Master's thesis

Tensor Network renormalization for Non-uniform Classical Spin System

(非一様な古典スピン系に対するテンソルネットワーク繰り込み)

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I appreciate to

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Chapter 1

Introduction

The effects of lattice defects in spin systems,bond dilution[1],random couplings[2],random exeternal fields[3], are studied vigorously for several decades because such non-uniformness and randomness is essential to describe and understand real magnetic materials.

Chapter 2

Tensor Network Method

2.1 Tensor Network group

In the tensor network methods,the partition function or the wave function is represented by large tensors ,and decomposed into small tensors by singular value decomposition.We can get new tensor by constracting decomposed small tensors.Decomposing and constracting are fundamental approah in the tensor network method.For example,I review Levin and Nave‘s approah.

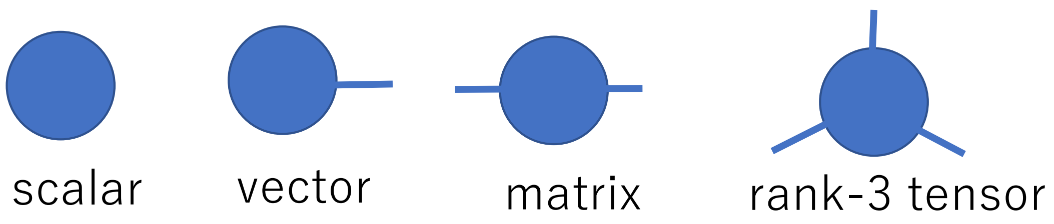

Figure 2.1. zzzzzzzzzzz

2.2 Tensor renormalization group

Levin and Nave developed the tensor renormalization group(TRG)[4]. They applicate two-dimensional ising model.

2.2.1 Singular value decomposition

Chapter 3

Higher Order Tensor Renormalization Group Algorithm

3.1 Higher order tensor renormalization group

The higher order tensor renormalization group(HOTRG)[5] is developed by For example, in the two dimensional ising model,

$$H = -\sum_{<ij>} \sigma^i \sigma^j \tag{3.1}$$

then,partition function is

$$Z = \sigma \prod_{<ij>} \tag{3.2}$$

T is inverse $\beta = 1/T$

Chapter 4

Application to percolation

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Chapter 5

Summary

Appendix A

Appendix

A.1 binder ratio

Hamiltonian of two dimensional ising model is

$$\mathrm{H} = -\sum_{<ij>} \sigma^i \sigma^j - h \sum_i \sigma_i \tag{A.1}$$

then,partition function is

$$Z = \Sigma_\sigma e^{-\beta \mathrm{H}} \tag{A.2}$$

free energy per one site(N:tne number of spins in the system)

$$f = -\frac{1}{N\beta} \sum_{<ij>} \sigma^i \sigma^j \tag{A.3}$$

then,partition function is

$$Z = \sigma \prod_{<ij>} \tag{A.4}$$

Bibliography

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