________ watts
27. Compute the inductive reactance of the inductor.

    X_L ______________ Ω
28. Compute the reactive power using Ohm's law.

    $VARs_L$ ______________
29. Compute the apparent power in the circuit using total values of voltage and current.

    VA ______________
30. Compute the circuit power factor using the values of true power and apparent power.

    PF ______________ %
31. Compute angle theta.

    ∠∅ ______________ °
32. Disconnect the circuit and return the components to their proper place.

Review Questions

To answer the following questions, it may be necessary to refer to the formulas shown in Figure 12-6.

1. A 5 mh inductor is connector to a 400 Hz line. What is the inductive reactance of the inductor?

____

2. A resistor and inductor are connected in parallel to a 120 volt, 60 Hz line. The circuit has a current flow of 3 amperes. The resistor has a resistance of 72 Ω. What is the inductance of the inductor?

____

3. A resistor with a resistance of 50 Ω is connected in parallel with an inductor with an inductance of 0.175 henry. The power source is 60 Hz. What is the impedance of the circuit?

____

4. A resistor and inductor are connected in parallel to a 277 volt, 60 Hz line. The resistor has a current of 12 amperes flowing through it, and the inductor has a current flow of 8 amperes flowing through it. What is the total current flow in the circuit?

____

5. A resistor and inductor are connected in parallel to a 400 Hz line. The inductor has a voltage drop of 136 volts across it. What is the voltage drop across the resistor?

____

6. An inductor has a current flow of 2 amperes when connected to a 240 volt, 50 Hz line. How much current will flow through the inductor if it is connected to a 240 volt, 60 Hz line?

____

7. An RL parallel circuit has an apparent power of 2,400 VA. The true power is 1,860 watts. What is the circuit power factor?

____

8. How many degrees out of phase are the voltage and current in question 7?

____

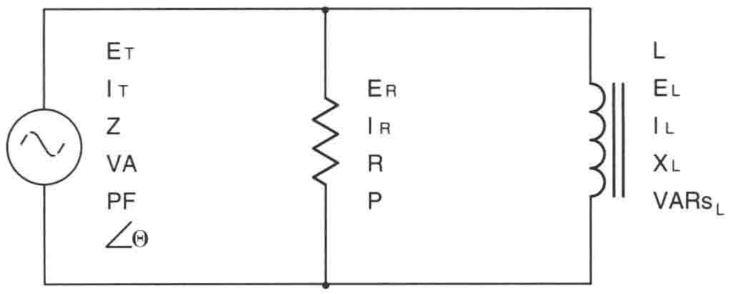
9. An RL parallel circuit has a power factor of 64%. The circuit voltage is 480 volts and the total current is 25.6 amperes. What is the true power in the circuit?

____

10. The voltage and current are 44° out of phase with each other in an RL parallel circuit. What is the circuit power factor?

____

Resistive-Inductive Parallel Circuits



$Z = \dfrac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}}$	$Z = \dfrac{VA}{I_T^2}$	$I_T = \sqrt{I_R^2 + I_L^2}$	$I_T = \dfrac{VA}{E_T}$	$I_T = \sqrt{\dfrac{VA}{Z}}$
$Z = \dfrac{E_T}{I_T}$ $\quad Z = \dfrac{E_T^2}{VA}$	$Z = R \times PF$	$I_T = \dfrac{E_T}{Z}$	$I_T = \dfrac{I_R}{PF}$	$E_T = E_R = E_L$
$VA = E_T \times I_T$	$PF = \dfrac{Z}{R}$	$E_L = I_L \times X_L$	$I_L = \sqrt{I_T^2 - I_R^2}$	$E_T = I_T \times Z$
$VA = I_T^2 \times Z$	$PF = \dfrac{P}{VA}$	$E_L = E_T = E_R$	$I_L = \dfrac{E_L}{X_L}$	$E_T = \dfrac{VA}{I_T}$
$VA = \dfrac{E_T^2}{Z}$	$PF = \dfrac{I_R}{I_T}$	$E_L = \sqrt{VARs_L \times X_L}$	$I_L = \dfrac{VARs_L}{E_L}$	$E_T = \sqrt{VA \times Z}$
$VA = \sqrt{P^2 + VARs_L^2}$	$PF = \text{Cos}\angle\Theta$	$E_L = \dfrac{VARs_L}{I_L}$	$I_L = \sqrt{\dfrac{VARs_L}{X_L}}$	$L = \dfrac{X_L}{2\pi f}$
$VA = \dfrac{P}{PF}$	$VARs_L = \sqrt{VA^2 - P^2}$	$VARs_L = E_L \times I_L$	$VARs_L = \dfrac{E_L^2}{X_L}$	
$VARs_L = I_L^2 \times X_L$		$E_R = I_R \times R$	$I_R = \sqrt{I_T^2 - I_L^2}$	
$X_L = \dfrac{E_L}{I_L}$	$X_L = \dfrac{1}{\sqrt{\left(\frac{1}{R}\right)^2 - \left(\frac{1}{Z}\right)^2}}$	$E_R = \sqrt{P \times R}$	$I_R = \dfrac{E_R}{R}$	
$X_L = \dfrac{E_L^2}{VARs_L}$	$X_L = 2\pi f L$	$E_R = \dfrac{P}{I_R}$	$I_R = \dfrac{P}{E_R}$	
$X_L = \dfrac{VARs_L}{I_L^2}$	$R = \dfrac{E_R}{I_R}$	$E_R = E_T = E_L$	$I_R = \sqrt{\dfrac{P}{R}}$	
	$R = \dfrac{E_R^2}{P}$	$R = \dfrac{P}{I_R^2}$	$I_R = I_T \times PF$	
$R = \dfrac{1}{\sqrt{\left(\frac{1}{Z}\right)^2 - \left(\frac{1}{X_L}\right)^2}}$	$P = \dfrac{E_R^2}{R}$	$R = \dfrac{Z}{PF}$	$P = \sqrt{VA^2 - VARs_L^2}$	
$P = E_R \times I_R$	$P = I_R^2 \times R$		$P = VA \times PF$	

Figure 12-6 RL parallel circuit formulas.

Unit 13 Capacitance

Objectives

After studying this unit, you should be able to

- Describe polarized and nonpolarized capacitors.
- Perform an ohmmeter test on a capacitor.
- Calculate values of capacitance and capacitive reactance.
- Determine the value of a capacitor by making electrical measurements.

Capacitors are the third major type of electrical load to be discussed. A basic capacitor is constructed by separating two metal plates with an insulating material called a dielectric (Figure 13-1). There are three factors that determine the amount of capacitance a capacitor will have:

1. *The surface area of the plates.* The greater the surface area, the more capacitance the capacitor will exhibit.
2. *The distance between the plates.* The closer the two plates are together, the greater the amount of capacitance.
3. *The type of dielectric.* Different types of dielectric can produce more or less capacitance.

Dielectric materials have a rating called the dielectric constant. Some common dielectric materials and their dielectric constants are shown in Figure 13-2. The dielectric constant of a material is determined by measuring the amount of increased capacitance when a particular material is employed. Air has a dielectric constant of 1 and is used as the base reference. Assume that a dielectric material is inserted between the plates of the capacitor without changing the distance between the plates. Now assume that the capacitor exhibits 10 times more capacitance with the dielectric material inserted between the plates instead of air. The material will have a dielectric constant of 10.

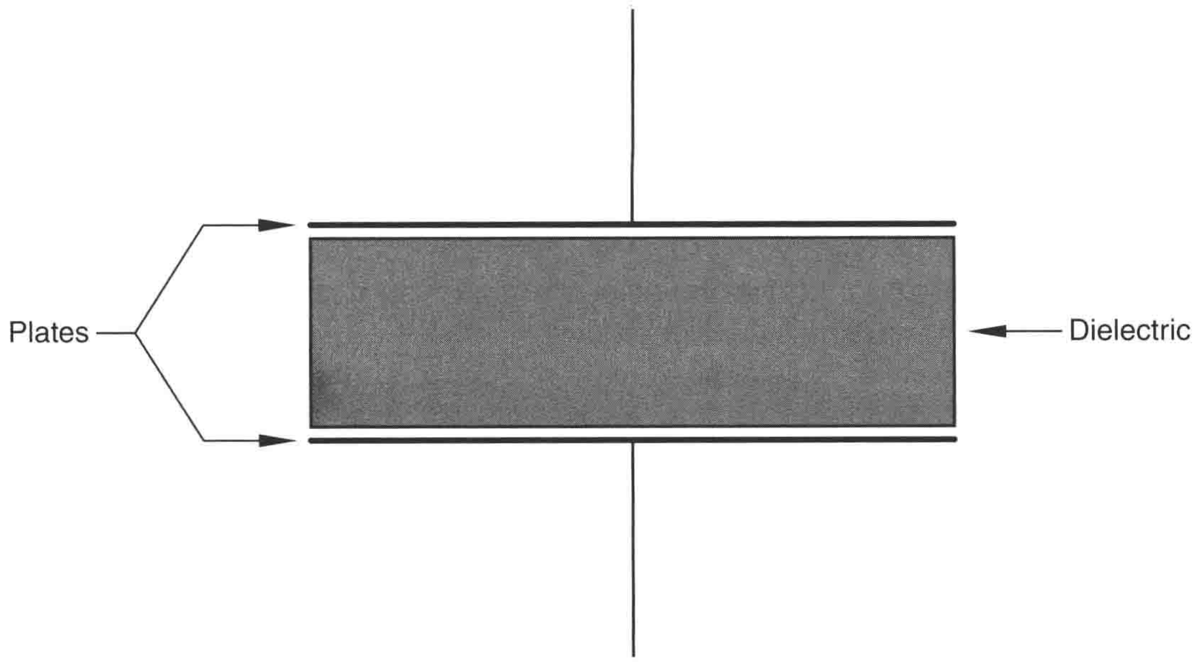


Figure 13-1 Basic capacitor is formed by separating two metal plates with an insulating material.

Material	Dielectric Constant
Air	1
Bakelite	4.0 to 10.0
Castor Oil	4.3 to 4.7
Cellulose Acetate	7.0
Ceramic	1,200
Dry Paper	3.5
Hard Rubber	2.8
Insulating Oils	2.2 to 4.6
Lucite	2.4 to 3.0
Mica	6.4 to 7.0
Mycalex	8.0
Paraffin	1.9 to 2.2
Porcelain	5.5
Pure Water	81
Pyrex Glass	4.1 to 4.9
Rubber Compounds	3.0 to 7.0
Teflon	2
Titanium Dioxide Compounds	90 to 170

Figure 13-2 Dielectric constant of different materials.

The dielectric or insulating material has the ability to change the amount of capacitance because of the manner in which a capacitor stores an electric charge. When a capacitor is charged, electrons are deposited on one plate and removed from the other, as seen in Figure 13-3. As electrons flow away from the capacitor plate connected to the positive battery terminal and to the capacitor plate connected to the negative battery terminal, a voltage is developed across the two capacitor plates. This electron flow will continue until the voltage across the plates is equal to the battery voltage. When the two voltages become equal, the current flow stops. If the battery is disconnected from the circuit, the capacitor is left in a charged state. The voltage across the capacitor plates produces stress on the dielectric material. Since one plate is now more positive and the other plate is more negative, the atoms of the dielectric become stressed

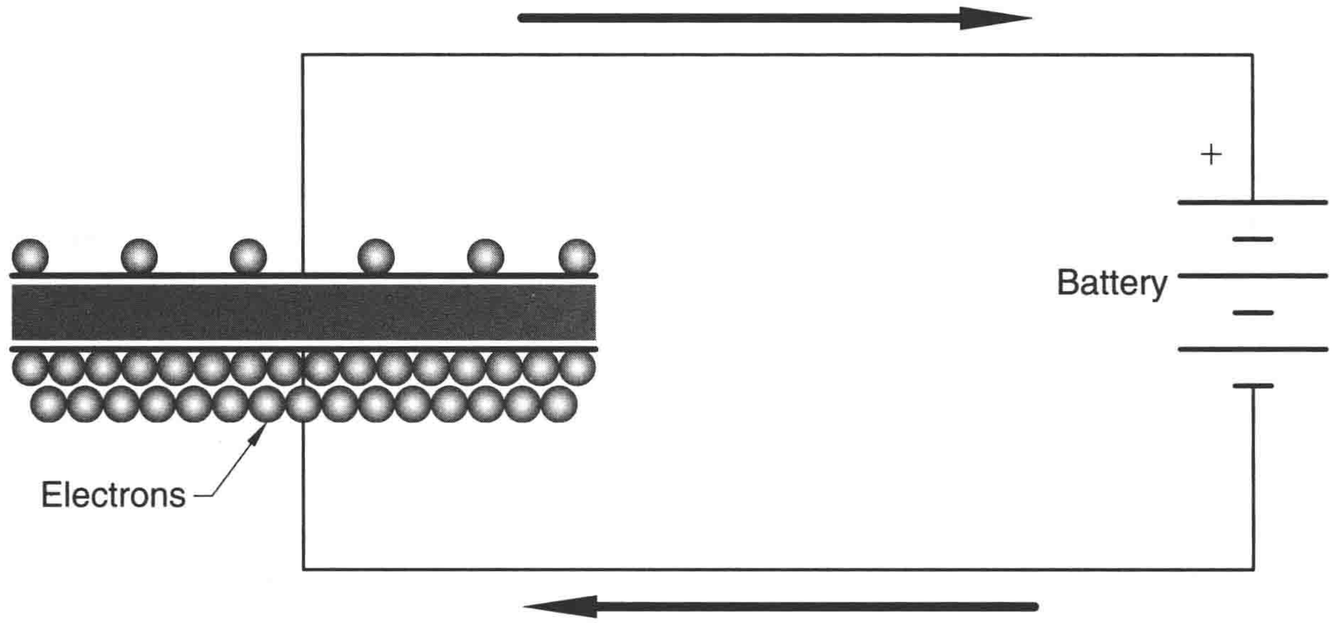


Figure 13-3 Charging a capacitor.

due to the attraction and repulsion of charged particles. The negative electrons in the atoms of the dielectric are attracted to the positive plate and repelled from the negative plate, as seen in Figure 13-4. This molecular stretching is called dielectric stress. The greater the potential difference between the two plates, the greater the dielectric stress. This is the reason that the voltage rating of a capacitor should never be exceeded. This molecular stretching is similar to stretching a rubber band. If the stress becomes too great (i.e., too much voltage), the dielectric will break down and short the capacitor. The dielectric stress produces an electrostatic field. Most of the capacitor's energy is stored in the electrostatic field.

The dielectric stress gives the capacitor the ability to produce almost infinite current. This is the reason that capacitors are one of the most dangerous components in the electrical field.

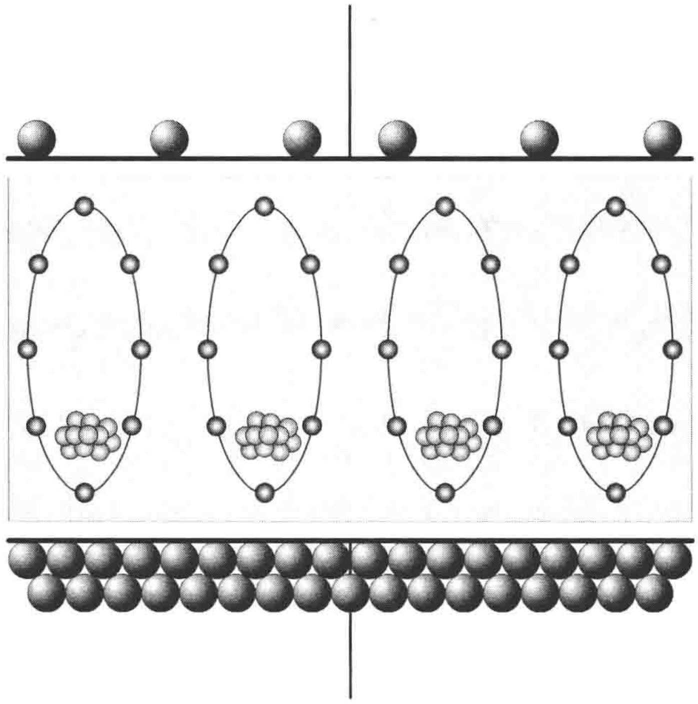


Figure 13-4 Atoms of the dielectric in a charged capacitor.

Never charge a capacitor and hand it to another person! Many people believe this to be a comical act, but in reality it is extremely dangerous. One of the basic characteristics of a capacitor is that it opposes a change of voltage. When a charged capacitor is connected across a load, it will produce any current necessary to prevent a voltage change. Assume that a capacitor has been charged to 500 volts. Now assume that the capacitor is connected across a 100 Ω load resistor. The capacitor will produce an initial current of 5 amps in an effort to prevent a voltage change (I = 500/100). If a person should contact the leads of a capacitor charged to 500 volts, the effect is the same as making contact with a 500 volt power line.

Caution

Never charge a capacitor and hand it to another person!

Capacitor Charge and Discharge

Capacitors charge and discharge at an exponential rate. This is the same rate of increase and decrease as the current flow through an inductor. Recall that each time constant of the exponential curve has a value of 63.2% of the whole. It is assumed that there are five time constants in the curve. An exponential charging curve is shown in Figure 13-5. The curve assumes the capacitor to be connected to a 100 volt source. The capacitor charge and

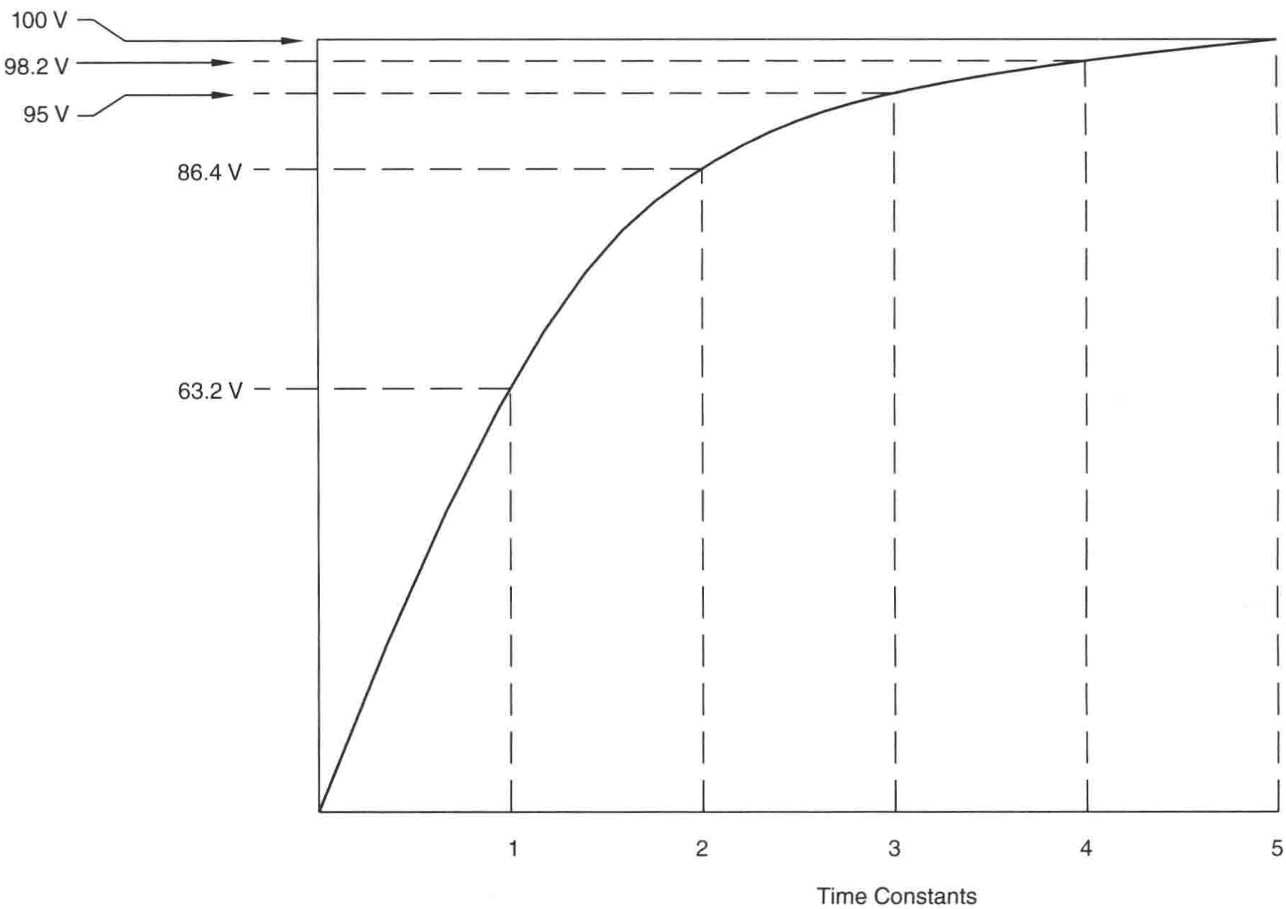


Figure 13-5 Capacitors charge at an exponential rate.

discharge time is determined by the amount of capacitance and resistance in the circuit. The formula ($T = RC$) can be used to determine the charge or discharge time of a capacitor where:

T = Time in seconds

R = Resistance in ohms

C = Capacitance in farads

The value of T is for one time constant.

Example: Determine the amount of time needed to charge a 100 μF capacitor connected in series with a 50 kΩ resistor.

$$T = 50{,}000 \times 0.000100 \ (100\ \mu F)$$

$$T = 5 \text{ sec per time constant}$$

$$\text{Total time} = 5 \text{ sec} \times 5 \text{ time constants}$$

$$\text{Total time} = 25 \text{ sec}$$

Capacitance Values

Capacitance is measured in units called farads. A farad is the amount of capacitance necessary to produce a charge of 1 coulomb with a voltage change of 1 volt across its plates. The farad is an extremely large amount of capacitance. A 1 farad capacitor with air as a dielectric and a spacing of 1 inch between the plates would have a plate area of approximately 1,100 square miles. Since the farad is such a large amount of capacitance, capacitance values are generally expressed as microfarads (μF), nanofarads (nF), and picofarads (pF).

$$\mu F = \frac{1}{1{,}000{,}000} \text{ or } \times 10^{-6}$$

$$nF = \frac{1}{1{,}000{,}000{,}000} \text{ or } \times 10^{-9}$$

$$pF = \frac{1}{1{,}000{,}000{,}000{,}000} \text{ or } \times 10^{-12}$$

The picofarad is sometimes called micro-microfarad because it is one-millionth of a microfarad.

Most formulas that use capacitance assume that the capacitance value is in farads. As a general rule, the value given must be converted into farads for use in a formula. A value of 25 μF can be converted to farads by moving the decimal point six places to the left so that 25 μF becomes 0.000025 farads. Or a value of 300 pF can be converted to farads by moving the decimal point twelve places to the left so that 300 pF becomes 0.000000000300 farads.

Another method of entering capacitance values with a scientific calculator is to enter the value using scientific notation. Most scientific calculators contain an EE key or EXP key, depending on the manufacturer. These keys can be used to enter a value in scientific notation. Micro is one-millionth or 10^{-6}. 25 μF can be entered as shown:

25 EE 6 +/−

300 pF can be entered as shown:

300 EE 12 +/−

Polarized and Nonpolarized Capacitors

Capacitors can be divided into two major categories: polarized and nonpolarized. Nonpolarized capacitors are often called AC capacitors because they can be connected to alternating current circuits. These capacitors are not polarity sensitive. Either of the capacitor terminals can be connected to a positive or negative voltage.

Polarized capacitors can be connected to direct current circuits only. One of their terminals will be marked with a + or − sign to indicate the proper polarity. If the polarity connection is reversed, the capacitor will be damaged and possibly explode. Polarized capacitors are often called electrolytic capacitors. The greatest advantage of an electrolytic capacitor is that it can exhibit a large amount of capacitance in a small case size. Electrolytic capacitors are generally used in electronic equipment.

Capacitor Voltage Rating

As stated previously, the voltage rating of a capacitor should never be exceeded. The voltage ratings are marked in different ways, often making it difficult for a student to determine exactly what the voltage rating is. Some common capacitor voltage ratings are:

VDC (Volts DC)

WVDC (Working Volts DC)

VAC (Volts AC)

The VDC and WVDC ratings are essentially the same. Polarized capacitors will be rated in DC volts because they must be connected to direct current. Nonpolarized capacitors are generally marked VAC or WVAC. If the voltage rating is given as VAC, it is an RMS value. If a nonpolarized capacitor is marked WVDC, it is a peak value, not an RMS value.

Example: Assume that an AC capacitor is marked 600 WVDC. What is the maximum RMS value of voltage that can be connected to the capacitor? To find the answer, assume 600 volts to be a peak value and multiply by 0.707 to find the RMS value.

$$E_{RMS} = 600 \times 0.707$$

$$E_{RMS} = 424.2 \text{ volts}$$

Capacitance in AC Circuits

When a capacitor is connected in a DC circuit, current will flow during the time the capacitor is charging or discharging. Once the capacitor has become charged, it becomes an open circuit. When a capacitor is connected into an AC circuit, the capacitor charges and discharges each time the current changes direction. Since the current is continually changing direction, current will appear to flow through the capacitor.

As the capacitor charges, the impressed voltage across the capacitor opposes the applied voltage, acting like a resistance to the flow of current. This opposition to current flow is called capacitive reactance and is symbolized X_C. The formula for determining capacitive reactance is:

$$X_C = \frac{1}{2\pi fC}$$

where

X_C = Capacitive reactance in ohms

$\pi = 3.1416$

f = Frequency in Hz

C = Capacitance in farads

Example: A 10 μF capacitor is connected to a 120 volt, 60 Hz line. How much current will flow in this circuit? The first step is to determine the capacitive reactance.

$$X_C = \frac{1}{2 \times \pi \times 60 \times 0.000010 \text{ farad}}$$

$$X_C = 265.26\ \Omega$$

The current flow can now be determined by replacing the value of resistance (R) with the value of capacitive reactance (X_C) in an Ohm's law formula:

$$I = \frac{E}{X_C}$$

$$I = \frac{120}{265.26}$$

$$I = 0.452 \text{ amp}$$

If the value of capacitive reactance is known, the capacitance can be determined using the formula:

$$C = \frac{1}{2\pi f X_C}$$

Example: A capacitor is connected in a 480 volt, 60 Hz circuit. The circuit has a current of 14 amperes. What is the capacitance of the capacitor? The first step is to determine the capacitive reactance using Ohm's law.

$$X_C = \frac{E}{I}$$

$$X_C = \frac{480}{14}$$

$$X_C = 34.29\ \Omega$$

Now that the value of X_C is known, the capacitance can be determined.

$$C = \frac{1}{2 \times \pi \times 60 \times 34.29}$$

$$C = 0.00007736 \text{ farad or } 77.36\ \mu F$$

Capacitors Connected in Series

When capacitors are connected in series, the total capacitance is reduced because it has the effect of increasing the distance between the plates. The formula for determining the total

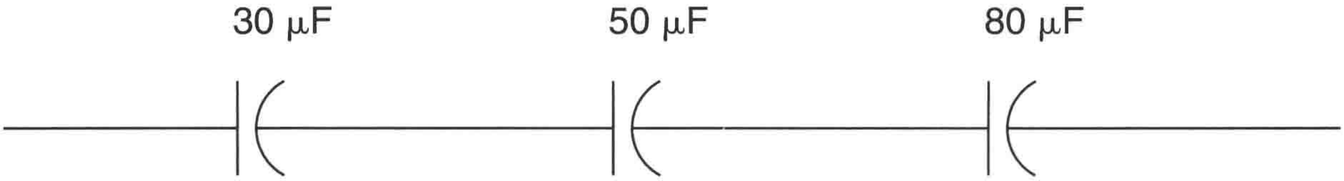


Figure 13-6 Capacitors connected in series.

capacitance of capacitors in series is similar to determining total resistance in a parallel circuit.

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_N}} \quad \text{(reciprocal method)}$$

or

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2} \quad \text{(product over sum method)}$$

or

$$C_T = \frac{C}{N}$$

This formula can be employed when all capacitor values are the same. The product over sum method can be employed to determine the value of two capacitors at a time.

Assume that a circuit contains three capacitors with values of 30 μF, 50 μF, and 80 μF connected in series, as shown in Figure 13-6. What is the total capacitance of this circuit?

$$C_T = \frac{1}{\frac{1}{30} + \frac{1}{50} + \frac{1}{80}}$$

$$C_T = 15.19\ \mu F$$

The total capacitive reactance is equal to the sum of the individual capacitive reactances in the circuit.

$$X_{C_T} = X_{C_1} + X_{C_2} + X_{C_3} + X_{C_N}$$

The advantage of connecting capacitors in series is that it increases the voltage rating of the connection. Assume that three capacitors rated at 100 μF and 100 volts each are connected in series. The total capacitance of the connection would be 33.33 μF ($C_T = \frac{100}{3}$), but the voltage rating would now be 300 volts instead of 100 volts.

Capacitors Connected in Parallel

When capacitors are connected in parallel, the total capacitance value will be the sum of the individual capacitors. Connecting capacitors in parallel has the effect of increasing the plate area of one capacitor. The formula for finding the total capacitance of parallel-connected capacitors is:

$$C_T = C_1 + C_2 + C_3 + C_N$$

The total capacitive reactance can be determined in a manner similar to determining parallel resistance.

$$X_{C_T} = \frac{1}{\frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{X_{C_3}} + \frac{1}{X_{C_N}}} \quad \text{(reciprocal method)}$$

or

$$X_{C_T} = \frac{X_{C_1} \times X_{C_2}}{X_{C_1} + X_{C_2}} \quad \text{(product over sum method)}$$

or

$$X_{C_T} = \frac{X_C}{N}$$

This formula can be employed when all values of capacitive reactance are the same. The product over sum method can be employed when two values of capacitive reactance are known.

Voltage and Current Relationships

In a pure capacitive circuit, the voltage and current are 90° out of phase with each other, as shown in Figure 13-7. In a pure capacitive circuit, the current *leads* the applied voltage instead of lagging the applied voltage. Since the current and voltage are 90° out of phase, there is no true power or watts produced in a pure capacitive circuit. The electrical energy is stored in the form of an electrostatic field during part of the cycle and returned to the circuit during another part. Like inductive circuits, the power in a capacitive circuit is in VARs and not watts. To distinguish between inductive and capacitive VARs, capacitive VARs will be noted as $VARs_C$. The formulas for determining capacitive VARs are the same basic formulas used for watts and inductive VARs.

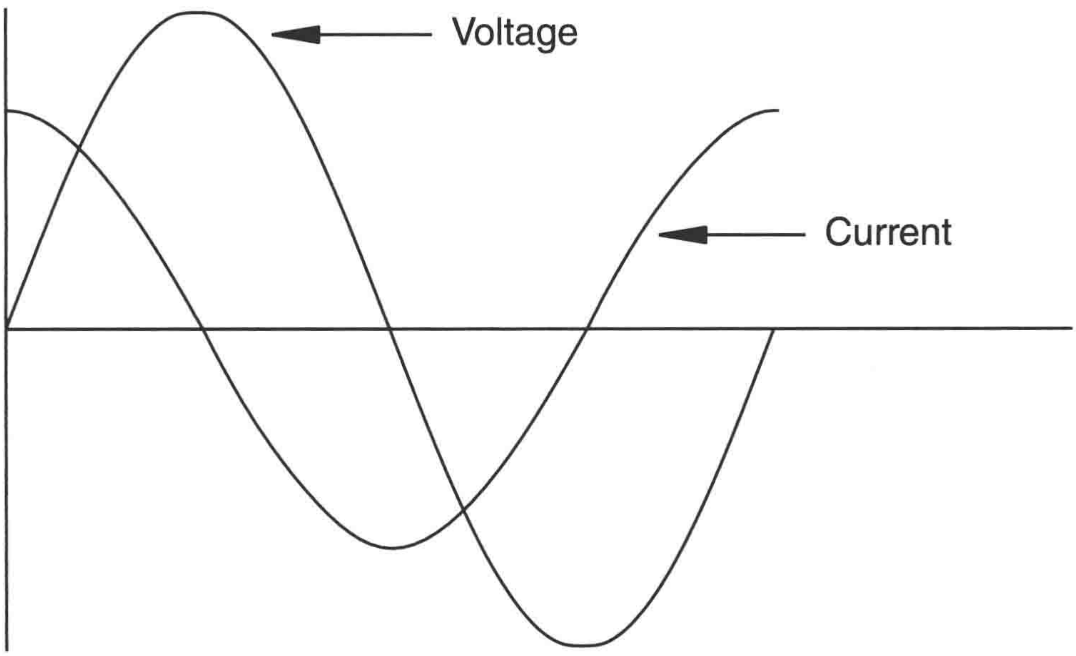


Figure 13-7 In a pure capacitive circuit, the current leads the voltage by 90°.

$$VARs_C = E_C \times I_C$$

$$VARs_C = I_C^2 \times X_C$$

$$VARs_C = \frac{E_C^2}{X_C}$$

Capacitor Testing

Testing capacitors is difficult at best. To really test a capacitor, two conditions must be assessed. One is the condition of the dielectric, and the other is the condition of the plate area. The first test can be made with an ohmmeter. The ohmmeter is used to check for a short circuit between the plates. If an analog ohmmeter is used, when the meter leads are connected to the capacitor the needle should swing upscale and then return to infinity. If the ohmmeter leads are reversed, the needle should move approximately twice as far upscale and then return to infinity. If a digital ohmmeter is employed, it should indicate either infinity or a very high resistance in the meg-ohm range.

The ohmmeter test can indicate a short circuit, but it cannot test the condition of the dielectric. To test the dielectric, rated voltage must be applied to the capacitor. As a general rule, a test instrument called a "HiPot" is used to test the dielectric. HiPot is an abbreviation for high potential. The HiPot permits rated voltage to be applied to the capacitor while measuring the leakage current through the dielectric. In theory, there should be no current flow through the dielectric. Leakage current is an indication that the dielectric is breaking down under load.

The second test involves measuring the amount of capacitance of the capacitor. If the plates are in good condition, the capacitance should be within a few percent of the marked capacitor rating. The capacitance value can be determined in several ways. One is to use a capacitance tester designed to measure the amount of capacitance. Another is to connect the capacitor in an AC circuit and measure the current flow. The capacitive reactance can then be computed using Ohm's law, and the capacitance value can be computed once the capacitive reactance is known.

LABORATORY EXERCISE

Name ____ Date __________

Materials Required

1 120-VAC power supply

1 AC voltmeter

1 AC ammeter, in-line or clamp-on. (If a clamp-on meter is employed, the use of a 10:1 scale divider is recommended.)

1 ohmmeter

1 7.5-μF capacitor 240 VAC or more

1 10-μF capacitor 240 VAC or more

1 25-μF capacitor 240 VAC or more

1. Use an ohmmeter to test the resistance of the following capacitors. Be sure to reverse the ohmmeter leads when checking each capacitor. List the amount of resistance for each capacitor.

   7.5 μF ________ Ω 10 μF ________ Ω 25 μF ________ Ω

Caution

If any of the capacitors tested indicate a low value of resistance (less than 100 kΩ), inform your instructor before continuing with this experiment.

2. Calculate the amount of capacitive reactance for the 7.5 μF capacitor. Assume a frequency of 60 Hz.

   $$X_C = \frac{1}{2\pi fC}$$

   (Note: A shortcut is to use a value of 377 for ($2\pi f$) if the frequency is 60 Hz.)

   X_C = ________________ Ω

3. Using the calculated value of capacitive reactance, compute the amount of current that should flow if the capacitor is connected to 120 VAC.

   I = ________________ amp(s)

4. Connect the circuit shown in Figure 13-8. Turn on the power and measure the amount of current flow. Compare the measured value with the computed value. Are the two answers approximately the same?

   I = ______________ amp(s) Yes/no ________________

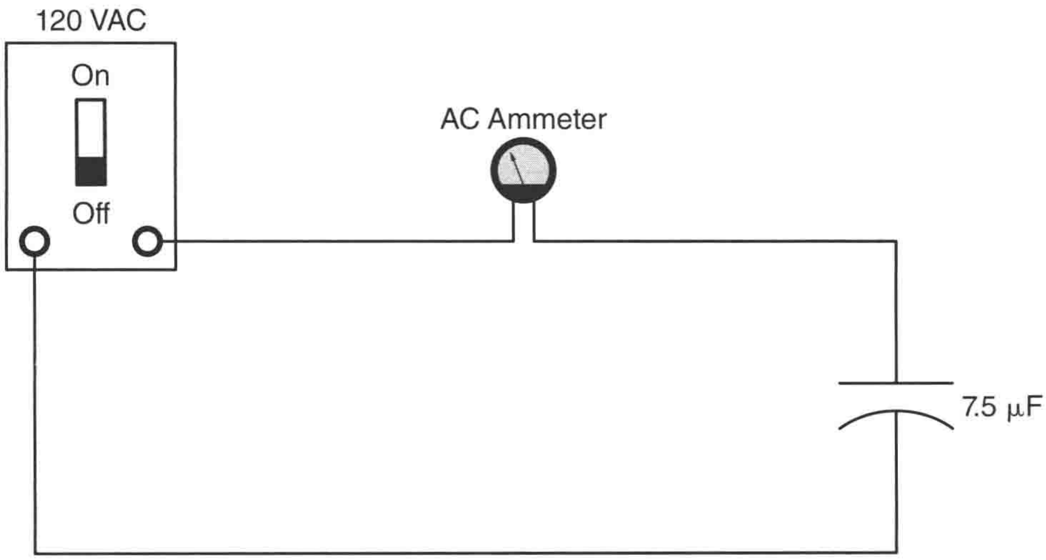


Figure 13-8 A capacitive circuit.

5. **Turn off the power supply.**
6. Replace the 7.5 μF capacitor with a 10 μF capacitor.
7. Turn on the power supply and measure the amount of current flow in the circuit.

   I = _______________ amp(s)
8. Using the measure value of current, compute the capacitive reactance for this circuit.

   $$X_C = \frac{E}{I}$$

   X_C = _____________ Ω
9. Using the computed value of capacitive reactance, compute the capacitance of the capacitor. (Note: The computed value will be in farads. Change your answer to microfarads.) Is the computed value approximately the same as the marked value on the capacitor?

   $$C = \frac{1}{2\pi f X_C}$$

   C = ______________ μF Yes/no _______________
10. **Turn off the power supply.**
11. Replace the 10 μF capacitor with the 25 μF capacitor.
12. Compute the amount of current that should flow if a voltage of 120 volts is applied to the circuit.

    I = _______________ amp(s)
13. Turn on the power supply and measure the current in the circuit. Is the measured current approximately the same as the computed value?

    I = _______________ amp(s) Yes/no _____________
14. **Turn off the power supply.**
15. Connect the three capacitors to form a series circuit as shown in Figure 13-9.

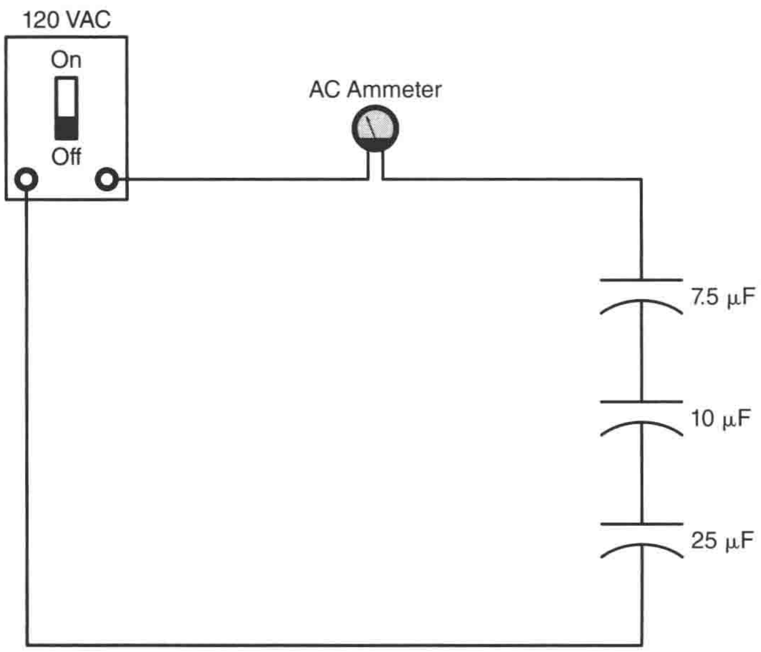


Figure 13-9 Capacitors connected in series.

16. Turn on the power supply and measure the current in the circuit.

    I = _______________ amp(s)

17. Use an AC voltmeter to measure the voltage drop across each capacitor.

    7.5 μF ________ volts 10 μF _________ volts 25 μF ________ volts

18. Using the measured values of voltage and current, compute the capacitive reactance of each capacitor. Recall that in a series circuit the current is the same in all parts of the circuit.

    7.5 μF ________ Ω 10 μF _________ Ω 25 μF ________ Ω

19. Using the formula ($X_{C_T} = X_{C_1} + X_{C_2} + X_{C_3} + X_{C_N}$), compute the capacitive reactance for the circuit.

    X_{C_T} = ___________________ Ω

20. Use Ohm's law and the values of applied voltage and circuit current to compute the total capacitive reactance for this circuit. Is this computed value approximately the same as the capacitive reactance value in step 19?

    X_{C_T} = ______________ Ω Yes/no ____________

21. **Turn off the power supply.**

22. Reconnect the three capacitors to form a parallel connection as shown in Figure 13-10.

23. Turn on the power supply and measure the current flow in the circuit.

    I = ________________ amp(s)

24. **Turn off the power supply.**

25. Calculate the capacitive reactance of the circuit using Ohm's law.

    X_{C_T} = ______________ Ω

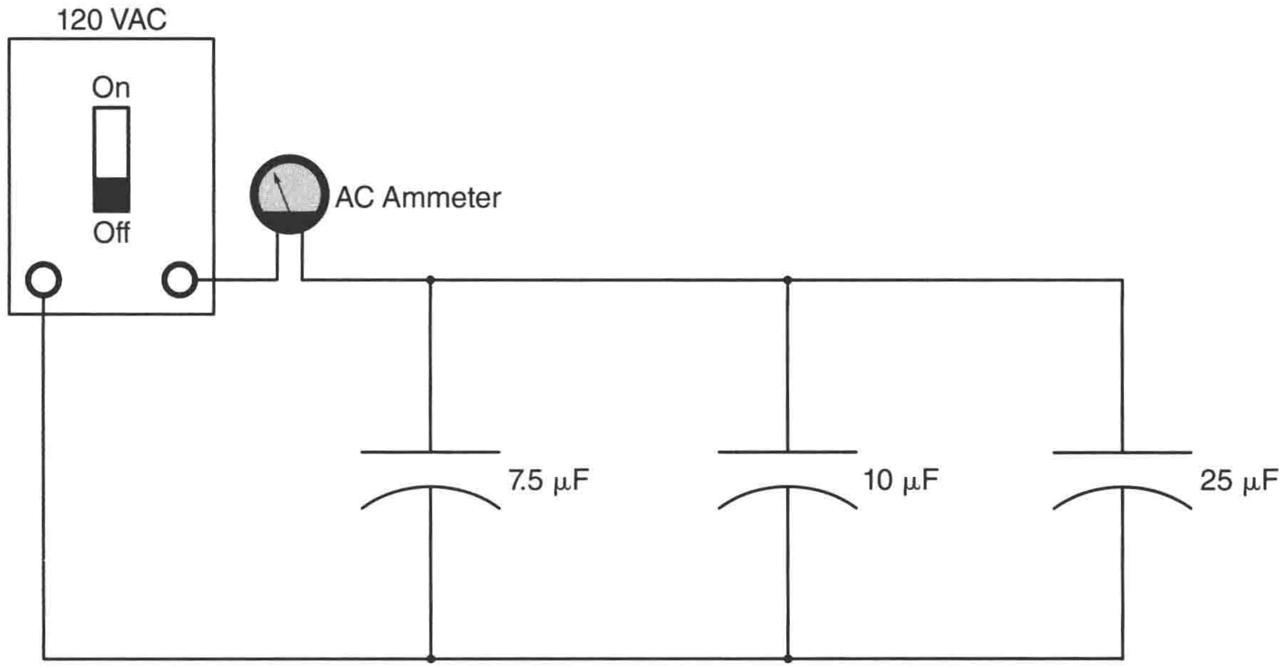


Figure 13-10 Capacitors connected in parallel.

26. Use the values of capacitive reactance computed in step 18 for each of the capacitors to compute the total amount of capacitive reactance for the circuit. Is the computed value approximately the same as the value computed in step 25?

$$X_{C_T} = \frac{1}{\frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{X_{C_3}} + \frac{1}{X_{C_N}}}$$

X_{C_T} = ______________ Ω Yes/no ____________

27. Using the computed value of capacitive reactance in step 26, compute the total amount of capacitance in the circuit.

C_T = ______________ μF

28. Add the values of capacitance listed on the three capacitors. Is the sum of the listed values approximately the same as the computed value in step 27?

C_T = ______________ μF Yes/no ____________

29. Disconnect the circuit and return the components to their proper place.

Review Questions

1. Name three factors that determine the capacitance of a capacitor.

____

____

____

2. A nonpolarized capacitor is connected in a DC circuit. Is there a danger of damaging the capacitor when the power is turned on?

____

3. A polarized capacitor is connected to an AC circuit. Is there a danger of damaging the capacitor when the power is turned on?

____

4. Four capacitors having values of 100 μF, 175 μF, 75 μF, and 50 μF are connected in series. What is the total capacitance of the circuit?

____

5. Assume that the circuit in question 4 is connected to a 240 volt AC, 60 Hz power supply. How much voltage would be dropped across the 100 μF capacitor?

____

6. Assume that the 175 μF capacitor in question 4 has a voltage rating of 35 WVDC. If the circuit is connected to 240 VAC, will the voltage rating of the capacitor be exceeded?

____

7. A 500 pF capacitor is connected to a 60 Hz circuit. What is the capacitive reactance of this capacitor?

____

8. Assume that a 500 pF capacitor is connected in a circuit with a frequency of 100 kHz. What is the capacitive reactance of the capacitor?

____

9. Assume that three capacitors with values of 5 μF, 12 μF, and 22 μF are connected in parallel. What is the total capacitance of the circuit?

____

10. Assume that the circuit in question 9 is connected to a 208-volt, 60 Hz power line. How much current will flow in the circuit?

____

Unit 14 Resistive-Capacitive Series Circuits

Objectives

After studying this unit, you should be able to

- Discuss the voltage and current relationship in an RC series circuit.
- Determine the phase angle of current in an RC series circuit.
- Determine the power factor in an RC series circuit.

In an AC circuit containing pure resistance, the current and voltage are in phase with each other. In an AC circuit containing a pure capacitive load, the current will lead the voltage by 90°. Since the current is the same at any point in a series circuit, the voltage drop across the resistor and the voltage drop across the capacitor will be 90° out of phase with each other, as shown in Figure 14-1.

Example Problem

A series circuit containing a resistor and capacitor is shown in Figure 14-2. It is assumed the circuit is connected to 120 VAC with a frequency of 60 Hz. The resistor has a resistance of 30 Ω and the capacitor has a capacitive reactance of 46 Ω. The following values will be computed:

Z - Total impedance of the circuit

I_T - Total circuit current

E_R - Voltage drop across the resistor

P - True power or watts

E_C - Voltage drop across the capacitor

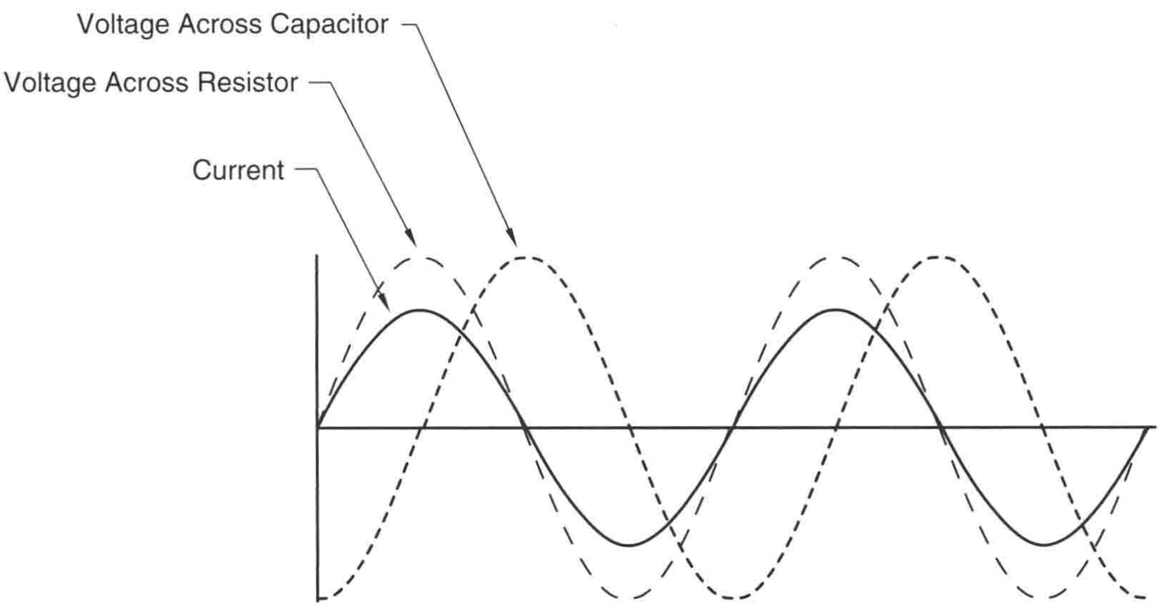


Figure 14-1 The voltage across the resistor and the voltage across the capacitor are 90° out of phase with each other.

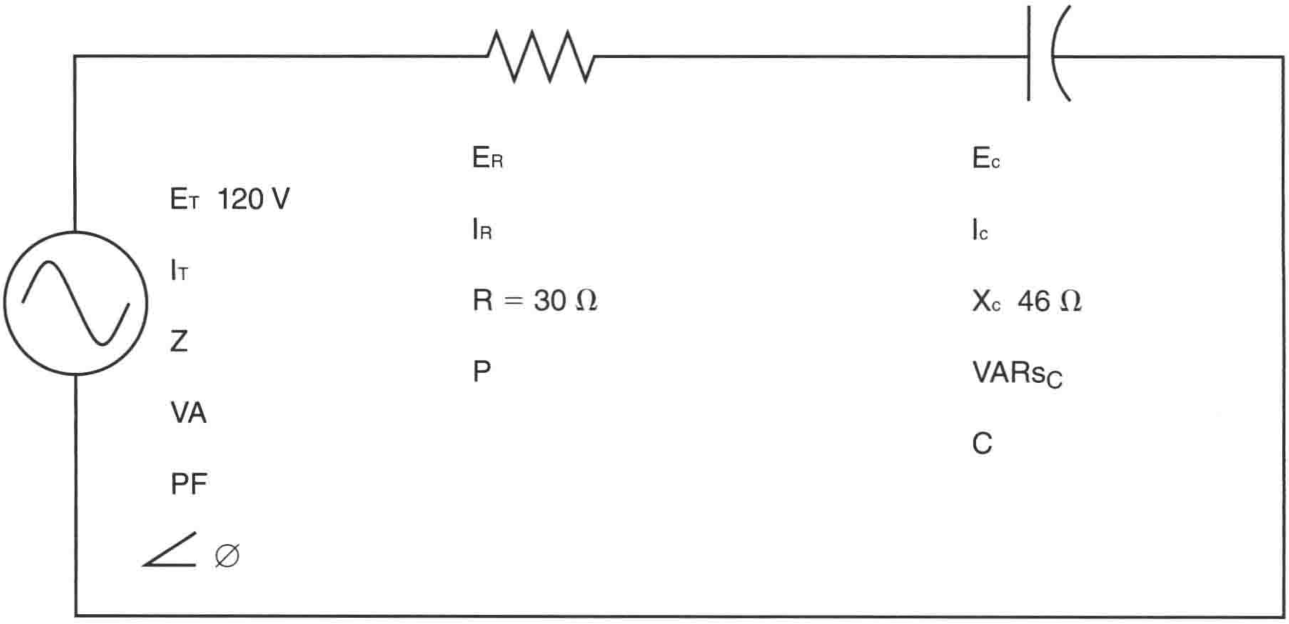


Figure 14-2 RC series circuit.

C - Capacitance of the capacitor

$VARs_C$ - Reactive power

VA - Volt amps or apparent power

PF - Power factor

$\angle\varnothing$ - Angle theta (the angle's degree amount that the current and voltage are out of phase with each other)

The first step in determining the missing values is to compute the total circuit impedance. Since the capacitive and resistive parts of the circuit are out of phase with each other by 90°, vector addition must be used to determine the total impedance. The formula for impedance determining the impedance in a series circuit containing resistance and capacitive reactance is $Z = \sqrt{R^2 + X_c^{\,2}}$.

$$Z = \sqrt{30^2 + 46^2}$$

$$Z = \sqrt{900 + 2{,}116}$$

$$Z = \sqrt{3{,}016}$$

$$Z = 54.92\ \Omega$$

The total circuit current can now be computed using Ohm's law.

$$I_T = \frac{E_T}{Z}$$

$$I_T = \frac{120}{54.92}$$

$$I_T = 2.18 \text{ amps}$$

The current is the same at any point in a series circuit. The values of I_C and I_L are, therefore, the same as I_T.

Now that the amount of current flowing through the resistor is known, the voltage drop across the resistor can be determined using Ohm's law.

$$E_R = I_R \times R$$

$$E_R = 2.18 \times 30$$

$$E_R = 65.4 \text{ volts}$$

The true power or watts can be computed using any of the power formulas. In this example the true power will be computed using E × I.

$$P = E_R \times I_R$$

$$P = 65.4 \times 2.18$$

$$P = 142.57 \text{ watts}$$

The voltage drop across the capacitor can be determined using Ohm's law and reactive values.

$$E_c = I_c \times X_c$$

$$E_c = 2.18 \times 46$$

$$E_c = 100.28 \text{ volts}$$

The reactive VARs can be computed in a manner similar to determining the value for watts or volt amps, except that reactive values are used.

$$VARs_C = E_c \times I_c$$

$$VARs_C = 100.28 \times 2.18$$

$$VARs_C = 218.61$$

The capacitance value of the capacitor can be computed using the formula:

$$C = \frac{1}{2\pi f X_c}$$

$$C = \frac{1}{2 \times 3.1416 \times 60 \times 46}$$

$$C = \frac{1}{377 \times 46}$$

$$C = 0.00005766 \text{ farad or } 57.66\ \mu F$$

The apparent power or volt amps can be computed using formulas similar to those for determining watts or VARs, except that total circuit values are used in the formula:

$$VA = E_T \times I_T$$

$$VA = 120 \times 2.18$$

$$VA = 261.6$$

The circuit power factor can be computed using the formula:

$$PF = \frac{P}{VA}$$

$$PF = \frac{142.57}{261.6}$$

$$PF = 0.545 \text{ or } 54.5\%$$

The power factor is the cosine of angle theta. In this circuit, the decimal power factor is 0.545. The cosine of angle theta is 0.545. To determine angle theta, find the angle that corresponds to a cosine of 0.545. Most scientific calculators contain trigonometric functions. To find what angle corresponds to one of the sin, cos, or tan functions, it is generally necessary to use the invert key, the arc key, or one of the keys marked $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$.

$$\cos \angle\varnothing = 0.545$$

$$\angle\varnothing = 56.97°$$

The current and voltage are 56.97° out of phase with each other in this circuit.

The circuit with all completed values is shown in Figure 14-3.

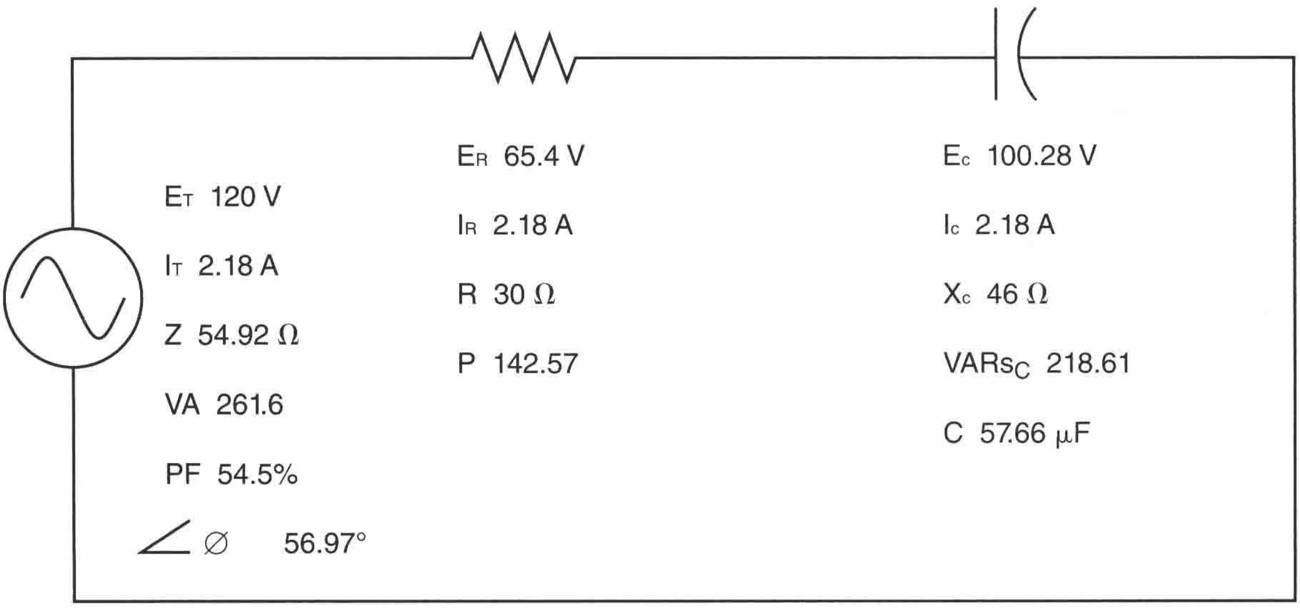


Figure 14-3 RC series circuit with all missing values.

LABORATORY EXERCISE

Name ______________________________ Date __________

Materials Required

1 25 μF AC capacitor with a minimum voltage rating of 240 VAC

1 10 μF AC capacitor with a minimum voltage rating of 240 VAC

1 100 ohm resistor

1 150 ohm resistor

AC ammeter (In-line or clamp-on type may be used. If a clamp-on type is employed, the use of a 10:1 scale divider is recommended.)

Connecting wires

AC voltmeter

1 120 volt AC power supply

Formulas for resistive–capacitive series circuits shown in Figure 14-6

1. Connect the circuit shown in Figure 14-4.
2. Calculate the capacitive reactance of the 25 μF capacitor assuming a frequency of 60 Hz.

   X_C = __________ Ω
3. Calculate the impedance of the circuit using the following formula:

   $$Z = \sqrt{R^2 + X_C^2}$$

   Z = __________ Ω
4. Assuming a voltage of 120 volts, calculate the total current flow in the circuit using the following formula:

   $$I_T = \frac{E_T}{Z}$$

   I_T = __________ A
5. Calculate the voltage drop across the 100 ohm resistor using Ohm's law.

   E_R = __________ volts
6. Calculate the voltage drop across the 25 μF capacitor using Ohm's law.

   E_C = __________ volts
7. Turn on the power and measure the current flow through the circuit. **Turn off the power.**

   I_T = __________ A
8. Is the calculated value in step 4 within 5% of the measured value of current?

   ____

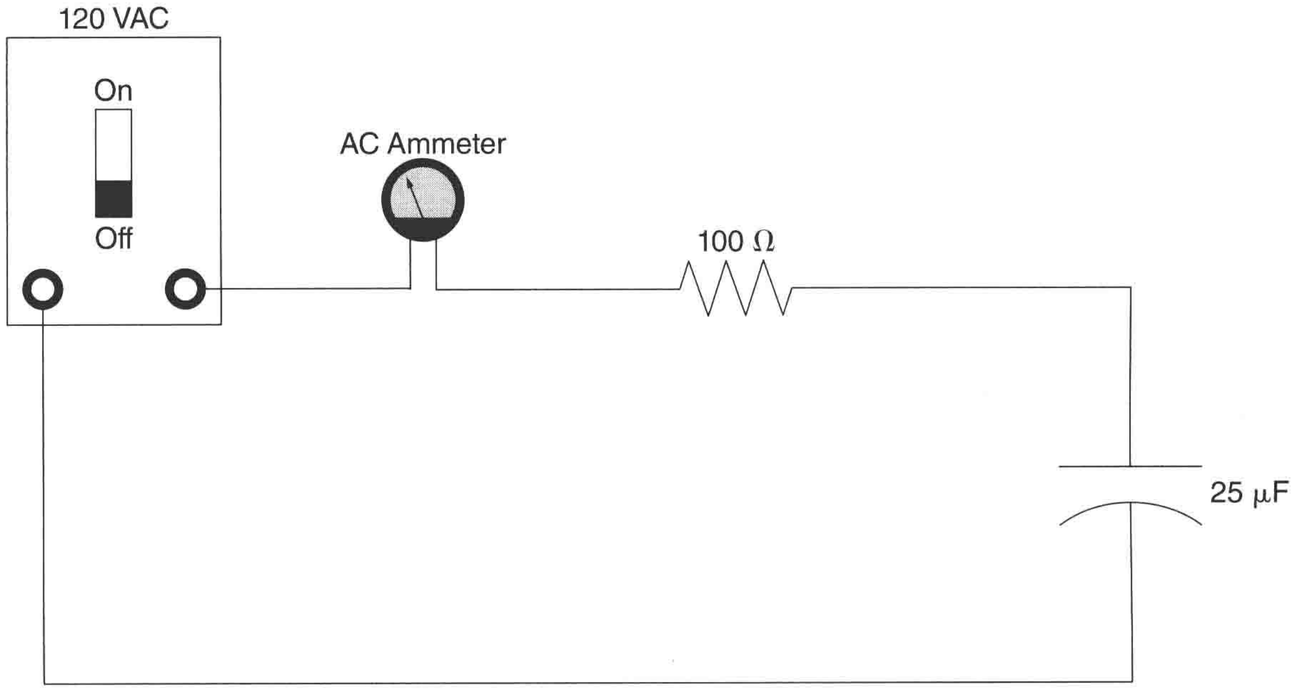


Figure 14-4 A 100 ohm resistor is connected in series with a 25 μF capacitor.

9. Turn on the power and measure the voltage drop across the resistor and capacitor. **Turn off the power.**

   E_R = __________ volts

   E_C = __________ volts

10. Compare the measured values of voltage drop across the resistor and capacitor with the calculated values in steps 5 and 6. Are the values within 5% of each other?

    ____

11. In a series circuit the current in all parts of the circuit must be the same. Therefore, the voltage drop across the resistor is 90° out of phase with the voltage drop across the capacitor. When vector addition is used, the sum of the two voltage drops should equal the applied voltage. Calculate the total or applied voltage using the following formula:

    $$E_T = \sqrt{E_R^2 + E_C^2}$$

    E_T = __________ volts

12. Is the computed value of voltage within 5% of the voltage that was applied to the circuit?

    ____

13. Determine the true power of the circuit using the circuit current and the voltage drop across the resistor.

    $P = E_R \times I$

    P = __________ watts

14. Determine the capacitive VARs using the circuit current and the voltage drop across the capacitor.

    $VARs_C = E_C \times I$

    $VARs_C$ = __________

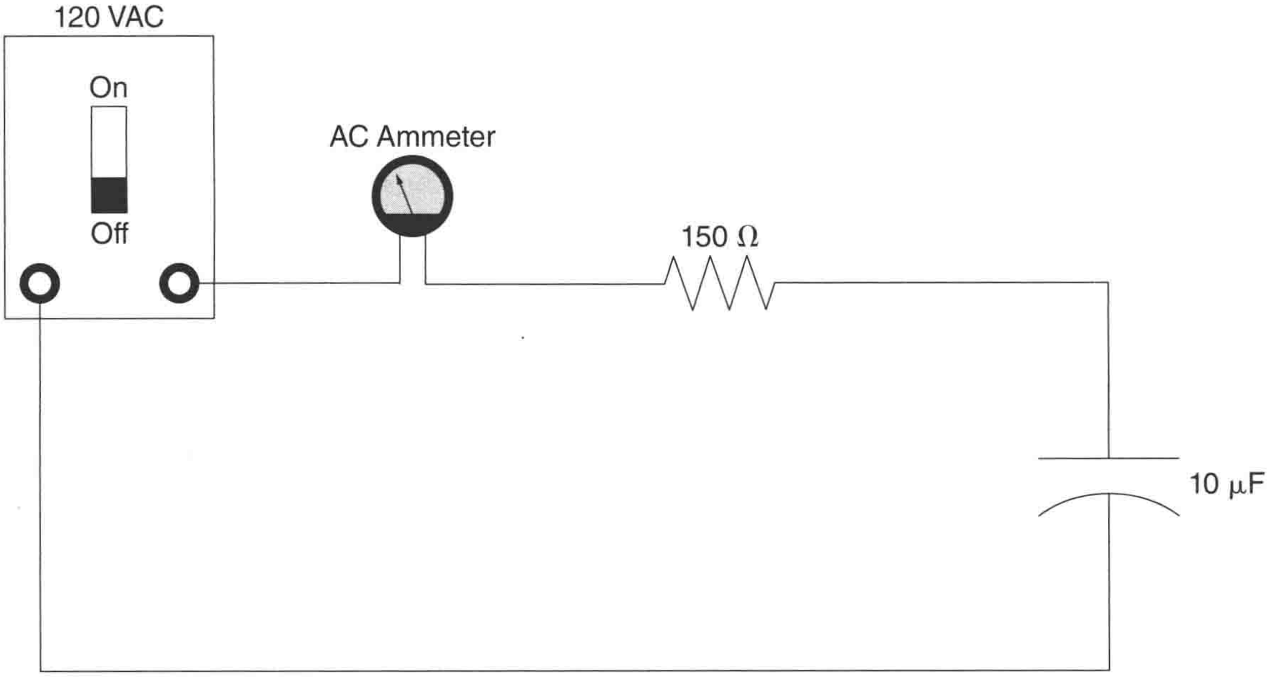


Figure 14-5 Replacing the circuit components.

15. Determine the apparent power in the circuit using the applied voltage and the circuit current.

    $VA = E_T \times I$

    VA = ________

16. Calculate the circuit power factor using the true power and apparent power.

    $$PF = \frac{P}{VA}$$

    PF = ________ %

17. Calculate angle theta.

    $\angle\varnothing = \text{Cos PF}$

    $\angle\varnothing$ = ________ °

18. Connect the circuit shown in Figure 14-5.

19. Calculate the capacitive reactance of the 10 μF capacitor assuming a frequency of 60 Hz.

    X_C = ________ Ω

20. Calculate the impedance of the circuit using the following formula:

    $$Z = \sqrt{R^2 + X_C^2}$$

    Z = ________ Ω

21. Assuming a voltage of 120 volts, calculate the total current flow in the circuit using the following formula:

    $$I_T = \frac{E_T}{Z}$$

    I_T = ________ A

22. Calculate the voltage drop across the 150 ohm resistor using Ohm's law.

    E_R = __________ volts

23. Calculate the voltage drop across the 25 μF capacitor using Ohm's law.

    E_C = __________ volts

24. Turn on the power and measure the current flow through the circuit. **Turn off the power.**

    I_T = __________ A

25. Is the calculated value in step #21 within 5% of the measured value of current?

    ____

26. Turn on the power and measure the voltage drop across the resistor and capacitor. **Turn off the power.**

    E_R = __________ volts

    E_C = __________ volts

27. Compare the measured values of voltage drop across the resistor and capacitor with the calculated values in steps 22 and 23. Are the values within 5% of each other?

    ____

28. In a series circuit the current is the same in any part of the circuit must be the same. Therefore, the voltage drop across the resistor is 90° out of phase with the voltage drop across the capacitor. When vector addition is used, the sum of the two voltage drops should equal the applied voltage. Calculate the total or applied voltage using the following formula:

    $$E_T = \sqrt{E_R^2 + E_C^2}$$

    E_T = __________ volts

29. Is the computed value of voltage within 5% of the voltage that was applied to the circuit?

    ____

30. Determine the true power of the circuit using the circuit current and the voltage drop across the resistor.

    $P = E_R \times I$

    P = __________ watts

31. Determine the capacitive VARs using the circuit current and the voltage drop across the capacitor.

    $VARs_C = E_C \times I$

    $VARs_C$ = __________

32. Determine the apparent power in the circuit using the applied voltage and the circuit current.

    $VA = E_T \times I$

    VA = __________

33. Calculate the circuit power factor using the true power and apparent power.

    $$PF = \frac{P}{VA}$$

    PF = __________ %

34. Calculate angle theta.

    $\angle\varnothing = \text{Cos PF}$

    $\angle\varnothing =$ _________ °

35. Disconnect the circuit and return the components to their proper place.

Review Questions

Refer to the formulas shown in Figure 14-6 to answer some of the following questions:

1. The current and voltage are 64° out of phase in an RC series circuit. The apparent power is 260 VA. What is the true power in the circuit?

    ____

2. The circuit in question 1 has a current flow of 1.25 amps. What is the applied voltage?

    ____

3. A capacitor and resistor are connected in series. The resistor has a resistance of 26 Ω and the capacitor has a capacitive reactance of 16 Ω. What is the impedance of the circuit?

    ____

4. An RC series circuit is connected to a 120 volt, 60 Hz line. The capacitor has a voltage drop of 84 volts. What is the voltage drop across the resistor?

    ____

5. A capacitor is using 1.6 kVARs and has a capacitive reactance of 24 Ω. How much current is flowing through the inductor?

    ____

6. An RC series circuit is connected to a 277 volt, 60 Hz line. The resistor has a power dissipation of 146 watts and the capacitor is using 138 VARs. How much current is flowing in the circuit?

    ____

7. An RC series circuit is connected to a 480 volt, 60 Hz line. The apparent power of the circuit is 8.2 kVA. What is the impedance of the circuit?

    ____

8. An RC series circuit has a power factor of 72%. How many degrees are the voltage and current out of phase with each other?

    ____

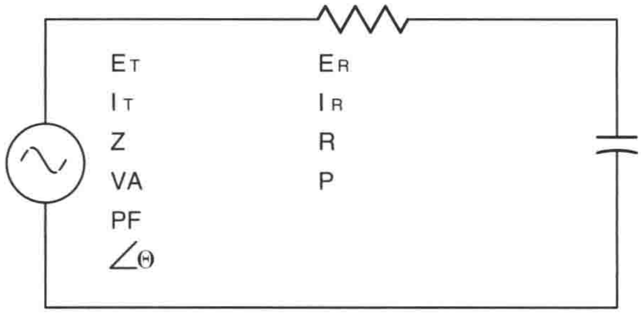
9. An RL series circuit is connected to a 240 volt, 60 Hz line. The capacitor has a current of 1.25 amps flowing though it. The X_C of the inductor is 22 Ω. The resistor has a resistance of 16 Ω. How much voltage is dropped across the resistor?

    ____

10. An RC series circuit has an apparent power of 650 VA and a true power of 475 watts. What is the reactive power in the circuit?

    ____

Resistive-Capacitive Series Circuits



$I_T = I_R = I_C$

$I_T = \frac{E_T}{Z}$

$I_T = \frac{VA}{E_T}$

$Z = \sqrt{R^2 + X_C^2}$ $\quad Z = \frac{VA}{I_T^2}$ $\quad Z = \frac{E_T^2}{VA}$ $\quad C = \frac{1}{2 \pi f X_C}$

$Z = \frac{E_T}{I_T}$ $\quad Z = \frac{R}{PF}$ $\quad P = E_R \times I_R$ $\quad E_R = I_R \times R$

$VA = E_T \times I_T$ $\quad PF = \frac{R}{Z}$ $\quad P = \sqrt{VA^2 - VARs_C^2}$ $\quad E_R = \sqrt{P \times R}$

$VA = I_T^2 \times Z$ $\quad PF = \frac{P}{VA}$ $\quad P = \frac{E_R^2}{R}$ $\quad E_R = \frac{P}{I_R}$

$VA = \frac{E_T^2}{Z}$ $\quad PF = \frac{E_R}{E_T}$ $\quad P = I_R^2 \times R$ $\quad E_R = \sqrt{E_T^2 - E_C^2}$

$VA = \sqrt{P^2 + VARs_C^2}$ $\quad PF = \text{Cos}\angle\Theta$ $\quad P = VA \times PF$ $\quad E_R = E_T \times PF$

$VA = \frac{P}{PF}$ $\quad R = \sqrt{Z^2 - X_C^2}$ $\quad E_C = I_C \times X_C$ $\quad I_C = I_R = I_T$

$I_R = I_T = I_C$ $\quad R = \frac{E_R}{I_R}$ $\quad E_C = \sqrt{E_T^2 - E_R^2}$ $\quad I_C = \frac{E_C}{X_C}$

$I_R = \frac{E_R}{R}$ $\quad R = \frac{E_R^2}{P}$ $\quad E_C = \sqrt{VARs_C \times X_C}$ $\quad I_C = \frac{VARs_C}{E_C}$

$I_R = \frac{P}{E_R}$ $\quad R = \frac{P}{I_R^2}$ $\quad E_C = \frac{VARs_C}{I_C}$ $\quad I_C = \sqrt{\frac{VARs_C}{X_C}}$

$I_R = \sqrt{\frac{P}{R}}$ $\quad R = Z \times PF$ $\quad VARs_C = E_C \times I_C$ $\quad E_T = \sqrt{E_R^2 + E_C^2}$

$X_C = \sqrt{Z^2 - R^2}$ $\quad X_C = \frac{E_C^2}{VARs_C}$ $\quad VARs_C = I_C^2 \times X_C$ $\quad E_T = I_T \times Z$

$X_C = \frac{E_C}{I_C}$ $\quad X_C = \frac{VARs_C}{I_C^2}$ $\quad VARs_C = \frac{E_C^2}{X_C}$ $\quad E_T = \frac{VA}{I_C}$

$VARs_C = \sqrt{VA^2 - P^2}$ $\quad X_C = \frac{1}{2 \pi f C}$ $\quad E_T = \frac{E_R}{PF}$

Figure 14-6 Formulas for RC series circuits.

Unit 15 RC Parallel Circuits

Objectives

After studying this unit, you should be able to

- Discuss the voltage and current relationship in an RC parallel circuit.
- Determine the phase angle of current in an RC parallel circuit.
- Determine the power factor in an RC parallel circuit.
- Discuss the differences between apparent power, true power, and reactive power.
- Find the impedance in an RC parallel circuit.

In any parallel circuit, the voltage must be the same across all branches. Since the current is in phase with the voltage in a pure resistive circuit, and the current leads the voltage by 90° in a pure capacitive circuit, the current flow through the capacitive branch will be out of phase with the current through the resistive branch. The total current will be out of phase with the applied voltage by some amount between 0° and 90° depending on the relative values of resistance and capacitive reactance.

Determining electrical values in an RC parallel circuit is very similar to determining values in a series circuit with a few exceptions. Probably the greatest difference is calculating the value of impedance when the values of R and X_C are known. Recall that vector addition can be used with the ohmic values of an RC series circuit to determine the impedance.

$$Z = \sqrt{R^2 + X_c^{\,2}}$$

The same basic concept is true for an RL parallel circuit, except that the reciprocal value of R and X_C must be used.

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}}$$

Example: A resistor and capacitor are connected in parallel. The resistor has a resistance of 50 Ω and the capacitor has a capacitive reactance of 60 Ω. Find the impedance of the circuit, seen in Figure 15-1.

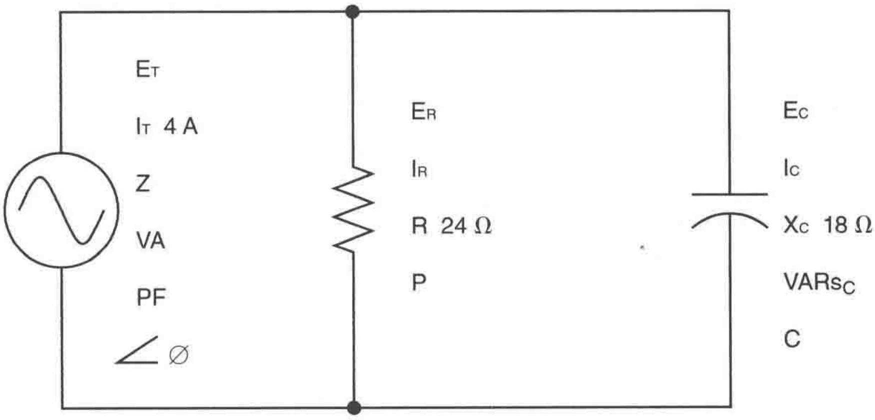


Figure 15-1 RC parallel circuit.

Solution:

$$Z = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{60}\right)^2}}$$

$$Z = \frac{1}{\sqrt{(0.02)^2 + (0.01667)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0002779}}$$

$$Z = \frac{1}{\sqrt{0.0006779}}$$

$$Z = \frac{1}{0.02604}$$

$$Z = 38.4\ \Omega$$

Example Circuit

An RC parallel circuit is connected to a 60 Hz line. The resistor has a resistance of 24 Ω and the capacitor has a capacitive reactance of 18 Ω. The circuit has total current flow of 4 amps. The following values will be computed:

E_T - Total circuit voltage

Z - Circuit impedance

VA - Apparent power

I_R - Current flow through the resistor

P - True power

I_C - Current flow through the inductor

$VARs_C$ - Reactive power

C - Capacitance of the capacitor

PF - Power factor

$\angle\varnothing$ - Angle theta

The first step in determining the missing values for this problem is to determine the circuit impedance using the values of R and X_C.

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{24}\right)^2 + \left(\frac{1}{18}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.001736 + 0.003086}}$$

$$Z = \frac{1}{0.06944}$$

$$Z = 14.4\ \Omega$$

Now that the value of impedance is known, the total voltage can be determined using Ohm's law.

$$E_T = I_T \times Z$$

$$E_T = 4 \times 14.4$$

$$E_T = 57.6 \text{ volts}$$

In a parallel circuit, the voltage is the same across all branches. Therefore, the voltage drops across the resistor and capacitor are 57.6 volts.

The apparent power can be computed using the formula:

$$VA = E_T \times I_T$$

$$VA = 57.6 \times 4$$

$$VA = 230.4$$

The current flowing through the resistor can be computed using Ohm's law.

$$I_R = \frac{E_R}{R}$$

$$I_R = \frac{57.6}{24}$$

$$I_R = 2.4 \text{ amps}$$

The true power in the circuit can be computed using resistive values.

$$P = E_R \times I_R$$

$$P = 57.6 \times 2.4$$

$$P = 138.24 \text{ watts}$$

The current flow through the capacitor can be computed using Ohm's law.

$$I_c = \frac{E_c}{X_c}$$

$$I_c = \frac{57.6}{18}$$

$$I_c = 3.2 \text{ amps}$$

The capacitive VARs can be computed using the inductive values and Ohm's law.

$$\text{VARs}_C = E_c \times I_c$$

$$\text{VARs}_C = 57.6 \times 3.2$$

$$\text{VARs}_C = 184.32$$

The capacitance of the capacitor can be computed using the formula:

$$C = \frac{1}{2\pi f X_c}$$

$$C = \frac{1}{377 \times 18}$$

$$C = 0.00014736 \text{ farad or } 147.36\,\mu\text{F}$$

The power factor can be determined by comparing the true power and apparent power.

$$PF = \frac{P}{VA}$$

$$PF = \frac{138.24}{230.4}$$

$$PF = 0.6 \text{ or } 60\%$$

The cosine of angle theta is equal to the decimal power factor value.

$$\cos \angle\varnothing = PF$$

$$\cos \angle\varnothing = 0.6$$

$$\angle\varnothing = 53.13^\circ$$

The circuit with all the missing values is shown in Figure 15-2.

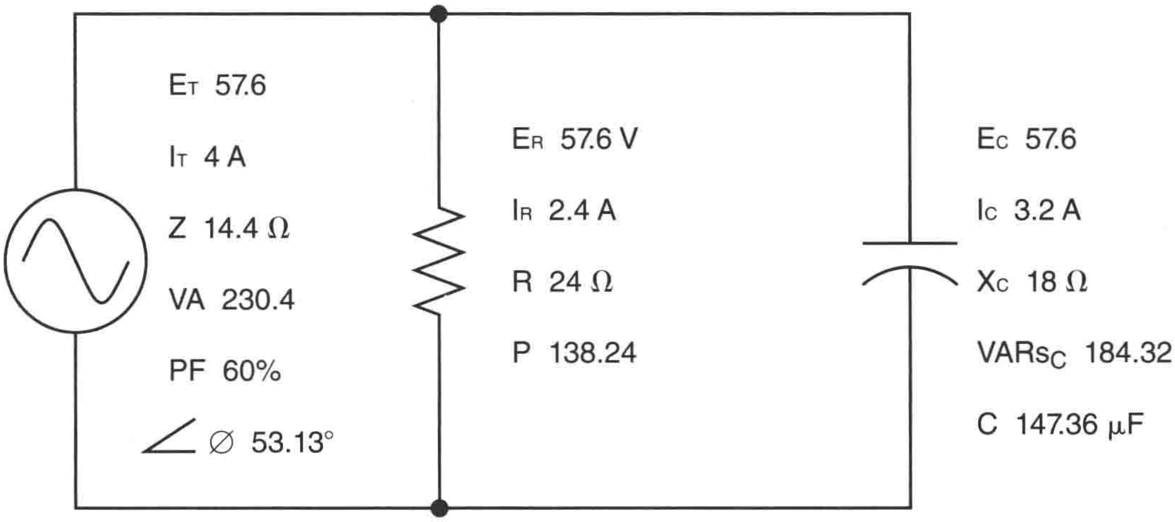


Figure 15-2 RC parallel circuit with all the missing values.

LABORATORY EXERCISE

Name ____ Date __________

Materials Required

1 25 μF AC capacitor rated at 240 VAC or greater

1 7.5 μF AC capacitor rated at 240 VAC or greater

1 100 ohm resistor

1 150 ohm resistor

AC ammeter (In-line or clamp-on may be used. If a clamp-on type is employed, the use of a 10:1 scale divider is recommended.)

Connecting wires

AC voltmeter

1 120 volt AC power supply

Formulas for resistive–capacitive parallel circuits shown in Figure 15-6

1. Connect the circuit shown in Figure 15-3.
2. Calculate the capacitive reactance of the capacitor assuming a frequency of 60 Hz.

   $X_C =$ __________
3. Calculate the impedance of the circuit using the following formula:

   $$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}}$$

   $Z =$ __________ Ω

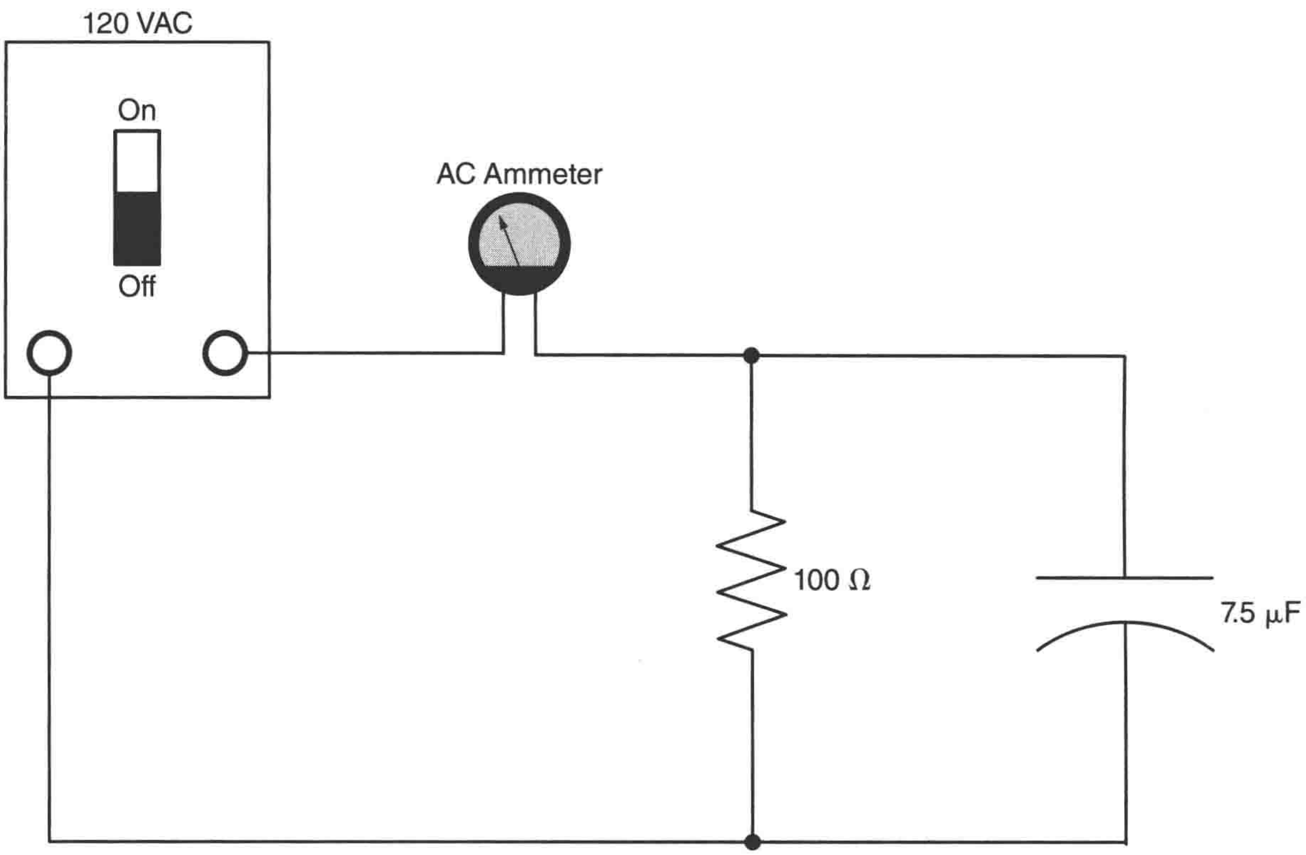


Figure 15-3 The AC ammeter measures the total circuit current.

4. Assume an applied voltage of 120 volts. Calculate the total circuit current using the following formula:

$$I_T = \frac{E}{Z}$$

I_T = __________ A

5. Assume an applied voltage of 120 volts. Calculate the current flow through the 100 ohm resistor using the following formula:

$$I_R = \frac{E}{R}$$

I_R = __________ A

6. Assume an applied voltage of 120 volts. Calculate the current that appears to flow through the capacitor using the following formula:

$$I_C = \frac{E}{X_C}$$

I_C = __________ A

7. Turn on the power supply and measure the total current in the circuit. **Turn off the power.**

I_C = __________ A

8. Compare the measured value with the calculated value in step 4. Are the two values within 5% of each other?
9. Reconnect the circuit as shown in Figure 15-4.

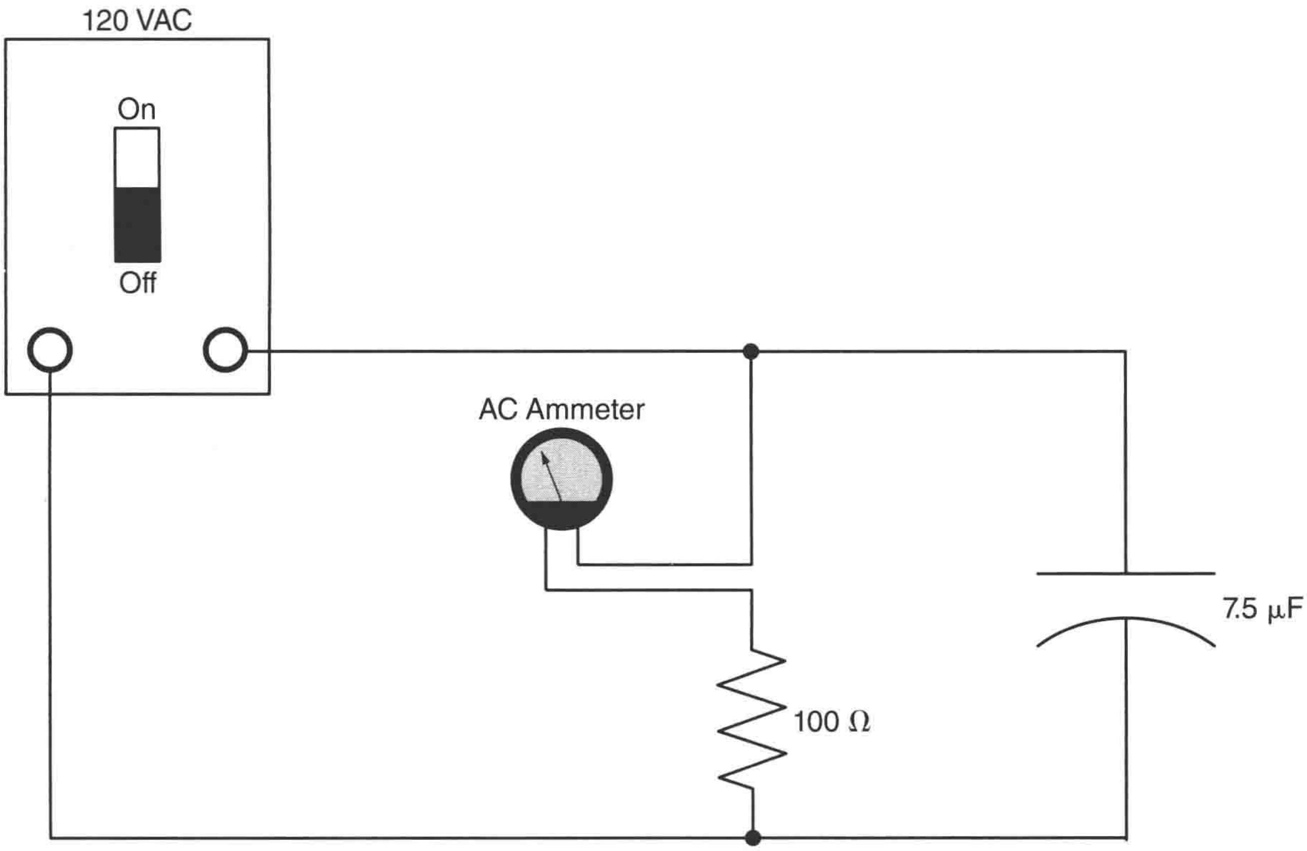


Figure 15-4 Measuring current through the resistive load.

10. Turn on the power and measure the current flow through the resistor. **Turn off the power.**

    I_R = __________ A

11. Compare the measured value with the calculated value in step 5. Are the two values within 5% of each other?

    ____

12. Reconnect the circuit as shown in Figure 15-5.
13. Turn on the power and measure the current that appears to flow through the capacitor. **Turn off the power.**

    I_C = __________ A

14. Compare the measured value with the calculated value in step 6. Are the two values within 5% of each other?

    ____

15. Calculate the true power in the circuit using the values of voltage and current that apply to the resistor.

    P = __________ watts

16. Calculate the reactive power using the values of voltage and current that apply to the capacitor.

    $VARs_C$ = __________

17. Calculate the apparent power in the circuit using the values of voltage and current that apply to the entire circuit.

    VA = __________

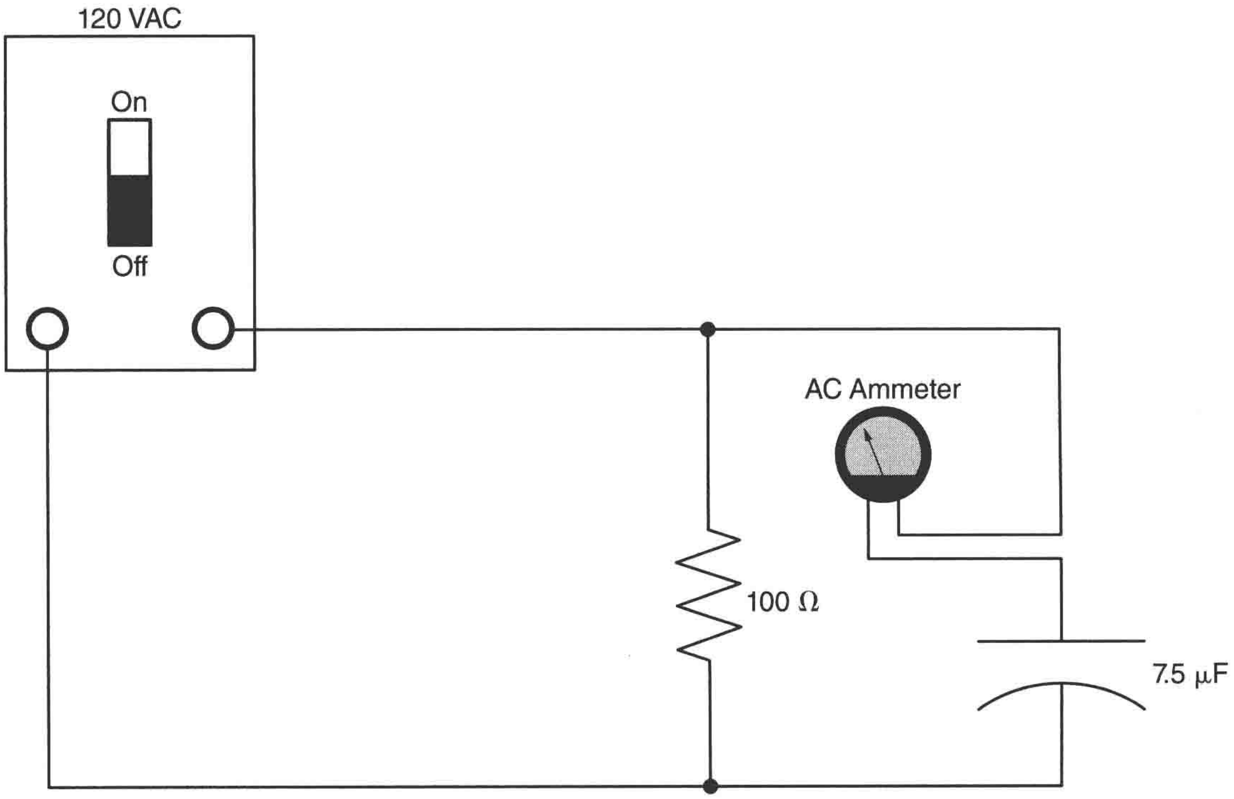


Figure 15-5 Measuring current through the capacitive load.

18. Calculate the circuit power factor using the following formula. Be sure to express the answer as a percentage.

$$PF = \frac{P}{VA}$$

PF = __________ %

19. Determine the degree of out-of-phase condition between the voltage and total current in the circuit by calculating the value of angle theta.

$\text{Cos } \angle\varnothing = PF$

$\angle\varnothing$ = __________ °

20. Replace the 100 ohm resistor in the circuit with a 150 ohm resistor, and replace the 7.5 μF capacitor with a 25 μF capacitor. Reconnect the ammeter to measure the total circuit current as shown in Figure 15-4.

21. Calculate the capacitive reactance of the capacitor assuming a frequency of 60 Hz.

X_C = __________ Ω

22. Calculate the impedance of the circuit using the following formula:

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_c}\right)^2}}$$

Z = __________ Ω

23. Assume an applied voltage of 120 volts. Calculate the total circuit current using the following formula:

$$I_T = \frac{E}{Z}$$

I_T = __________ A

24. Assume an applied voltage of 120 volts. Calculate the current flow through the 150 ohm resistor using the following formula:

$$I_R = \frac{E}{R}$$

I_R = __________ A

25. Assume an applied voltage of 120 volts. Calculate the current that appears to flow through the capacitor using the following formula:

$$I_C = \frac{E}{X_C}$$

I_C = __________ A

26. Turn on the power supply and measure the total current in the circuit. **Turn off the power.**

I_T = __________ A

27. Compare the measured value with the calculated value in step 4. Are the two values within 5% of each other?

28. Reconnect the circuit to measure the current through the resistor as shown in Figure 15-4.

29. Turn on the power and measure the current flow through the resistor. **Turn off the power.** I_R = __________ A

30. Compare the measured value with the calculated value in step #5. Are the two values within 5% of each other?

____

31. Reconnect the circuit to measure the current that appears to flow through the capacitor as shown in Figure 15-5.

32. Turn on the power and measure the current that appears to flow through the capacitor. **Turn off the power.**

I_C = __________ A

33. Compare the measured value with the calculated value in step 6. Are the two values within 5% of each other?

____

34. Calculate the true power in the circuit using the values of voltage and current that apply to the resistor.

P = __________ watts

35. Calculate the reactive power using the values of voltage and current that apply to the capacitor.

$VARs_C$ = __________

36. Calculate the apparent power in the circuit using the values of voltage and current that apply to the entire circuit.

VA = __________

37. Calculate the circuit power factor using this formula: Be sure to express the answer as a percent.

$$PF = \frac{P}{VA}$$

PF = __________ %

38. Determine the degree of out of phase condition between the voltage and total current in the circuit by calculating the value of angle theta.

    Cos $\angle\varnothing$ = PF

    $\angle\varnothing$ = __________ °

39. Disconnect the circuit and return the components to their proper place.

Review Questions

To answer the following questions, it may be necessary to refer to the formulas shown in Figure 15-6.

1. A 50 μF capacitor is connected to a 400 Hz line. What is the capacitive reactance of the capacitor?

   ____

2. A resistor and capacitor are connected in parallel to a 120 volt, 60 Hz line. The circuit has a current flow of 3 amperes. The resistor has a resistance of 68 Ω. What is the capacitance of the capacitor?

   ____

3. A resistor with a resistance of 50 Ω is connected in parallel with a capacitor with a capacitance of 35 μF. The power source is 60 Hz. What is the impedance of the circuit?

   ____

4. A resistor and capacitor are connected in parallel to a 277 volt, 60 Hz line. The resistor has a current of 16 amperes flowing through it, and the capacitor has a current flow of 28 amperes flowing through it. What is the total current flow in the circuit?

   ____

5. A resistor and capacitor are connected in parallel to a 1,000 Hz line. The capacitor has a voltage drop of 200 volts across it. What is the voltage drop across the resistor?

   ____

6. A capacitor has a current flow of 1.6 amperes when connected to a 240 volt, 50 Hz line. How much current will flow through the capacitor if it is connected to a 240 volt, 60 Hz line?

   ____

7. An RC parallel circuit has an apparent power of 21 kVA. The true power is 18.6 kW. What is the circuit power factor?

   ____

8. How many degrees out of phase are the voltage and current in question 7?

   ____

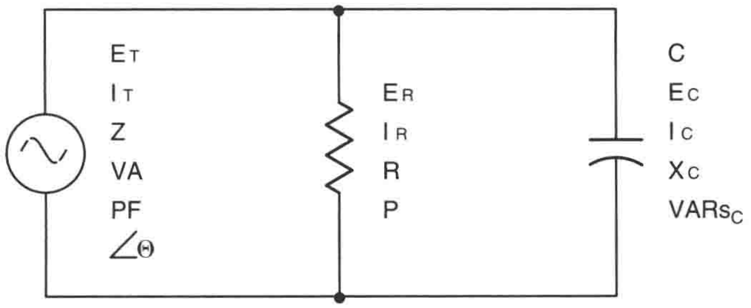
9. An RC parallel circuit has a power factor of 84%. The circuit voltage is 480 volts and the total current is 28 amperes. What is the true power in the circuit?

   ____

10. The voltage and current are 24° out of phase with each other in an RC parallel circuit. What is the circuit power factor?

    ____

Resistive-Capacitive Parallel Circuits



$$I_T = \sqrt{I_R^2 + I_C^2}$$
$$I_T = \frac{E_T}{Z}$$
$$I_T = \frac{VA}{E_T}$$
$$I_T = \frac{I_R}{PF}$$
$$I_T = \sqrt{\frac{VA}{Z}}$$
$$E_R = I_R \times R$$
$$E_R = \sqrt{P \times R}$$
$$E_R = \frac{P}{I_R}$$
$$E_R = E_T = E_C$$
$$E_R = E_T \times PF$$
$$I_C = \sqrt{I_T^2 - I_R^2}$$
$$I_C = \frac{E_C}{X_C}$$
$$I_C = \frac{VARs_C}{E_C}$$
$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C}\right)^2}}$$

$$VA = E_T \times I_T$$
$$VA = I_T^2 \times Z$$
$$VA = \frac{E_T^2}{Z}$$
$$VA = \sqrt{P^2 + VARs_C^2}$$
$$VA = \frac{P}{PF}$$
$$I_R = \sqrt{I_T^2 - I_C^2}$$
$$I_R = \frac{E_R}{R}$$
$$I_R = \frac{P}{E_R}$$
$$I_R = \sqrt{\frac{P}{R}}$$
$$X_C = \frac{E_C}{I_C}$$
$$X_C = \frac{1}{\sqrt{\left(\frac{1}{Z}\right)^2 - \left(\frac{1}{R}\right)^2}}$$
$$X_C = \frac{1}{2 \pi f C}$$
$$I_C = \sqrt{\frac{VARs_C}{X_C}}$$
$$Z = \frac{VA}{I_T^2}$$

$$PF = \frac{Z}{R}$$
$$PF = \frac{P}{VA}$$
$$PF = \frac{E_R}{E_T}$$
$$PF = \cos\angle\Theta$$
$$R = \frac{E_R}{I_R}$$
$$R = \frac{1}{\sqrt{\left(\frac{1}{Z}\right)^2 - \left(\frac{1}{X_C}\right)^2}}$$
$$R = \frac{E_R^2}{P}$$
$$R = \frac{P}{I_R^2}$$
$$R = \frac{Z}{PF}$$
$$X_C = \frac{E_C^2}{VARs_C}$$
$$X_C = \frac{VARs_C}{I_C^2}$$
$$C = \frac{1}{2 \pi f X_C}$$
$$Z = \frac{E_T^2}{VA}$$
$$Z = \frac{E_T}{I_T}$$

$$E_T = E_R = E_C$$
$$E_T = I_T \times Z$$
$$E_T = \frac{VA}{I_T}$$
$$E_T = \sqrt{VA \times Z}$$
$$P = E_R \times I_R$$
$$P = \sqrt{VA^2 - VARs_C^2}$$
$$P = \frac{E_R^2}{R}$$
$$P = I_R^2 \times R$$
$$P = VA \times PF$$
$$E_C = I_C \times X_C$$
$$E_C = E_T = E_R$$
$$E_C = \sqrt{VARs_C \times X_C}$$
$$E_C = \frac{VARs_C}{I_C}$$
$$VARs_C = I_C^2 \times X_C$$
$$VARs_C = \frac{E_C^2}{X_C}$$
$$VARs_C = E_C \times I_C$$
$$VARs_C = \sqrt{VA^2 - P^2}$$
$$Z = R \times PF$$

Figure 15-6 RC parallel circuit formulas.

Unit 16 Resistive-Inductive-Capacitive Series Circuits

Objectives

After studying this unit, you should be able to

- Determine values of watts and VARs for circuit components.
- Determine circuit power factor.
- Measure values of current and voltage in an RLC series circuit.
- Compute the phase angle difference between current and voltage in an RLC series circuit.
- Connect an RLC series circuit.

In a pure resistive circuit, the current and voltage are in phase with each other. In a pure inductive circuit, the voltage leads the current by 90°, and in a pure capacitive circuit, the voltage lags the current by 90°. Since the current is the same at any point in a series circuit, the voltage drops across the different components are out of phase with each other, as shown in Figure 16-1. Since the voltage across the capacitor lags the current by 90° and the voltage across the inductor leads the current by 90°, they are 180° out of phase with each other. Since these two voltages are 180° out of phase with each other, one tends to cancel the effects of the other. The result is that the larger voltage is reduced and the smaller is eliminated as far as the circuit is concerned. Assume that an RLC series circuit contains a resistor with a voltage drop of 100 volts, an inductor with a voltage drop of 150 volts, and a capacitor with a voltage drop of 120 volts. The capacitive voltage reduces the inductive voltage to the point that the circuit is essentially an RL series circuit that has a resistor with a voltage drop of 100 volts and an inductor with a voltage drop of 30 volts (Figure 16-2).

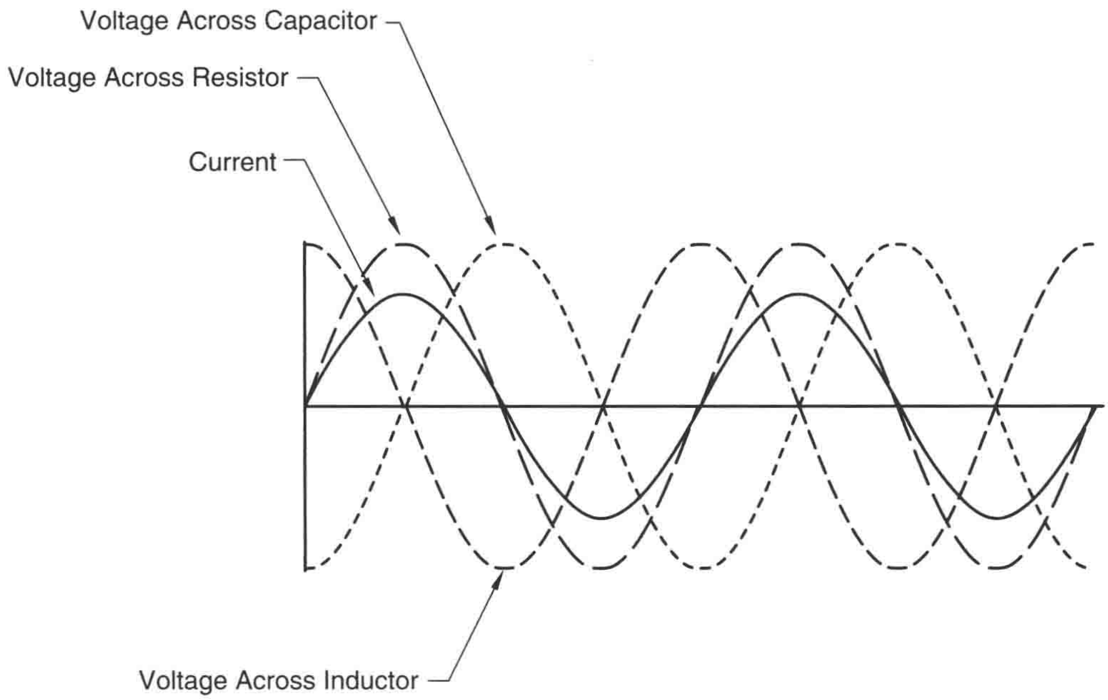


Figure 16-1 Current and voltage relationships in an RLC series circuit.

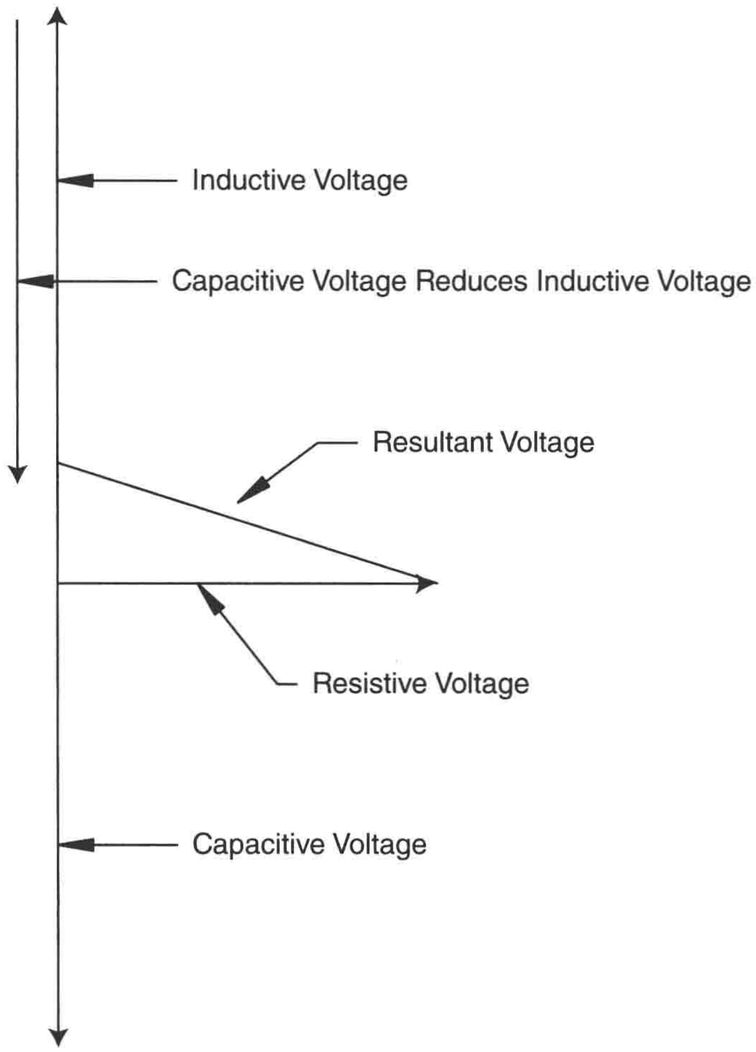


Figure 16-2 Inductive and capacitive voltage drops oppose each other.

Since the inductive and capacitive voltages oppose each other, it is possible for these components to have a greater voltage drop than the voltage applied to the circuit. The circuit shown in Figure 16-3 contains a resistor, inductor, and capacitor connected in series. The resistor has a resistance of 24 Ω, the inductor has an inductive reactance of 40 Ω, and the capacitor has a capacitive reactance of 35 Ω. The circuit has a total voltage of 208 volts. The following values will be computed:

Z - Total circuit impedance

I - Circuit current

E_R - Voltage drop across the resistor

P - True power or watts

E_L - Voltage drop across the inductor

$VARs_L$ - Reactive power component of the inductor

E_C - Voltage drop across the capacitor

$VARs_C$ - Reactive power component of the capacitor

VA - Apparent power

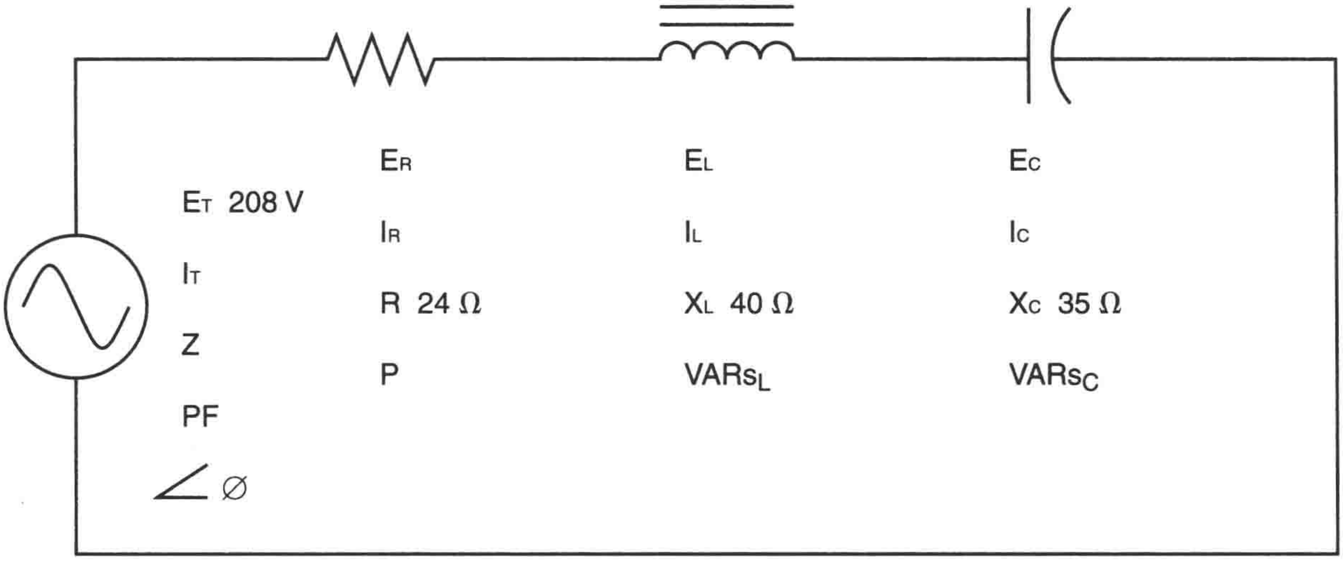


Figure 16-3 RLC series circuit.

PF - Power factor

∠Ø - Angle theta

The first step is to determine the total circuit impedance using the formula:

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

$$Z = \sqrt{24^2 + (40 - 35)^2}$$

$$Z = \sqrt{601}$$

$$Z = 24.51\ \Omega$$

Now that the impedance is known, the total circuit current can be computed using Ohm's law.

$$I_T = \frac{E_T}{Z}$$

$$I_T = \frac{208}{24.51}$$

$$I_T = 8.49 \text{ amps}$$

In a series circuit, current is the same through all parts. Therefore, I_R, I_L, and I_C have a value of 8.49 amps.

The voltage drop across the resistor can be computed using Ohm's law.

$$E_R = I_R \times R$$

$$E_R = 8.49 \times 24$$

$$E_R = 203.76 \text{ volts}$$

True power can be determined using the formula:

$$P = E_R \times I_R$$

$$P = 203.76 \times 8.49$$

$$P = 1{,}729.92 \text{ watts}$$

The voltage drop across the inductor can be computed using Ohm's law and inductive values.

$$E_L = I_L \times X_L$$

$$E_L = 8.49 \times 40$$

$$E_L = 339.6 \text{ volts}$$

Note that the voltage across the inductor is greater than the applied voltage to the circuit.

The reactive power for the inductor can be computed using the formula:

$$VARs_L = E_L \times I_L$$

$$VARs_L = 339.6 \times 8.49$$

$$VARs_L = 2{,}883.2$$

The voltage drop across the capacitor can be determined using Ohm's law and capacitive values.

$$E_c = I_c \times X_c$$

$$E_c = 8.49 \times 35$$

$$E_c = 297.15 \text{ volts}$$

The reactive power for the capacitor can be computed using the formula:

$$VARs_C = E_c \times I_c$$

$$VARs_C = 297.15 \times 8.49$$

$$VARs_C = 2{,}522.8$$

The apparent power can be computed using the total values of voltage and current.

$$VA = E_T \times I_T$$

$$VA = 208 \times 8.49$$

$$VA = 4{,}288.72$$

The power factor can be determined using the formula:

$$PF = \frac{P}{VA}$$

$$PF = \frac{1{,}729.92}{4{,}288.72}$$

$$PF = 0.4034 \text{ or } 40.34\%$$

The power factor is the cosine of angle theta.

$$\angle\varnothing = \cos 0.0434$$

$$\angle\varnothing = 66.21^\circ$$

The circuit with all the missing values is shown in Figure 16-4. Formulas for RLC series circuits are shown in Figure 16-6.

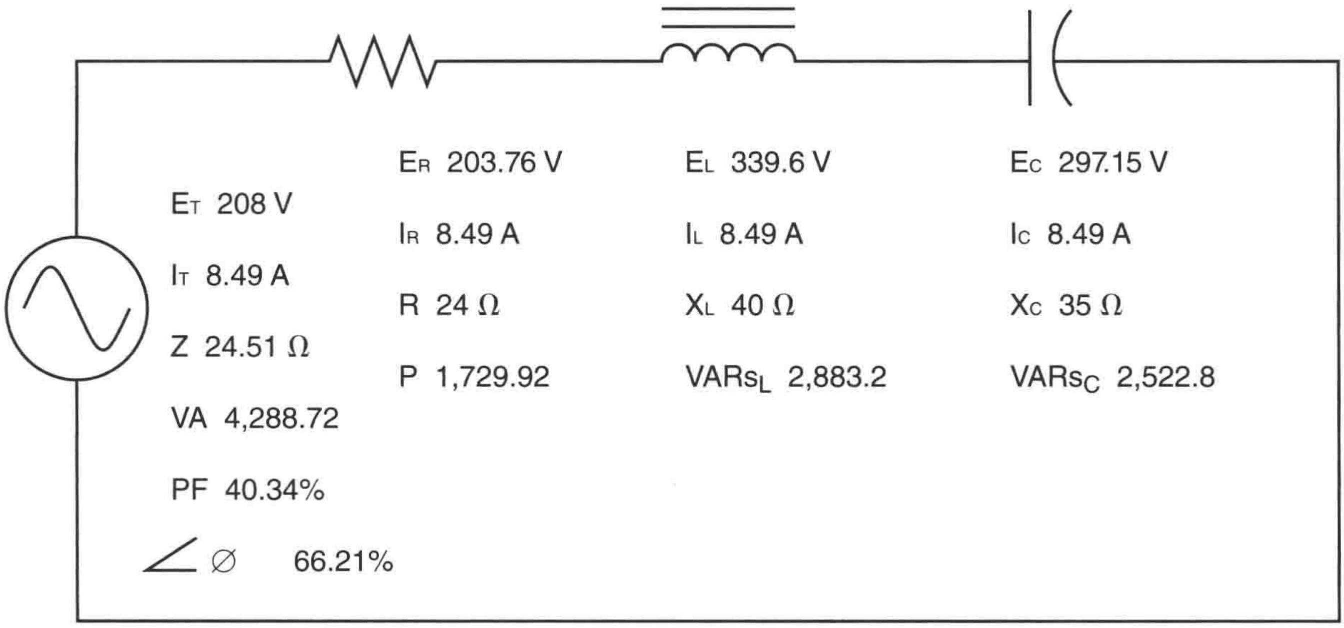


Figure 16-4 RLC series circuit with all the missing values.

LABORATORY EXERCISE

Name ______________________________ Date __________

Materials Required

Formulas for RLC series circuits as shown in Figure 16-6

1 120 volt power supply

1 100 ohm resistor

1 150 ohm resistor

1 0.5 kVA control transformer with two windings rated at 240 volts each and one winding rated at 120 volts

1 25 μF AC capacitor with a voltage rating not less than 240 volts

1 10 μF AC capacitor with a voltage rating not less than 240 volts

1 AC ammeter (An in-line or clamp-on type meter may be used. If a clamp-on type is employed the use of a 10:1 scale divider is recommended.)

1 AC voltmeter

1. Connect the circuit shown in Figure 16-5.
2. Turn on the power supply and measure the current flow in the circuit.

   I_T ______________ amps
3. Measure the voltage drop across the 100 ohm resistor, terminals X_1 and X_2 of the transformer, and the 25 μF capacitor with an AC voltmeter. **Turn off the power.**

   $E_{R\ (Resistor)}$ __________ $E_{L\ (Transformer)}$ __________ $E_{C\ (Capacitor)}$ __________
4. Does any component exhibit a greater voltage drop than the amount of voltage being applied to the circuit? If yes, which component?

   Yes/no __________ Component ______________
5. Using the measured value of voltage and current, compute the apparent power, true power, $VARs_L$, and $VARs_C$.

   VA ______________ P ______________

   $VARs_L$ ______________ $VARs_C$ ______________
6. Compute the circuit power factor.

   PF = ______________ %
7. How many degrees are the voltage and current out of phase with each other?

   ∠∅ = ______________ °

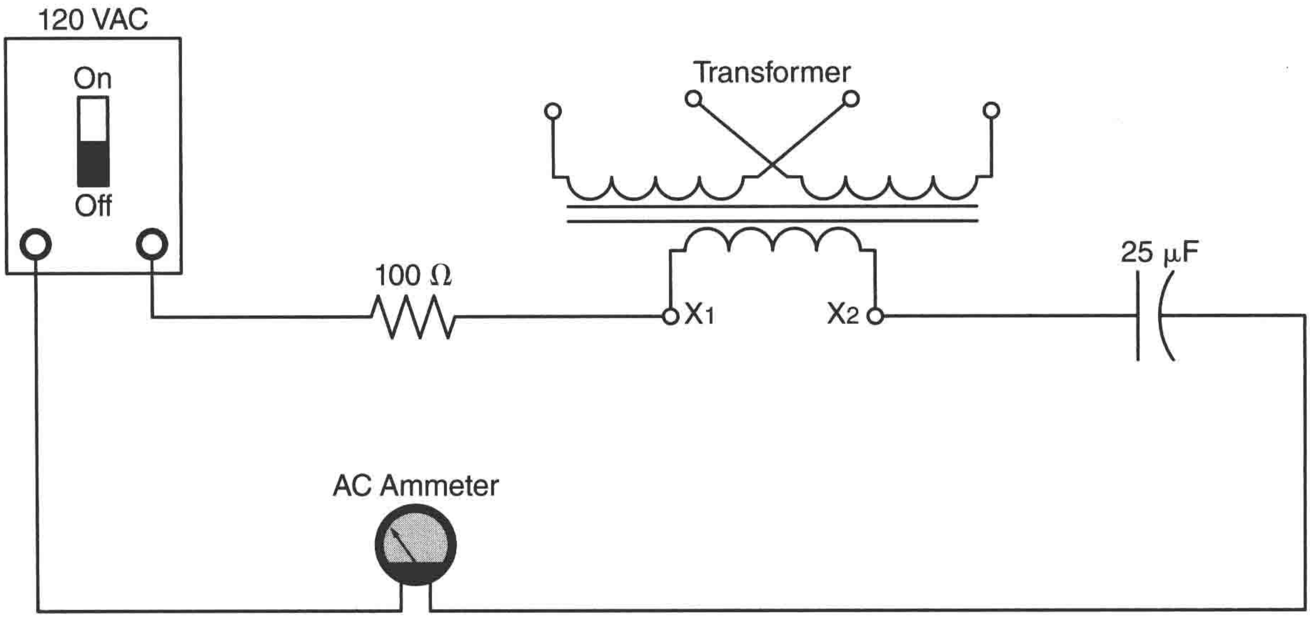


Figure 16-5 Connecting an RLC series circuit.

8. Replace the 25 μF AC capacitor with a 10 μF AC capacitor, and replace the 100 ohm resistor with a 150 ohm resistor.
9. Turn on the power supply and measure the current flow in the circuit.

   I_T ______________ amps
10. Measure the voltage drop across the 150 ohm resistor, terminals X_1 and X_2 of the transformer, and the 10 μF capacitor with an AC voltmeter. **Turn off the power.**

    $E_{R\ (Lamp)}$ __________ $E_{L\ (Transformer)}$ __________ $E_{C\ (Capacitor)}$ __________
11. Does any $\text{component}_{(s)}$ exhibit a greater voltage drop than the amount of voltage being applied to the circuit? If yes which component?

    Yes/no __________ $\text{Component}_{(s)}$ ______________
12. Using the measured value of voltage and current, compute the apparent power, true power, $VARs_L$, and $VARs_C$.

    VA ______________ P ______________

    $VARs_L$ ______________ $VARs_C$ ______________
13. Compute the circuit power factor.

    PF = ______________ %
14. How many degrees are the voltage and current out of phase with each other?

    $\angle\varnothing$ = ______________ °
15. Disconnect the circuit and return the components to their proper place.

Review Questions

Refer to the formula sheet in Figure 16-6 to answer the following questions:

1. A 120 volt, 60 Hz circuit contains a resistor, inductor, and capacitor connected in series. The total impedance of the circuit is 16.5 Ω. How much current is flowing through the inductor?

   ____
2. A circuit has a 48 Ω resistor connected in series with an inductor with an inductive reactance of 96 Ω and a capacitor with a capacitive reactance of 76 Ω. What is the total impedance of the circuit?

   ____
3. An RLC series circuit has a resistor with a power consumption of 124 watts. The inductor has a reactive power of 366 $VARs_L$ and the capacitor has a reactive power of 288 $VARs_C$. What is the circuit power factor?

   ____
4. An RLC series circuit has an apparent power of 1,564 VA and a power factor of 77%. What is the true power in the circuit?

   ____
5. In an RLC series circuit, the resistor has a voltage drop of 121 volts, the inductor has a voltage drop of 216 volts, and the capacitor has a voltage drop of 189 volts. What is the voltage applied to the circuit?

   ____

6. What is the power factor of the circuit in question 5?

____

7. An RLC series circuit has an applied voltage of 277 volts. The inductor has a voltage drop of 355 volts and the capacitor has a voltage drop of 124 volts. What is the voltage drop across the resistor?

____

8. The resistor of an RLC series circuit has a resistance of 16 Ω and a voltage drop of 56 volts. The capacitor has a capacitive reactance of 24 Ω. What is the reactive power of the capacitor?

____

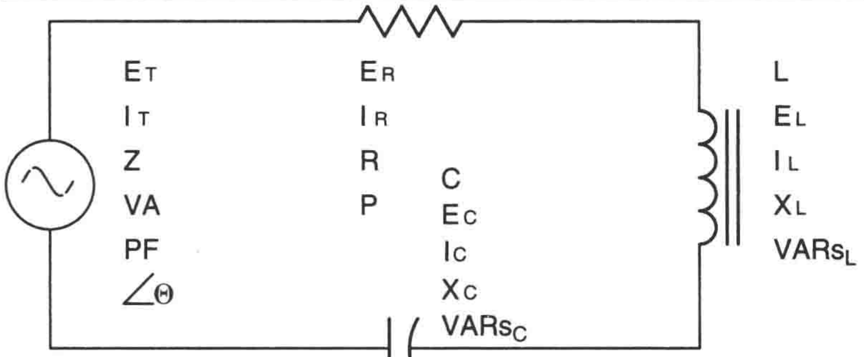
9. An RLC series circuit has a power factor of 84%. The resistor has a voltage drop of 96 volts. What is the total voltage applied to the circuit?

____

10. The resistor of an RLC series circuit has a resistance of 48 Ω. The inductor has an inductive reactance of 88 Ω and the capacitor has a capacitive reactance of 62 Ω. The apparent power of the circuit is 780 VA. What is the total current in the circuit?

____

Resistive-Inductive-Capacitive Series Circuits



$E_T = \sqrt{E_R^2 + (E_L - E_C)^2}$	$P = \sqrt{VA^2 - (VARs_L - VARs_C)^2}$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$Z = \dfrac{R}{PF}$	
$E_T = I_T \times Z$	$E_T = \dfrac{E_R}{PF}$	$P = I_R^2 \times R$	$Z = \dfrac{E_T}{I_T}$	$I_T = I_R = I_L = I_C$
$PF = \dfrac{R}{Z}$	$E_T = \dfrac{VA}{I_T}$	$P = VA \times PF$	$Z = \dfrac{VA}{I_T^2}$	$I_T = \dfrac{E_T}{Z}$
$PF = \dfrac{P}{VA}$	$P = E_R \times I_R$	$P = \dfrac{E_R^2}{R}$	$E_R = \dfrac{P}{I_R}$	$I_T = \dfrac{VA}{E_T}$
$PF = \dfrac{E_R}{E_T}$	$R = \sqrt{Z^2 - (X_L - X_C)^2}$	$E_R = I_R \times R$	$E_R = E_T \times PF$	$I_T = \sqrt{\dfrac{VA}{Z}}$
$PF = \cos\angle\Theta$	$R = \dfrac{E_R}{I_R}$	$E_R = \sqrt{P \times R}$	$E_R = \sqrt{E_T^2 - (E_L - E_C)^2}$	
$I_R = I_T = I_C = I_L$	$R = Z \times PF$	$X_L = \dfrac{E_L^2}{VARs_L}$	$E_C = I_C \times X_C$	
$I_R = \dfrac{E_R}{R}$	$R = \dfrac{E_R^2}{P}$	$X_L = \dfrac{VARs_L}{I_L^2}$	$E_C = \sqrt{VARs_C \times X_C}$	
$I_R = \dfrac{P}{E_R}$	$R = \dfrac{P}{I_R^2}$	$X_L = \dfrac{E_L}{I_L}$	$E_C = \dfrac{VARs_C}{I_C}$	
$I_R = \sqrt{\dfrac{P}{R}}$	$X_C = \dfrac{1}{2 \pi f C}$	$X_L = 2 \pi f L$	$VA = \sqrt{P^2 + (VARs_L - VARs_C)^2}$	
$I_C = I_R = I_T = I_L$	$X_C = \dfrac{E_C^2}{VARs_C}$	$L = \dfrac{X_L}{2 \pi f}$	$VA = \dfrac{P}{PF}$	
$I_C = \dfrac{E_C}{X_C}$	$X_C = \dfrac{E_C}{I_C}$	$C = \dfrac{1}{2 \pi f X_C}$	$VA = I_T^2 \times Z$	
$I_C = \dfrac{VARs_C}{E_C}$	$X_C = \dfrac{VARs_C}{I_C^2}$	$VARs_C = I_C^2 \times X_C$	$VA = \dfrac{E_T^2}{Z}$	
$I_C = \sqrt{\dfrac{VARs_C}{X_C}}$	$VARs_C = \dfrac{E_C^2}{X_C}$	$VARs_C = E_C \times I_C$	$VA = E_T \times I_T$	

Figure 16-6 Formulas for RLC series circuits.

Unit 17 Resistive-Inductive-Capacitive Parallel Circuits

Objectives

After studying this unit, you should be able to

- Calculate the impedance in an RLC parallel circuit.
- Determine the phase angle difference between voltage and current.
- Determine the circuit power factor.
- Connect an RLC parallel circuit.

In a parallel circuit, the voltage is the same across all branches. In a resistive circuit, the voltage and current are in phase with each other. In an inductive circuit, the current lags the voltage, and in a capacitive circuit, the current leads the voltage. Since the voltage is the same across all components, the currents through the resistive, inductive, and capacitive branches are out of phase with each other.

Determining the Impedance of an RLC Parallel Circuit

The impedance of a parallel RLC circuit can be determined in a manner similar to determining the impedance of an RLC series circuit. The difference lies in the fact that the reciprocal values of resistance, inductive reactance, and capacitive reactance must be used.

Example: An RLC parallel circuit has a resistor valued at 40 Ω, an inductor with an inductive reactance of 60 Ω, and a capacitor with a capacitive reactance of 30 Ω. What is the total circuit impedance?

The impedance can be determined using the formula:

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_c}\right)^2}}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{40}\right)^2 + \left(\frac{1}{60} - \frac{1}{30}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.000625 + 0.0002778}}$$

$$Z = \frac{1}{\sqrt{0.0009028}}$$

$$Z = 33.282\ \Omega$$

An RLC parallel circuit contains a 75 Ω resistor, a 0.32 henry inductor, and a 15 μF capacitor (Figure 17-1). The circuit has an applied voltage of 240 volts and the frequency is 60 Hz.

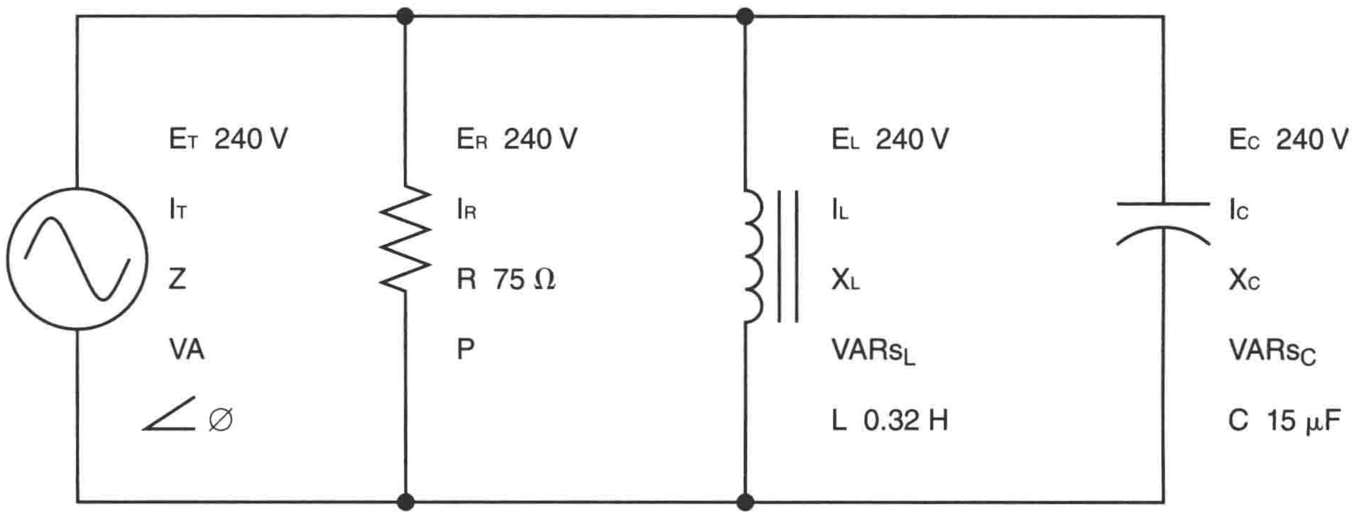


Figure 17-1 IRLC parallel circuit.

The following values will be determined:

X_L - Inductive reactance

X_C - Capacitive reactance

Z - Circuit impedance

I_T - Total circuit current

VA - Apparent power

I_R - Current flow through the resistor

P - True power

I_L - Current flow through the inductor

$VARs_L$ - Reactive power of the inductor

I_C - Current flow through the capacitive branch

$VARs_C$ - Reactive power of the capacitor

PF - Circuit power factor

$\angle\varnothing$ - Angle theta

The first step is to determine the values of inductive reactance and capacitive reactance. Recall that in a circuit with a frequency of 60 Hz, $2\pi f$ has a value of 377.

$$X_L = 2\pi fL$$

$$X_L = 377 \times 0.32$$

$$X_L = 120.64\ \Omega$$

$$X_c = \frac{1}{2\pi fC}$$

$$X_c = \frac{1}{377 \times 15EE-6}$$

$$X_c = 176.83\ \Omega$$

Now that the values of inductive reactance and capacitive reactance are known, the total circuit impedance can be determined.

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_c}\right)^2}}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{75}\right)^2 + \left(\frac{1}{120.64} - \frac{1}{176.83}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.000453 + 0.00000694}}$$

$$Z = \frac{1}{\sqrt{0.00046}}$$

$$Z = 46.625\ \Omega$$

The total circuit current can now be computed using Ohm's law.

$$I_T = \frac{E_T}{Z}$$

$$I_T = \frac{240}{46.625}$$

$$I_T = 5.147 \text{ amps}$$

The apparent power can be determined using the total values for voltage and current.

$$VA = E_T \times I_T$$

$$VA = 240 \times 5.147$$

$$VA = 1{,}235.28$$

Since all the branches of a parallel circuit have the same voltage applied across them, the current flow through the resistor can be determined using Ohm's law.

$$I_R = \frac{E_R}{R}$$

$$I_R = \frac{240}{75}$$

$$I_R = 3.2 \text{ amps}$$

True power can be computed using the formula:

$$P = E_R \times I_R$$

$$I = \frac{E}{R}$$

$$P = 240 \times 3.2$$

$$P = 768 \text{ watts}$$

The current through the inductive branch of the circuit can be determined using the following formula:

$$I_L = \frac{E_L}{X_L}$$

$$I_L = \frac{240}{120.64}$$

$$I_L = 1.989 \text{ amps}$$

Reactive power for the inductive branch can be computed using the inductive values of voltage and current.

$$VARs_L = 240 \times 1.989$$

$$VARs_L = 477.36$$

The current through the capacitive branch can be calculated using the following formula:

$$I_c = \frac{E_c}{X_c}$$

$$I_c = \frac{240}{176.83}$$

$$I_c = 1.357 \text{ amps}$$

The reactive power for the capacitive branch can be determined using capacitive values of voltage and current.

$$VARs_C = 240 \times 1.357$$

$$VARs_C = 325.68$$

The power factor can be computed using the values for apparent power and true power.

$$PF = \frac{P}{VA}$$

$$PF = \frac{768}{1{,}235.28}$$

$$PF = 0.6217 \text{ or } 62.17\%$$

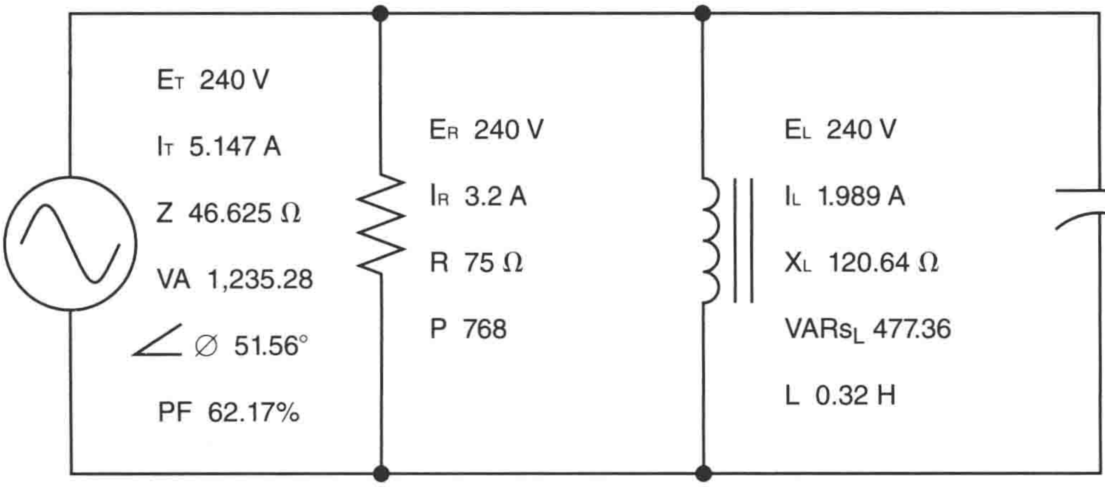


Figure 17-2 RLC parallel circuit with all the missing values.

The power factor is the cosine of angle theta.

$$\angle\varnothing = \text{inv}\cos 0.6217$$

$$\angle\varnothing = 51.56°$$

The circuit with all missing values is shown in Figure 17-2. Formulas for RLC parallel circuits are shown in Figure 17-7.

LABORATORY EXERCISE

Name ______________________________ Date __________

Materials Required

Formulas for RLC parallel circuits as shown in Figure 17-7

1 120-volt power supply

1 0.5-kVA control transformer with 2 windings rated at 240 volts each and one winding rated at 120 volts.

1 100-ohm resistor

1 150-ohm resistor

1 25-μF AC capacitor rated not less than 240 volts

1 10-μF AC capacitor rated not less than 240 volts

1 AC voltmeter

1 AC ammeter (in-line or clamp-on type may be used. If a clamp-on type meter is used, a 10:1 scale divider is recommended.)

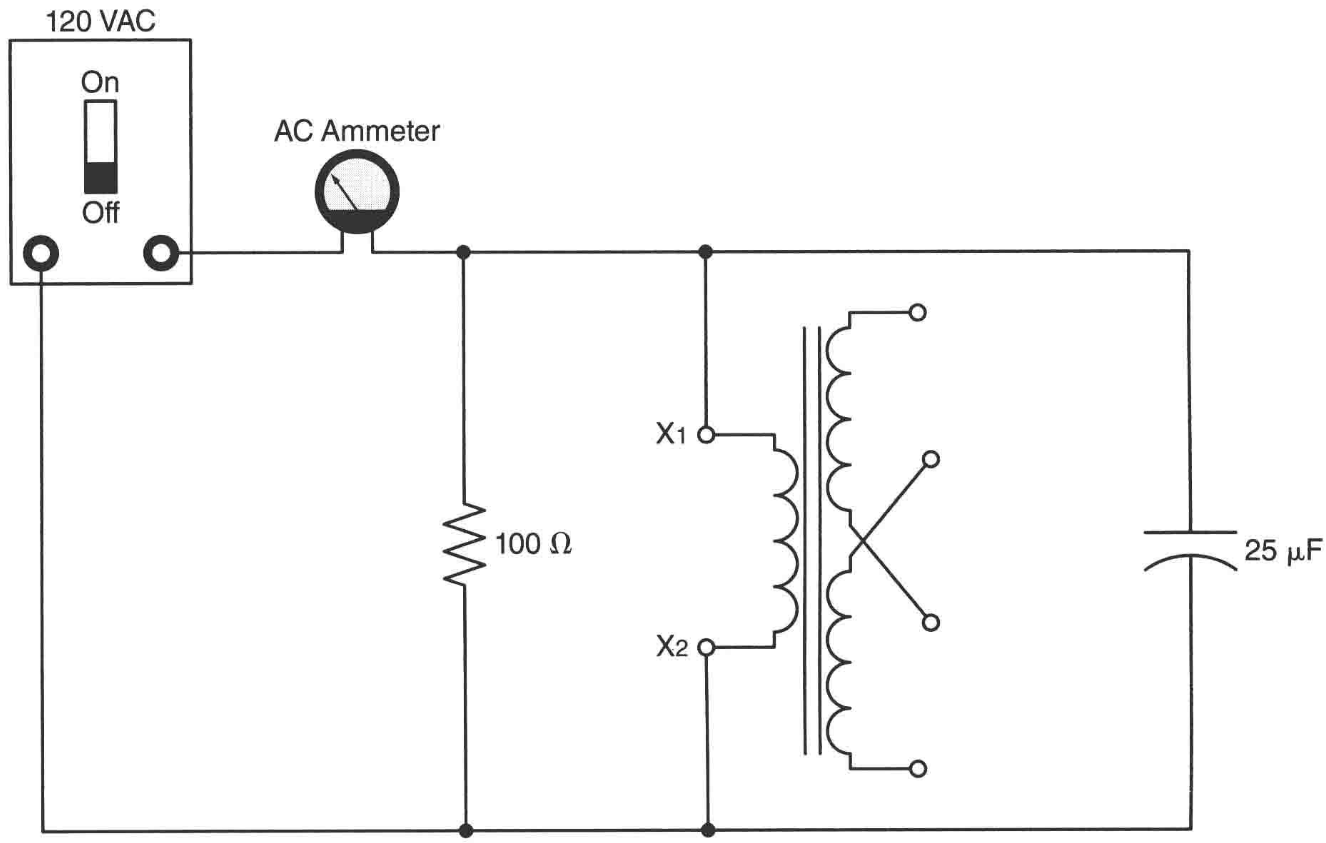


Figure 17-3 RLC parallel circuit.

1. Connect the circuit shown in Figure 17-3.
2. Turn on the power supply and measure the voltage with an AC voltmeter.

   E_T _______________ volts
3. Measure the total current flow in the circuit.

   I_T _______________ amps
4. Compute the apparent power in the circuit. ($VA = E_T \times I_T$)

   VA _______________
5. **Turn off the power supply** and reconnect the circuit to measure the current through the resistive branch as shown in Figure 17-4.
6. Turn on the power and measure the current flow through the resistive branch. **Turn off the power.**

   I_R _______________ amps
7. Compute the true power in the circuit using Ohm's law. ($P = E_R \times I_R$)

   P _______________ watts
8. Reconnect the circuit to measure the current flow through the inductive part of the circuit (Figure 17-5).
9. Turn on the power supply and measure the current through the inductive branch of the circuit. **Turn off the power.**

   I_L _______________ amps
10. Compute the reactive power in the inductive branch. ($VARs_L = E_L \times I_L$)

    $VARs_L$ _______________

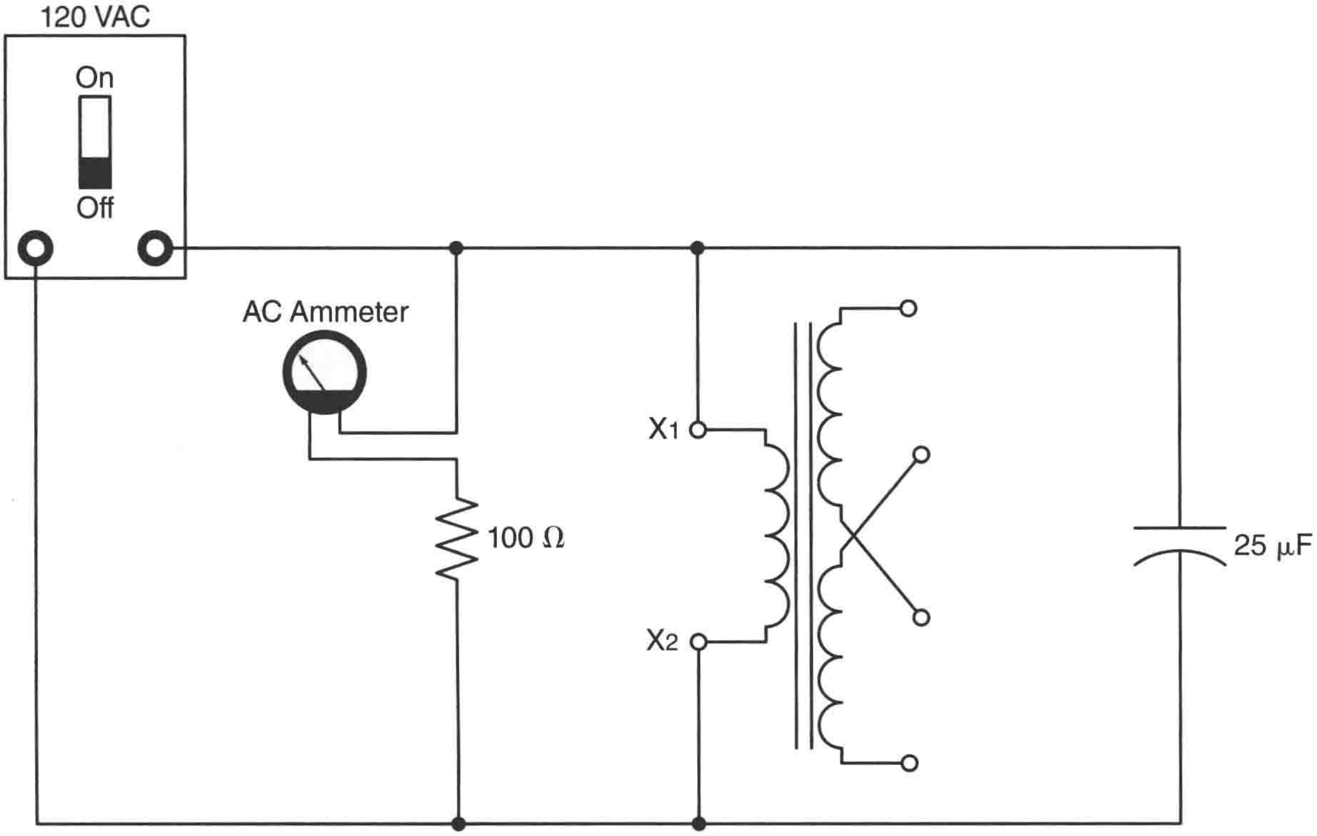


Figure 17-4 Measuring current through the resistive branch.

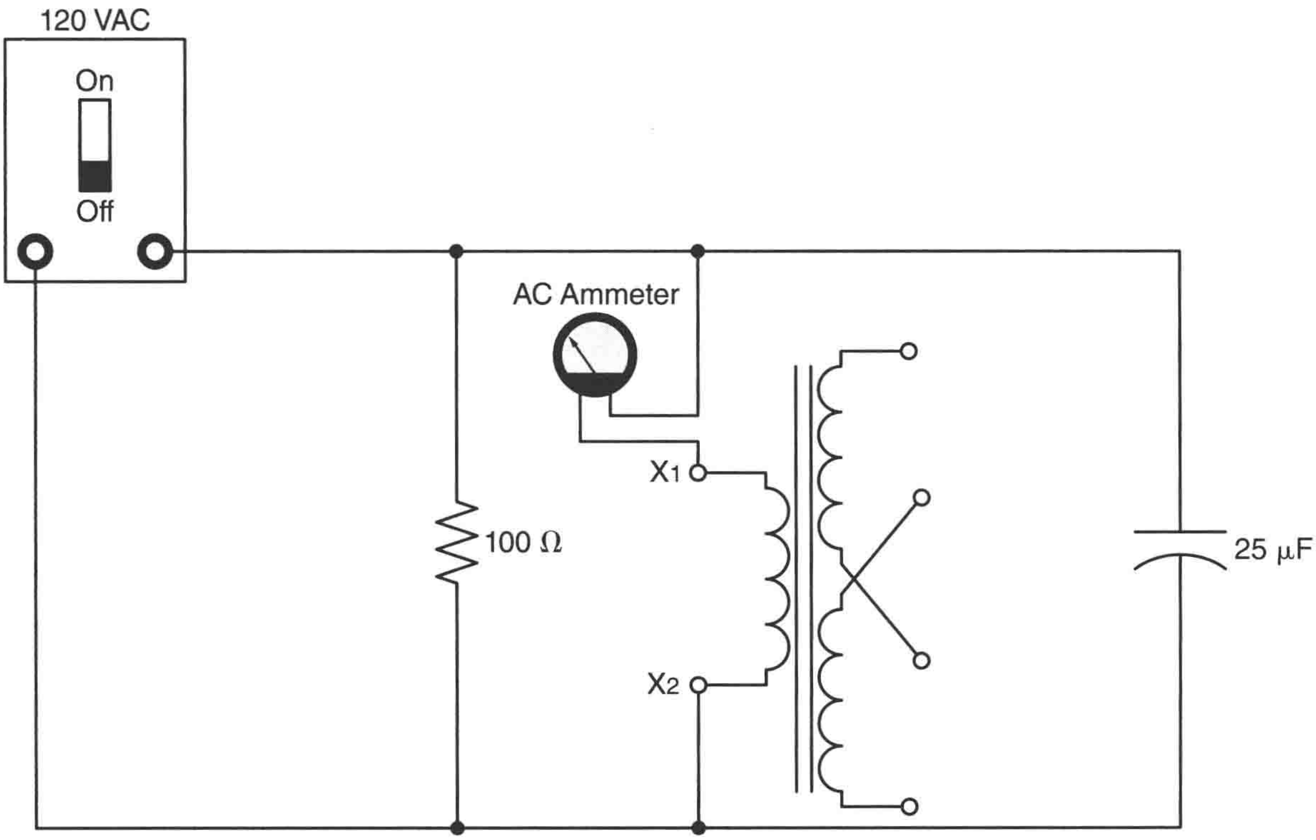


Figure 17-5 Measuring current through the inductive branch.

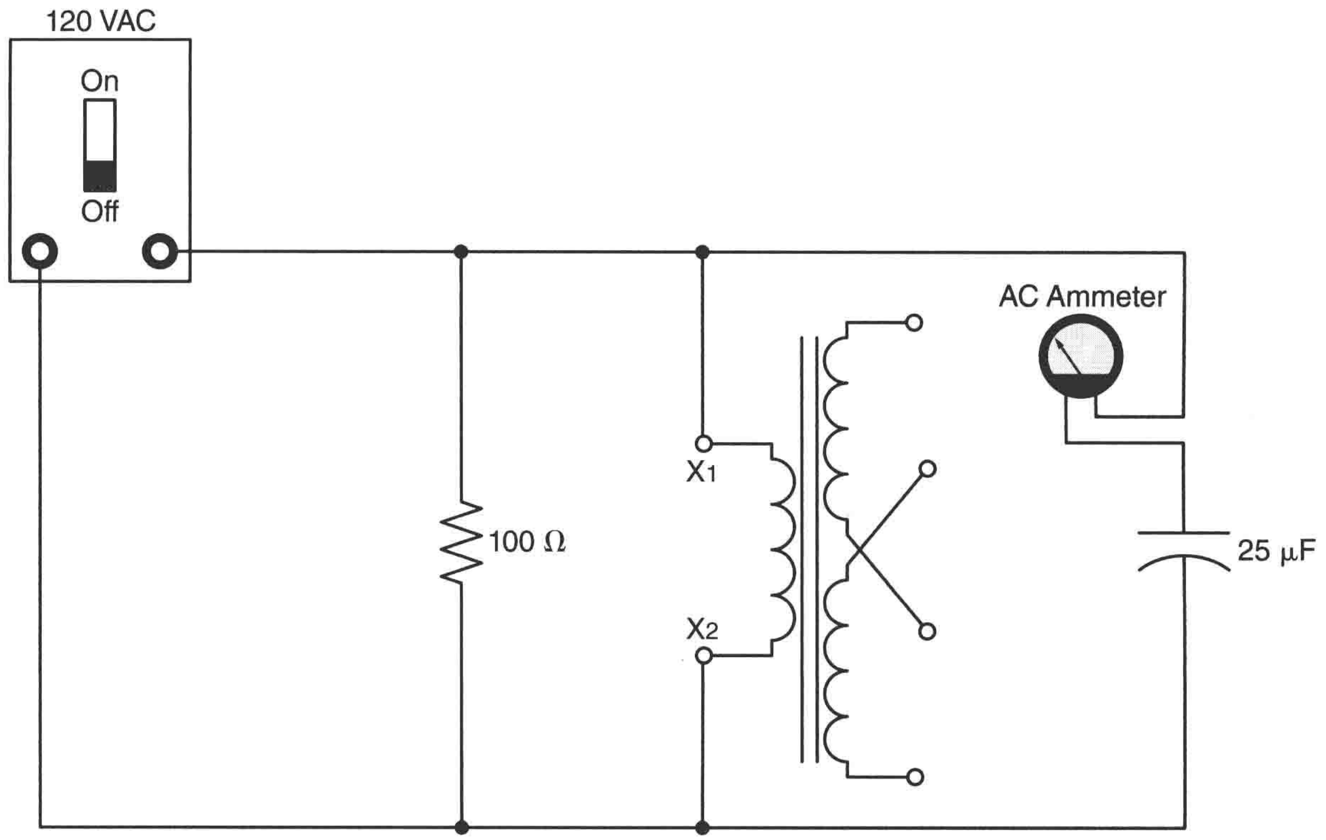


Figure 17-6 Measuring current through the capacitive branch.

11. Reconnect the ammeter to measure the current flow through the capacitive branch (Figure 17-6).
12. Turn on the power supply and measure the current flow through the capacitive branch. **Turn off the power.**

    I_C _______________ amps
13. Compute the reactive power through the capacitive branch. ($VARs_C = E_C \times I_C$)

    $VARs_C$ _______________
14. Using the values determined for apparent power and true power compute the circuit power factor. ($PF = \frac{P}{VA}$)

    PF _______________ %
15. Does this circuit have a leading or lagging power factor? Explain you answer.

    Leading/lagging ____________________

    ____

    ____

    ____
16. How many degrees are the voltage and current out of phase with each other in this circuit? ($\cos \angle\varnothing = PF$)

    $\angle\varnothing$ _______________ °
17. Replace the 100-ohm resistor in the circuit with a 150-ohm resistor.
18. Replace the 25-μF AC capacitor with a 10-μF capacitor.
19. Turn on the power supply and measure the voltage with an AC voltmeter.

    E_T _______________ volts

20. Measure the total current flow in the circuit. **Turn off the power.**

    I_T ______________ amps

21. Compute the apparent power in the circuit. ($VA = E_T \times I_T$)

    VA ______________

22. Reconnect the circuit to measure the current through the resistive branch.

23. Turn on the power and measure the current flow through the resistive branch. **Turn off the power.**

    I_R ______________ amps

24. Compute the true power in the circuit using Ohm's law. ($P = E_R \times I_R$)

    P ______________ watts

25. Reconnect the circuit to measure the current flow through the inductive part of the circuit.

26. Turn on the power supply and measure the current through the inductive branch of the circuit. **Turn off the power.**

    I_L ______________ amps

27. Compute the reactive power in the inductive branch. ($VARs_L = E_L \times I_L$)

    $VARs_L$ ______________

28. Reconnect the ammeter to measure the current flow through the capacitive branch.

29. Turn on the power supply and measure the current flow through the capacitive branch. **Turn off the power.**

    I_C ______________ amps

30. Compute the reactive power through the capacitive branch. ($VARs_C = E_C \times I_C$)

    $VARs_C$ ______________

31. Using the values determined for apparent power and true power compute the circuit power factor. ($PF = \frac{P}{VA}$)

    PF ______________ %

32. Does this circuit have a leading or lagging power factor? Explain you answer.

    Leading/lagging ______________

    ____

    ____

    ____

33. How many degrees are the voltage and current out of phase with each other in this circuit? ($\cos \angle\varnothing = PF$)

    $\angle\varnothing$ ______________ °

34. Disconnect the circuit and return the components to their proper place.

Review Questions

Use the formulas shown in Figure 17-7 to answer some of the following questions:

1. An RLC parallel circuit has a resistive current of 4.8 amps, an inductive current of 7.6 amps, and a capacitive current of 6.6 amps. What is the total current flow in the circuit?

   ____

2. Assume that the circuit in question 1 is connected to a 480 volt, 60 Hz line. What is the inductance of the inductor?

____

3. What is the impedance of an RLC parallel circuit if the resistive branch has a resistance of 112 Ω, the inductive branch has an inductive reactance of 86 Ω, and the capacitive branch has a capacitive reactance of 94 Ω?

____

4. An RLC parallel circuit has a total current of 15.8 amps. The inductive branch has a current flow of 9.6 amps, and the capacitive branch has a current flow of 12.4 amps. How much current is flowing in the resistive branch?

____

5. An RLC parallel circuit has a true power of 138 watts. The inductive VARs are 218 and the capacitive VARs are 86. What is the circuit power factor?

____

6. An RLC parallel circuit has a total current of 12 amps and an apparent power of 6,280 VA. How much voltage is across the capacitive branch of the circuit?

____

7. The capacitive branch of an RLC parallel circuit has a voltage drop of 208 volts. The reactive power is 2,268 VARs. What is the capacitive reactance of this branch?

____

8. An RLC parallel circuit has a total current flow of 23.8 amperes. The circuit power factor is 56.5%. How much current is flowing through the resistive branch of the circuit?

____

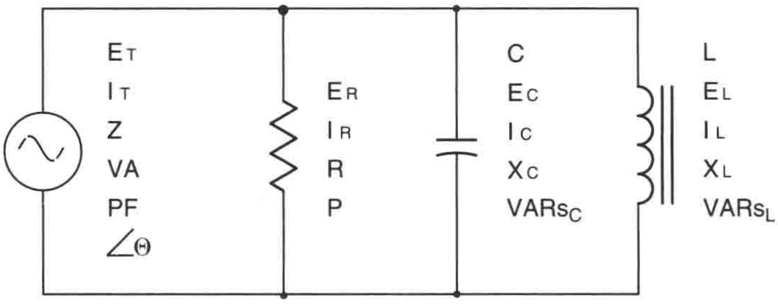
9. An RLC parallel circuit has an apparent power of 22.5 kVA. The total current is 46.6 amperes. What is the circuit impedance?

____

10. An RLC parallel circuit is connected to a 240 volt, 60 Hz line. The apparent power of the circuit is 3,865 VA. The inductive VARs are 1,892 and the capacitive VARs are 1,563. What is the resistance of the resistive branch?

____

Resistive-Inductive-Capacitive Parallel Circuits



$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} \qquad Z = R \times PF \qquad I_T = \sqrt{I_R^2 + (I_L - I_C)^2} \qquad E_R = E_T = E_L = E_C \qquad E_R = I_R \times R$$

$$Z = \frac{VA}{I_T^2} \qquad I_T = \frac{VA}{E_T} \qquad I_T = \frac{I_R}{PF} \qquad E_R = \sqrt{P \times R}$$

$$Z = \frac{E_T}{I_T} \qquad Z = \frac{E_T^2}{VA} \qquad E_T = E_R = E_L = E_C \qquad I_T = \sqrt{\frac{VA}{Z}} \qquad E_R = \frac{P}{I_R}$$

$$VA = E_T \times I_T \qquad E_T = I_T \times Z \qquad I_T = \frac{E_T}{Z} \qquad R = \frac{E_R}{I_R} \qquad VARs_C = E_C \times I_C$$

$$VA = I_T^2 \times Z \qquad E_T = \sqrt{VA \times Z} \qquad I_R = \frac{E_R}{R} \qquad R = \frac{E_R^2}{P} \qquad VARs_C = I_C^2 \times X_C$$

$$VA = \frac{E_T^2}{Z} \qquad VA = \frac{P}{PF} \qquad E_T = \frac{VA}{I_T} \qquad I_R = \sqrt{\frac{P}{R}} \qquad R = \frac{Z}{PF} \qquad VARs_C = \frac{E_C^2}{X_C}$$

$$VA = \sqrt{P^2 + (VARs_L - VARs_C)^2} \qquad I_R = \sqrt{I_T^2 - (I_L - I_C)^2} \qquad R = \frac{P}{I_R^2} \qquad X_C = \frac{VARs_C}{I_C^2}$$

$$PF = \text{Cos}\angle\Theta \qquad PF = \frac{Z}{R} \qquad I_R = I_T \times PF \qquad X_C = \frac{E_C^2}{VARs_C}$$

$$PF = \frac{P}{VA} \qquad PF = \frac{I_R}{I_T} \qquad E_L = I_L \times X_L \qquad I_R = \frac{P}{E_R} \qquad R = \frac{1}{\sqrt{\left(\frac{1}{Z}\right)^2 - \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} \qquad X_C = \frac{E_C}{I_C}$$

$$I_L = \frac{E_L}{X_L} \qquad I_L = \frac{VARs_L}{E_L} \qquad E_L = E_R = E_T = E_C \qquad P = \sqrt{VA^2 - (VARs_L - VARs_C)^2} \qquad X_C = \frac{1}{2\pi f C}$$

$$I_L = \sqrt{\frac{VARs_L}{X_L}} \qquad E_L = \sqrt{VARs_L \times X_L} \qquad P = VA \times PF \qquad E_C = \frac{VARs_C}{I_C} \qquad C = \frac{1}{2\pi f X_C}$$

$$X_L = 2\pi f L \qquad X_L = \frac{E_L}{I_L} \qquad E_L = \frac{VARs_L}{I_L} \qquad P = E_R \times I_R \qquad E_C = I_C \times X_C \qquad I_C = \frac{E_C}{X_C}$$

$$X_L = \frac{VARs_L}{I_L^2} \qquad X_L = \frac{E_L^2}{VARs_L} \qquad VARs_L = E_L \times I_L \qquad P = I_R^2 \times R \qquad E_C = E_T = E_R = E_L \qquad I_C = \frac{VARs_C}{E_C}$$

$$L = \frac{X_L}{2\pi f} \qquad VARs_L = \frac{E_L^2}{X_L} \qquad VARs_L = I_L^2 \times X_L \qquad P = \frac{E_R^2}{R} \qquad E_C = \sqrt{VARs_C \times X_C} \qquad I_C = \sqrt{\frac{VARs_C}{X_C}}$$

Figure 17-7 RLC parallel circuit formulas.

Unit 18 Power Factor Correction

Objectives

After studying this unit, you should be able to:

- Discuss the relationship between apparent power and true power.
- Determine the power factor of a circuit.
- Discuss the importance of power factor correction.
- Determine the amount of capacitance needed to correct a lagging power factor.

Power factor is a ratio of the apparent power and the true power or watts. In an alternating current circuit, the voltage and current can become out of phase with each other. Inductive loads cause the current to lag the applied voltage, and capacitive loads cause the current to lead the applied voltage. Power factor is always expressed as a percent. Basically, it indicates the portion of the power supplied by the power utility that is actually being utilized by the connected load. The utility company must supply the apparent power or volt-amperes. Wattmeters measure the true power or watts. Electric power is purchased in units of kilowatt hours. When the power factor falls below a certain percent, the electric utility generally charges a surcharge to make up the difference.

Example: Assume that a company has a power factor of 56%. Now assume that the wattmeter is indicating that power is being consumed at a rate of 50 kilowatts per hour. With a power factor of 56%, the power utility must actually supply 89.3 kVA (50 / 0.56). Assuming a three-phase voltage of 480 volts, the utility company is supplying 140.94 amperes, computed using following formula:

$$I = \frac{VA}{E \times \sqrt{3}}$$

If the power factor were to be correct to 100%, the apparent power and true power would become the same and the power company would have to furnish only 78.9 amperes to operate the load.

Although a power factor of 100% or unity is the ideal situation, as a general rule industrial customers generally try to correct to about 95%. The amount of capacitance needed to correct the power factor to 100% is much greater than that need for 95% correction.

Example: Assume that a single-phase motor is operating with a 60% lagging power factor. Also assume that the voltage is 240 volts single-phase at 60 Hz and that an ammeter indicates a current of 12.5 amperes. Determine the following:

- Apparent power (VA)
- True power (P)
- Reactive power ($VARs_L$). The VARs are inductive because the power factor is lagging.
- The amount of capacitance needed to correct the power factor to 100%
- The amount of capacitance needed to correct the power factor to 95%

Determining Capacitance for 100% PF

The apparent power can be computed using the formula:

$$VA = E_{Applied} \times I$$

$$VA = 240 \times 12.5$$

$$VA = 3{,}000$$

The true power can be determined using the formula:

$$P = E \times I \times PF$$

$$P = 240 \times 12.5 \times 0.60$$

$$P = 1{,}800 \text{ watts}$$

The inductive VARs can be found using the formula:

$$VARs_L = \sqrt{VA^2 - P^2}$$

$$VARs_L = \sqrt{3{,}000^2 - 1{,}800^2}$$

$$VARs_L = 2{,}400$$

To correct the power factor to unity or 100%, an equal amount of capacitive VARs would have to be added to the circuit. To determine the amount of capacitance needed, first determine the amount of capacitive current necessary to produce 2,400 capacitive VARs.

$$I_c = \frac{VARs_C}{E}$$

$$I_c = \frac{2{,}400}{240}$$

$$I_c = 10 \text{ amperes}$$

Now that the amount of current needed is known, the amount of capacitive reactance needed to produce that much current can be calculated.

$$X_c = \frac{E}{I_c}$$

$$X_c = \frac{240}{10}$$

$$X_c = 24\ \Omega$$

The amount of capacitance needed to produce a capacitive reactance of 24 Ω at 60 Hz can be computed using the formula:

$$C = \frac{1}{2\pi f X_c}$$

$$C = \frac{1}{377 \times 24}$$

$$C = 0.0001105 \text{ farad or } 110.5\ \mu F$$

Determining Capacitance for 95% PF

To determine the amount of capacitance needed to produce a 95% power factor, it is first necessary to determine what the apparent power should be to produce a 95% power factor. This can be computed using the formula:

$$VA = \frac{P}{PF}$$

$$VA = \frac{1{,}800}{0.95}$$

$$VA = 1{,}894.75$$

The next step is to determine the amount of reactive power needed to produce an apparent power of 1,894.75 VA. When correcting power factor, it is generally desirable to leave the power factor lagging as opposed to leading.

$$VARs_L = \sqrt{VA^2 - P^2}$$

$$VARs_L = \sqrt{1{,}894.75^2 - 1{,}800^2}$$

$$VARs_L = 591.67$$

At present, the reactive power is 2,400 $VARs_L$. To reduce the circuit to 591.6 $VARs_L$, 1,808.4 capacitive VARs will have to be added to the circuit (2,400 − 591.6).

The current needed to produce 1,808.4 $VARs_C$ can be determined using the formula:

$$I_c = \frac{VARs_C}{E}$$

$$I_c = \frac{1{,}808.4}{240}$$

$$I_c = 7.53 \text{ amperes}$$

Now that the amount of current needed is known, the amount of capacitive reactance needed to produce that much current can be calculated.

$$X_c = \frac{E}{I_c}$$

$$X_c = \frac{240}{7.53}$$

$$X_c = 31.87\ \Omega$$

The amount of capacitance needed to produce a capacitive reactance of 31.87 Ω at 60 Hz can be computed using the formula:

$$C = \frac{1}{2\pi f X_c}$$

$$C = \frac{1}{377 \times 31.87}$$

$$C = 0.00008328 \text{ farad or } 83.28\ \mu\text{F}$$

Note that substantially less capacitance is needed to correct the power factor to 95%. The current will now drop from 12.5 amperes to 7.89 amperes (1,894.75 VA / 240 V).

LABORATORY EXERCISE

Name ____ Date __________

Materials Required

1 120-volt AC power supply

1 0.5-kVA control transformer with 2 windings rated at 240 volts each and one winding rated at 120 volts

1 100-ohm resistor

1 150-ohm resistor

1 25-µf AC capacitor rated at not less than 240 volts

1 10-µf AC capacitor rated at not less than 240 volts

1 7.5-µf AC capacitor rated at not less than 240 volts

1 AC voltmeter

1 AC ammeter (an in-line or clamp-on type meter may be used. If a clamp-on type is employed, the use of a 10:1 scale divider is recommended.)

In this experiment, the low-voltage winding (X_1 and X_2) of a transformer will be connected in series with 60 ohms of resistance to produce a load with a lagging power factor. A 100 ohm resistor is connected in parallel with a 150 ohm resistor to produce a total of 60 ohms. The apparent power, true power, and reactive power of the circuit will then be calculated. After these values have been determined, the amount of capacitance needed to correct the power factor will be computed.

1. Connect the circuit shown in Figure 18-1.
2. Turn on the power and measure the total current.

   I_T ______________ amps.

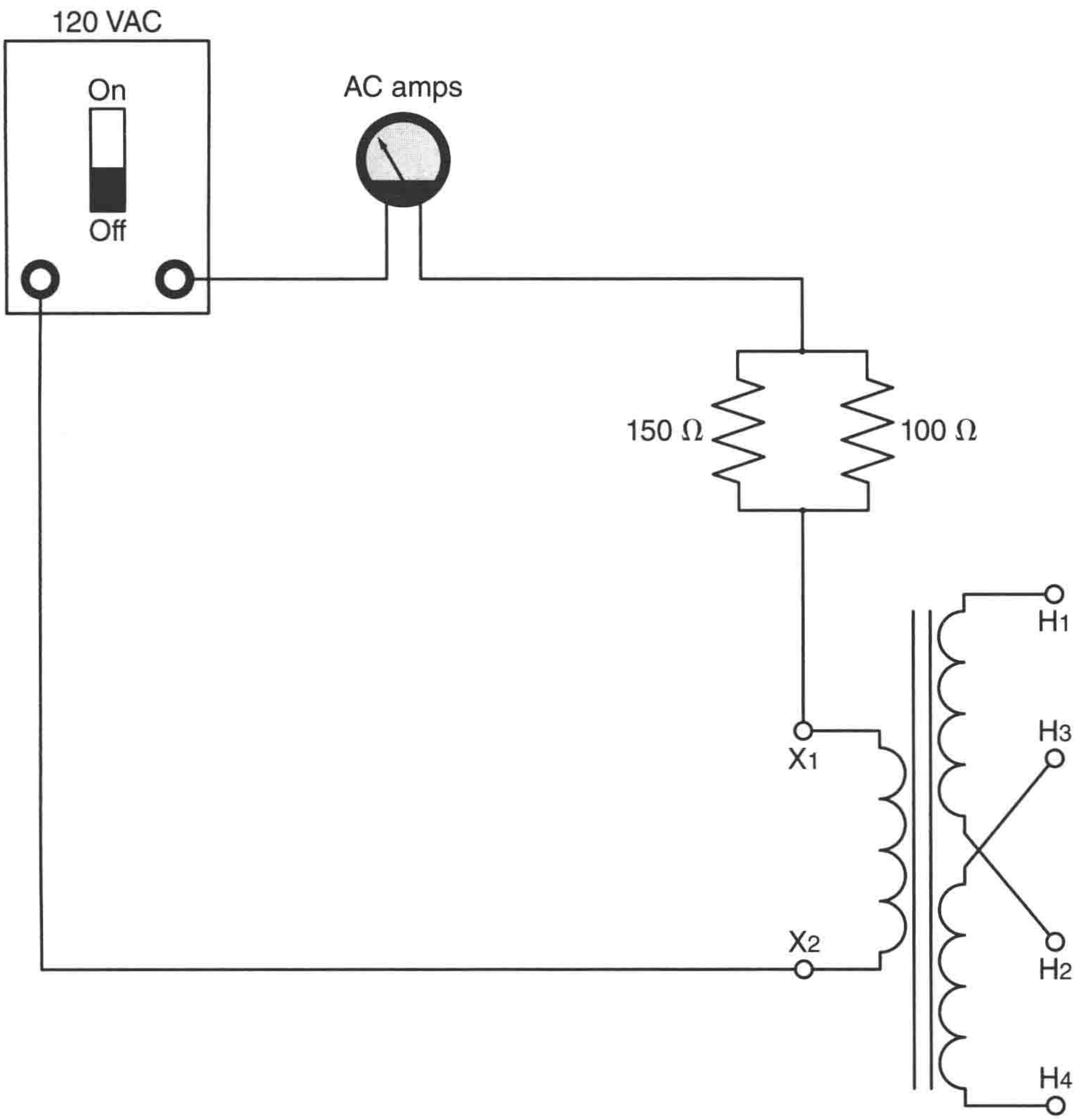


Figure 18-1 The 100 ohm and 150 ohm resistors are connected in parallel to produce a resistance of 60 ohms.

3. Measure the voltage applied to the circuit.

   E_T = ______________ volts

4. Measure the voltage drop across the 60 ohm resistance.

   E_R ______________ volts

5. Measure the voltage across the X_1 and X_2 winding of the transformer. **Turn off the power.**

   E_L ______________ volts

6. Compute the apparent power in the circuit using the total voltage and the total circuit current. ($VA = E_T \times I$)

   VA ______________

7. Compute the true power in the circuit using the voltage drop across the 60 watt lamp and the circuit current. ($P = E_R \times I$)

   P ______________ watts

8. Compute the inductive VARs in the circuit using the voltage drop across the transformer winding and the circuit current. ($VARs_L = E_L \times I$)

   $VARs_L$ ______________

9. Compute the circuit power factor. ($PF = \frac{P}{VA}$)

   PF ______________ %

10. Determine the phase angel difference between the voltage and current. ($\cos PF = \angle\varnothing$)

    $\angle\varnothing$ ______________ °

11. To determine the amount of capacitance needed to correct the power factor to 95%, first calculate the apparent power needed to produce a 95% power factor. (*Note*: A value of 0.95 should be used for power factor in the formula, because the power factor desired is 95%.)

    ($VA = \frac{P}{PF}$)

    VA ______________

12. Determine the inductive VARs necessary to produce the apparent power determined in step 12. ($VARs = \sqrt{VA^2 - P^2}$)

    $VARs_L$ ______________

13. To determine the capacitive VARs needed, subtract the needed VARs in step 13 from the calculated VARs in step 8.

    $VARs_C$ ______________

14. Compute the amount of capacitive necessary to produce the capacitive VARs determined in step 13. ($X_C = \frac{E^2}{VARs_C}$)

    X_C ______________ Ω

15. Compute the amount of capacitance needed to produce the capacitive reactance determined in step 14. ($C = \frac{1}{2\pi fX}$)

    C ______________ μF

16. From the capacitors provided in this experiment, select the capacitor that is the nearest value to the capacitor value determined in step 16. Do not go over the computed value. Connect the capacitor across the circuit as shown in Figure 18-2.

17. Turn on the power supply and measure the total circuit current. **Turn off the power.**

    I_T ______________ amps

18. Compute the apparent power of the circuit using the current value measured in step 17.

    VA ______________

19. Assuming that the true power has remained constant, compute the circuit power factor using the apparent power value determined in step 18.

    PF ______________ %

20. Was there an improvement in the circuit power factor?

    Yes/no ______________

21. Did the total current decrease after the capacitor was added to the circuit?

    Yes/no ______________

22. Disconnect the circuit and return the components to their proper place.

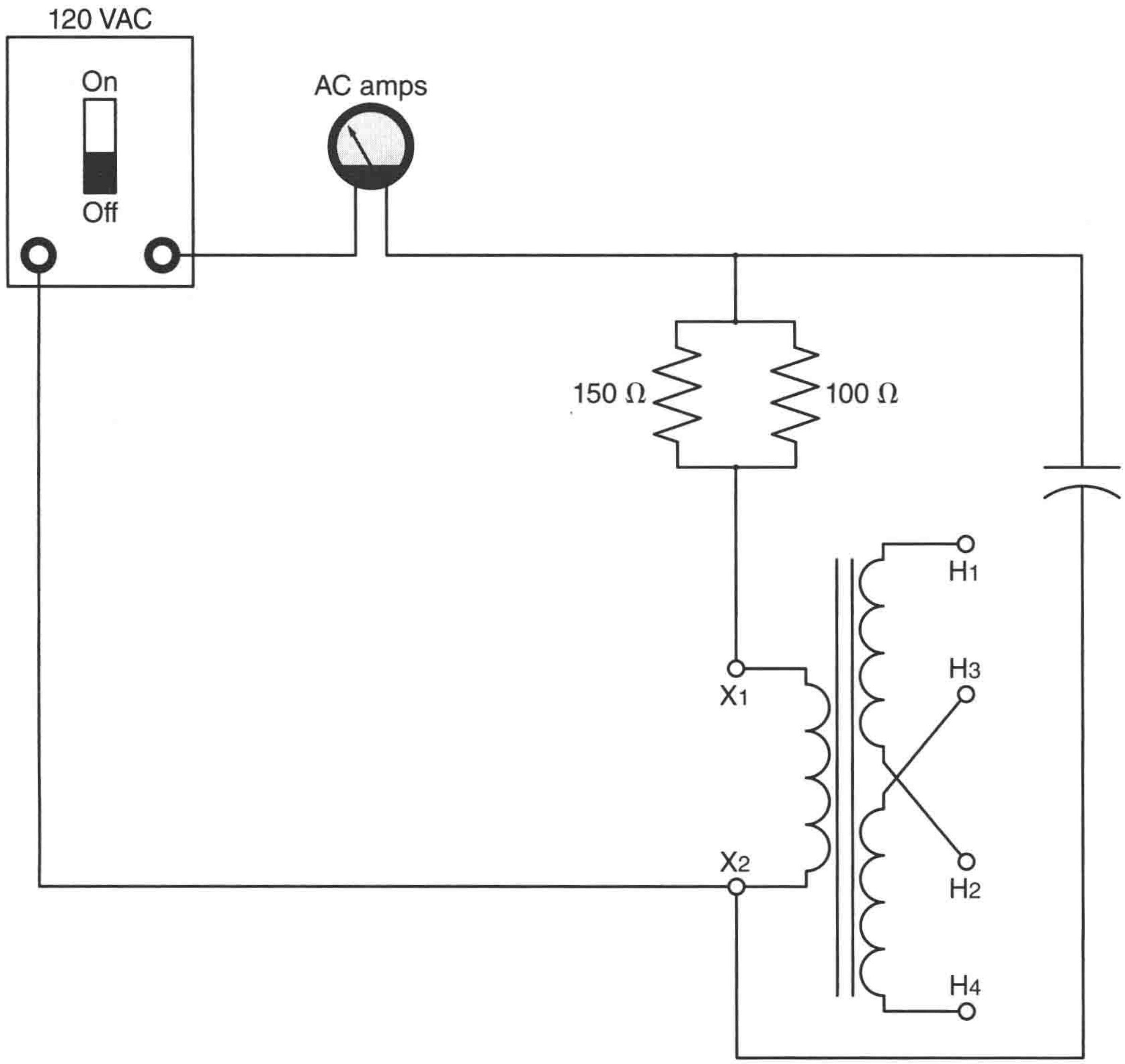


Figure 18-2 Adding capacitance to the circuit.

Review Questions

1. A single-phase motor is connected to a 120 volt, 60 Hz line. The motor has a current draw of 15.7 amperes. A watt meter indicated the motor is actually using 1,340 watts. Find the following:

   Apparent power:

   ____

   Power factor of the motor:

   ____

   Reactive power of the motor:

   ____

   Capacitance needed to correct the power factor to 100%:

   ____

   Capacitance needed to correct the power factor to 95%:

   ____

2. An inductive load is connected to a 208 volt, 60 Hz line. The circuit has a current draw of 6.25 amperes. The circuit power factor is 45%. If a 10 μF capacitor is connected in parallel with the inductive load, what will the new power factor be? Determine the following values:

   Apparent power:

   ____

   True power:

   ____

   Reactive power before the capacitor is connected:

   ____

   New power factor:

   ____

SECTION 4

Transformers and Motors

Unit 19 Transformer Basics

Objectives

After studying this unit, you should be able to:

- Discuss the construction of an isolation transformer.
- Determine the winding configuration with an ohmmeter.
- Connect a transformer and make voltage measurements.
- Compute the turns-ratio of the windings.

A transformer is a magnetically operated machine that can change values of voltage, current, and impedance without a change of frequency. Transformers are the most efficient machines known. Their efficiencies commonly range from 90% to 99% at full load. Transformers can be divided into several classifications such as:

- Isolation
- Auto
- Current

A basic law concerning transformers is that all values of a transformer are proportional to its turns-ratio. This does not mean that the exact number of turns of wire on each winding must be known to determine different values of voltage and current for a transformer. What must be known is the ratio of turns. For example, assume a transformer has two windings. One winding, the primary, has 1,000 turns of wire and the other, the secondary, has 250 turns of wire, as shown in Figure 19-1. The turns-ratio of this transformer is 4 to 1 or 4:1 (1,000/250 = 4). This indicates there are four turns of wire on the primary for every one turn of wire on the secondary.

Helpful Hint

A basic law concerning transformers is that all values of a transformer are proportional to its turns-ratio.

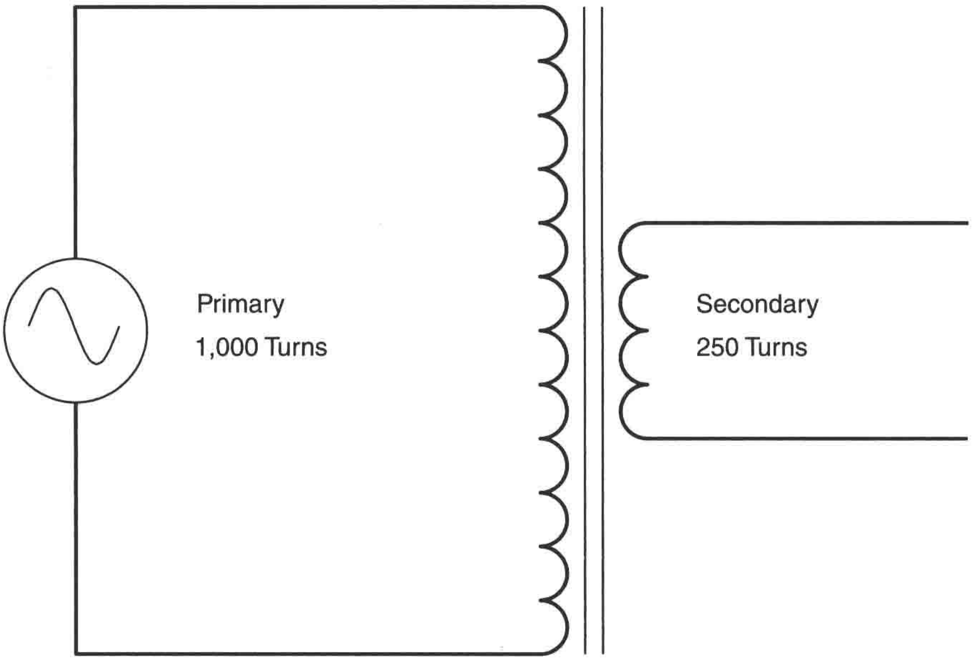


Figure 19-1 All values of a transformer are proportional to its turns-ratio.

Transformer Formulas

There are different formulas that can be used to find the values of voltage and current for a transformer. The following is a list of standard formulas:

where:

N_P = Number of turns in the primary
N_S = Number of turns in the secondary
E_P = Voltage of the primary
E_S = Voltage of the secondary
I_P = Current in the primary
I_S = Current in the secondary

$$\frac{E_P}{E_S} = \frac{N_P}{N_S}$$

$$\frac{E_P}{S_P} = \frac{I_S}{I_P}$$

$$\frac{N_P}{N_S} = \frac{I_S}{I_P}$$

or

$$E_P \times N_S = E_S \times N_P$$

$$E_P \times I_P = E_S \times I_S$$

$$N_P \times I_P = N_S \times I_S$$

The primary winding of a transformer is the power input winding. It is the winding that is connected to the incoming power supply. The secondary winding is the load winding or output winding. It is the side of the transformer that is connected to the driven load, seen in Figure 19-2. Any winding of a transformer can be used as a primary or secondary winding provided its voltage or current rating is not exceeded. Transformers can also be operated at a lower voltage than their rating indicates, but they cannot be connected to a higher voltage. Assume the transformer shown in Figure 19-2, for example, has a primary voltage rating of 480 volts and the secondary has a voltage rating of 240 volts. Now assume that the primary winding is connected to a 120 volt source. No damage would occur to the transformer, but the secondary winding would produce only 60 volts.

Isolation Transformers

The transformers shown in Figures 19-1 and 19-2 are *isolation* transformers. This means that the secondary winding is physically and electrically isolated from the primary winding. There is no electrical connection between the primary and secondary winding. *This transformer is magnetically coupled, not electrically coupled.* This "line isolation" is often a very desirable characteristic. Since there is no electrical connection between the load and power supply, the transformer becomes a filter between the two. The isolation transformer will attenuate any voltage spikes that originate on the supply side before they are transferred to the load side. Some isolation transformers are built with a turns-ratio of 1:1. A transformer of this type will have the same input and output voltage and is used for the purpose of isolation only.

The reason that the transformer can greatly reduce any voltage spikes before they reach the secondary is because of the rise time of current through an inductor. The current in an inductor rises at an exponential rate, as shown in Figure 19-3. As the current increases in value, the expanding magnetic field cuts through the conductors of the coil and induces a voltage that is opposed to the applied voltage. The amount of induced voltage is proportional to the rate of change of current. This simply means that the faster current attempts to increase, the greater the opposition to that increase will be. Spike voltages and currents are generally of very short duration, which means that they increase in value very rapidly (Figure 19-4). This rapid change of value causes the opposition to the change to increase just as rapidly. By the time the spike has been transferred to the secondary winding of the transformer, it has been eliminated or greatly reduced (Figure 19-5).

Another purpose of isolation transformers is to remove or isolate some piece of electrical equipment from circuit ground. It is sometimes desirable that a piece of electrical equipment not be connected directly to circuit ground. This is often done as a safety

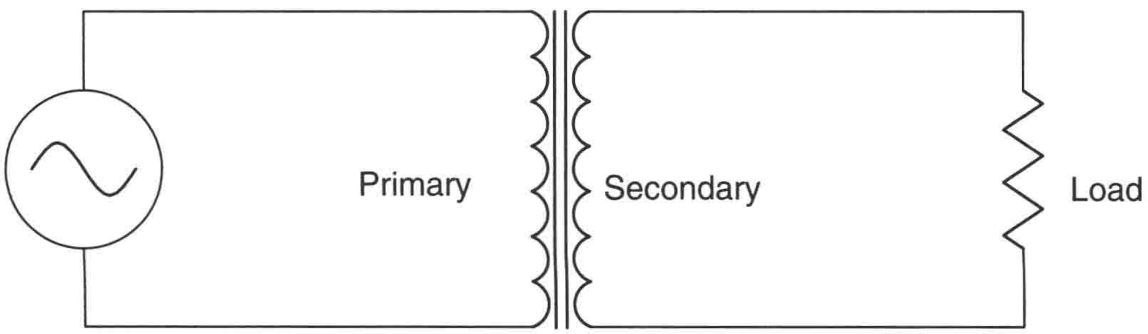


Figure 19-2 Isolation transformer.