The magneto scalar field and its role

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1. Abstract

In the early 2000s, there was a minor flare-up in the history of physics due to the article by C. Monstein and J.P. Wesley (Monstein & Wesley, 2002), when they claimed to have experimentally supported the existence of longitudinal electromagnetic waves. Around this time, the first alternative electrodynamics was born with the help of quaternions (van Vlaenderen & Waser, 2001). Later, following K. Rębilas (Rębilas, 2008), it turned out that the experimental results can be explained within the framework of classical electrodynamics. Lev Borisovich Okun in his article (Okun, 1989) processed a total of 30 articles and came to the conclusion that this would violate the Pauli exclusion principle. So this cannot occur in nature.

The laws of the theory of electrodynamics were formulated in quaternion form. Formalism shows that Maxwell's equations can be derived from only a single quaternion equation. I distinguish between modified and unmodified Maxwell equations according to whether or not the Lorenz gauge is applied. I present the shape of the quaternion as the continuity equation, generalizing the “traditional” continuity equation with three equations. A magneto scalar field set in equations that propagates at the speed of light. I lay the ground pillars of the static magneto scalar field. I also deal with magneto scalar waves, the energy conservation, the constitutive relations in vacuum, the invariance to the Lorentz transformation, the Lagrangian density and some other topics.

Keywords: quaternion, electrodynamics, magneto scalar potential

2. Introduction

Classical electrodynamics is the cornerstone of modern physics. Classical electrodynamics provides the basis for many physical models. Michael Faraday wrote and published a series of thirty articles in the Philosophical Transactions of the Royal Society between 1832 and 1856 under the following titles: Experimental Researches in Electricity (Al-Khalili, 2015). James Clerk Maxwell (Maxwell, 1865) Based on Faraday's empirical results, he formulated classical electrodynamics in the form of 20 partial differential equations. Oliver Heaviside (Heaviside, 2007) using vector calculus, and also the introduction of the $\vec{A}$ vector potential and the φ scalar potential rewrote Maxwell's equations. Ludvig Lorenz (Lorenz, 1867) realized that the wave equations of $\vec{A}$ and φ they can be produced with an additional constraint, namely through the Lorenz gauge named after him The gauge transformation is: $\varphi \rightarrow \varphi - \frac{\partial \chi}{\partial t}$ és $\vec{A} \rightarrow \vec{A} + \nabla \chi$, where χ is a gauge function. By definition $\vec{E}$ electric field strength vector and the $\vec{B}$ magnetic induction are invariant under the gauge transformation. Nowadays, classical electrodynamics is considered a complete and closed theory, no need for reinterpretation. However, I have found some experiments that are difficult or impossible to explain with classical electrodynamics. In (here), I list these experiments and conclusions. Falsifiability states that an empirical theory cannot be verified but can be disproved by contrary test results (Popper, 1972). I distinguish between modified and unmodified Maxwell's equations according to whether I use the Lorenz gauge or not. The modified Maxwell equations predict the existence of the longitudinal electric field strength vector and the magneto scalar potential, as well as the local violation of charge conservation. In this study, I will delve into the modified Maxwell equations.

3. The initial point

Any physical theory that does not describe the desired physical effect needs to be modified. However, there are supposed errors in a theory that turn out to be correct after all. However, there are effects that are difficult or not at all to explain with classical electrodynamics. I deal with these topics in this chapter.

3.1. The supposed overdetermination of Maxwell's equation

Maxwell's equations appear to be overdetermined, containing six unknowns and eight equations. Maxwell's divergence equations are generally thought to be redundant, and both equations are taken as initial conditions for the rotational equations. For this reason, the two divergence equations are not usually solved in electromagnetic simulations. There is a circular fallacy in this explanation, and the two non-redundant but fundamental divergence equations cannot be ignored in electromagnetic simulations (Changli, 2015).

3.2. C. Monstein's and J. P. Wesley's experiment

In the early 2000s, there was a minor flare-up in the history of physics, under the influence of C. Monstein and J.P. Wesley (Monstein & Wesley, 2002) claimed to have experimentally supported the existence of longitudinal electromagnetic waves. Later then K. Rębilas (Rębilas, 2008) it turned out that the experimental results can be explained within the framework of classical electrodynamics. In the article by C. Monstein and J.P. Wesley, a curl-free vector potential is a requirement. However, in the experiments of C. Monstein and J.P. Wesley, they did not experimentally confirm the local violation of the charge conservation. The question may arise in us, whether we can change the nature of the electromagnetic interaction just by properly designed experimental conditions?

3.3. Lee M. Hively's and Andrew S. Loebl's experiment

Lee M. Hively and Andrew S. Loebl claimed experimental support for the existence of longitudinal electromagnetic waves in the macroscopic range (Hively & Loebl, Classical and extended electrodynamics, 2019). According to their published article, the curl-free vector potential is a requirement and their experimental results are explained with the modified Maxwell equations. However, in their experiment, they did not experimentally confirm the local violation of the charge conservation. The question may arise in us, whether we can change the nature of the electromagnetic interaction just by properly designed experimental conditions?

3.4. Quantum mechanical aspect

The principle of conservation of charge was supported by many experiments. The experiments were characterized by a long observation time, which means that the processes could have been averaged. The question may arise, if we repeat the measurements on the same system at sufficiently short intervals, will we experience fluctuations in the total amount of charge in a closed system? One of the consequences of the modified Maxwell equations is the local violation of charge conservation. It says that charges can transform into electromagnetic-scalar waves and vice versa.

Lev Borisovich Okun in his article (Okun, 1989) processed a total of 30 articles and came to the conclusion that this would violate the Pauli exclusion principle. So this cannot occur in nature.

3.5. The initial point

In the literature of longitudinal electromagnetic interaction, Stueckelberg's work is often cited as a starting point (Stueckelberg, 1938). The following Lagrangian density is associated to him:

$$L = -\frac{\varepsilon_0 \cdot c^2}{4} \cdot F_{\mu\upsilon} \cdot F^{\mu\upsilon} + J^\mu \cdot A_\mu$$
$$-\frac{\gamma \cdot \varepsilon_0 \cdot c^2}{2} \cdot \left(\partial_\mu A^\mu\right)^2 - \frac{\gamma \cdot \varepsilon_0 \cdot c^2 \cdot k^2}{2} \cdot \left(A_\mu \cdot A^\mu\right) \tag{1}$$

For the full Lagrangian density, the parameter γ should be chosen as 1 and the parameter k as 0, where k is the Compton wavenumber of photons of mass m (Reed & Hively, 2020). The reality is that Stueckelberg's name is not associated with this Lagrangian density.
In field theory, the Stueckelberg action describes a massive spin-1 field as an R (the real numbers are the Lie algebra of U(1)). Yang–Mills theory coupled to a real scalar fieldφ. This scalar field takes on values in a real 1D affine representation of R with m as the coupling strength.

$$L = -\frac{1}{4} \cdot F_{\mu\upsilon} \cdot F^{\mu\upsilon} + \frac{1}{2} \cdot (\partial^\mu \varphi + m \cdot A^\mu) \cdot \left(\partial_\mu \varphi + m \cdot A_\mu\right) \tag{2}$$

This is a special case of the Higgs mechanism, where practically λ and thus the mass of the Higgs scalar excitation were taken to be infinite, so the Higgs is decoupled and can be ignored, which results in a non-linear, affine representation of the field, instead of a linear representation - in today's terminology, this a U(1) nonlinear σ-model. By choosing the measure φ=0, we get the Proca effect (Wikipedia, 2022).

4. Quaternion mathematics – major relationships

Many literatures deal with the application of quaternions. I will highlight one of them and will use their marking system in the future (Hong & Kim, 2019). Let's denote quaternions as follows:

$$\tilde{A} = \left(A_0, A_1 \cdot \underline{i} + A_2 \cdot \underline{j} + A_3 \cdot \underline{k}\right) \tag{3}$$

, where A_0, A_1, A_2, A_3 are real numbers and $\underline{i}$, $\underline{j}$, $\underline{k}$ are the quaternion unit vectors for which the following relations hold:

$$\underline{i}^2 = \underline{j}^2 = \underline{k}^2 = -1 \tag{4}$$

$$\underline{i} \cdot \underline{j} = \underline{k} \,\&\, \underline{k} \cdot \underline{i} = \underline{j} \,\&\, \underline{j} \cdot \underline{k} = \underline{i} \,\&\, \underline{i} \cdot \underline{j} \cdot \underline{k} = -1 \tag{5}$$

$$\underline{i} \cdot \underline{j} = -\underline{j} \cdot \underline{i} \,\&\, \underline{j} \cdot \underline{k} = -\underline{k} \cdot \underline{j} \,\&\, \underline{k} \cdot \underline{i} = -\underline{i} \cdot \underline{k} \tag{6}$$

Equation (3) can be divided into two parts: scalar (s) and quaternion vector ($\vec{v}$):

$$\tilde{A} = (s, \vec{v}) \tag{7}$$

If $\tilde{A} = \left(A_0, \vec{A}\right)$ and $\tilde{B} = \left(B_0, \vec{B}\right)$ two quaternions, then their product is formed as follows:

$$\tilde{A} \cdot \tilde{B} = \left(A_0, \vec{A}\right) \cdot \left(B_0, \vec{B}\right) = \left(A_0 \cdot B_0 - \vec{A} \cdot \vec{B}, A_0 \cdot \vec{B} + \vec{A} \cdot B_0 + \vec{A} \times \vec{B}\right) \tag{8}$$

, where $\vec{A} \cdot \vec{B}$ is the scalar product of the two vectors and $\vec{A} \times \vec{B}$ is the vector product of the two vectors. Quaternions can be extended to a set of complex numbers, that is, complex quaternions can be interpreted:

$$\tilde{A} = \left(a + i \cdot b, \vec{c} + i \cdot \vec{d}\right) \tag{9}$$

, where a, b are real numbers, $\vec{c}$, $\vec{d}$ are vectors, and i is the complex unit. Being complex quaternions, the complex conjugation operation can be:

$$\tilde{A}^* = \left(a - i \cdot b, \vec{c} - i \cdot \vec{d}\right) \tag{10}$$

In addition, the magnitude of a quaternion can be interpreted as follows:

$$|\tilde{A}| = \sqrt{\tilde{A} \cdot \tilde{A}^*} = \sqrt{A_0^2 + A_1^2 + A_2^2 + A_3^2} \tag{11}$$

The Nabla quaternion operator and it's complex conjugate can be defined as follows:

$$\tilde{\nabla} = \left(\frac{i}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) \,\&\, \tilde{\nabla}^* = \left(-\frac{i}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) \tag{12}$$

, where i is the imaginary unit and c is the speed of light in vacuum. Using equation (12), we can define the D'Alambert quaternion operator in the following way:

$$
\begin{gathered}
\tilde{\nabla}^{*} \cdot \tilde{\nabla} = \left(-\frac{i}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot \left(\frac{i}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) = \\
\left(\frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2} - \vec{\nabla} \cdot \vec{\nabla}, \left(-\frac{i}{c} \cdot \frac{\partial}{\partial t}\right)\vec{\nabla} + \left(\frac{i}{c} \cdot \frac{\partial}{\partial t}\right)\vec{\nabla} + \vec{\nabla} \times \vec{\nabla}\right) \\
\frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2} - \vec{\nabla} \cdot \vec{\nabla} = \tilde{\square}
\end{gathered} \tag{13}
$$

5. Quaternion electrodynamics

5.1. The modified Maxwell equations

The maxwell equations can be derived from a single quaternion (wave) equation that connects two quaternion quantities: The $\tilde{A}$ quaternion vector potential and the $\tilde{J}$ quaternion current density.

$$\tilde{A} = \left(i \cdot \frac{\varphi}{c}, \vec{A}\right) \;\&\; \tilde{A}^* = \left(-i \cdot \frac{\varphi}{c}, \vec{A}\right) \tag{14}$$

$$\tilde{J} = \left(i \cdot c \cdot \rho, \vec{J}\right) \tag{15}$$

, where φ the scalar potential, $\vec{A}$ is the vector potential, ρ is the charge density. Start by forming the product of the complex conjugated quaternion vector potential and the quaternion Nabla operator according to equation (12):

$$\tilde{G} = \tilde{\nabla}^* \cdot \tilde{A}^* = \left(-\frac{i}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot \left(-i \cdot \frac{\varphi}{c}, \vec{A}\right) = \left(-\left(\frac{1}{c^2} \cdot \frac{\partial \varphi}{\partial t} + \vec{\nabla} \cdot \vec{A}\right), \frac{i}{c} \cdot \left(-\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi\right) + \left(\vec{\nabla} \times \vec{A}\right)\right) \tag{16}$$

At this point, three familiar relationships are immediately apparent: the Lorenz gauge (if the first term in parentheses is chosen to be zero), the electric field strength and the magnetic induction (Jackson, 1999).

$$\Lambda = \left(\frac{1}{c^2} \cdot \frac{\partial \varphi}{\partial t} + \vec{\nabla} \cdot \vec{A}\right) \tag{17}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi \tag{18}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{19}$$

The Lorenz gauge is not assumed in this case and we examine what it leads to. Using equation (17), (18), (19) simplifies to the following:

$$\tilde{G} = \left(-\Lambda, \frac{i}{c} \cdot \vec{E} + \vec{B}\right) \tag{20}$$

Construct the complex conjugate of equation (18) and multiply by the complex conjugate of the quaternion Nabla operator. Then the equation will be proportional to the quaternion current density $\tilde{J}$. The proportionality factor is μ_0 the vacuum permeability:

$$\tilde{\nabla}^* \cdot \tilde{G}^* = \tilde{\Box}\tilde{A} = \mu_0 \cdot \tilde{J} \tag{21}$$

$$\tilde{\nabla}^* \cdot \tilde{G}^* = \left(-\frac{i}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) \cdot \left(-\Lambda + \frac{i}{c} \cdot \vec{E} + \vec{B}\right) \tag{22}$$

$$\left(\frac{i}{c}\cdot\left(\vec{\nabla}\cdot\vec{E}+\frac{\partial\Lambda}{\partial t}\right)-\left(\vec{\nabla}\cdot\vec{B}\right),\left(-\vec{\nabla}\Lambda-\frac{1}{c^2}\cdot\frac{\partial\vec{E}}{\partial t}+\vec{\nabla}\times\vec{B}\right)-\frac{i}{c}\cdot\left(\frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\times\vec{E}\right)\right)$$

Disassemble equation (22) into it's scalar and vectorial components and form newer equations. Assume that magnetic monopoles do not exist.

$$\text{scalar part}\left(\tilde{\nabla}^*\cdot\tilde{G}^*\right)=\text{scalar part}\left(\mu_0\cdot\tilde{J}\right) \tag{23}$$

Let's use the relationship between the speed of light in vacuum, the vacuum permeability and vacuum permittivity:

$$\frac{1}{c^2}=\mu_0\cdot\varepsilon_0 \tag{24}$$

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0}-\frac{\partial\Lambda}{\partial t} \tag{25}$$

$$\vec{\nabla}\cdot\vec{B}=0 \tag{26}$$

Equation (25) shows that the electric field strength has a source. It's source can be not only the charge density, but also the time-varying magneto scalar potential (Λ).

$$\text{vectorial part}\left(\tilde{\nabla}^*\cdot\tilde{G}^*\right)=\text{vectorial part}\left(\mu_0\cdot\tilde{J}\right) \tag{27}$$

$$\frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\times\vec{E}=0 \tag{28}$$

$$\vec{\nabla}\times\vec{B}=\mu_0\cdot\vec{J}+\frac{1}{c^2}\cdot\frac{\partial\vec{E}}{\partial t}+\vec{\nabla}\Lambda \tag{29}$$

Equation (29) shows that the curl of the magnetic induction generates not only the time-varying electric field strength but also the gradient of the magneto scalar potential.

So far I have dealt with the differential form of the modified Maxwell equations. In the next step, I define their integral form. Let's start from equations (25), (26), (28), (29). According to the Gauss theorem:

$$\int_V \vec{\nabla}\cdot\vec{F}dV=\oint_S \vec{F}\cdot\vec{n}dS \tag{30}$$

According to the Stokes theorem:

$$\int_S \left(\vec{\nabla}\times\vec{F}\right)\cdot\vec{n}d\vec{S}=\oint_C \vec{F}d\vec{r} \tag{31}$$

According to the gradient theorem:

$$\int_C \vec{\nabla}\varphi d\vec{r}=\oint_t \left(\vec{\nabla}\varphi\right)\cdot\vec{v}dt \tag{32}$$

, where

- $\vec{F}$ is a vector potential
- φ is a scalar potential
- $\vec{v}$ is a vector potential

Take the volume integral of both sides of equation (25) We apply the Gauss theorem. Then we arrive at the following equation.

$$\oint_S \vec{E}\cdot\vec{n}dS = \frac{1}{\varepsilon_0}\int_V \rho dV - \frac{\partial}{\partial t}\int_V \Lambda dV \tag{33}$$

Take the volume integral of both sides of equation 26). We apply the Gauss theorem. Then we arrive at the following equation.

$$\oint_S \vec{B}\cdot\vec{n}dS = 0 \tag{34}$$

Take the surface integral of both sides of equation 28). We apply Stokes' theorem. Then we arrive at the following equation.

$$\oint_C \vec{E}d\vec{r} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot\vec{n}dS \tag{35}$$

Take the surface integral of both sides of equation (29) We apply Stokes' theorem. Then we arrive at the following equation.

$$\mu_0\int_S \vec{J}\cdot\vec{n}\mathrm{d}S + \frac{1}{c^2}\cdot\frac{\partial}{\partial t}\int_S \vec{E}\cdot\vec{n}dS + \int_S \vec{\nabla}\Lambda\vec{n}dS = \oint_C \vec{B}d\vec{r} \tag{36}$$

With this, I derived the integral form of the modified Maxwell equations.

5.2. The Lorentz force

The quaternion Lorentz force can be written from the product of two quaternions: the $\tilde{v}$ velocity quaternion and the complex conjugate of the $\tilde{G}$ quaternion.

$$\tilde{F} = q\cdot\tilde{v}\cdot\tilde{G}^* = \mathrm{q}\cdot(i\cdot c,\vec{v})\left(-\Lambda,-\frac{i}{c}\cdot\vec{E}+\vec{B}\right) = \left(i\cdot\frac{P}{c},\vec{F}\right)$$
$$\tilde{F} = \left(-i\cdot c\cdot\Lambda + \frac{i}{c}\cdot\vec{v}\cdot\vec{E} - \vec{v}\cdot\vec{B}, -\Lambda\cdot\vec{v} + \vec{E} + i\cdot c\cdot\vec{B} + \vec{v}\times\left(-\frac{i}{c}\cdot\vec{E}+\vec{B}\right)\right) \tag{37}$$

, where q the charge, $\vec{v}$ the velocity vector, P the power, $\vec{F}$ the Lorentz force. The velocity vector and the magnetic induction are perpendicular to each other, therefore $\vec{v}\cdot\vec{B} = 0$. Let's disassemble equation (37) into it's scalar and vectorial components and form newer equations.

$$\text{scalar part}\left(\tilde{F}\right) = \text{scalar part}\left(i \cdot \frac{P}{c}, \vec{F}\right) \tag{38}$$

$$P = q \cdot \left(\vec{v} \cdot \vec{E} - \Lambda \cdot c^2\right) \tag{39}$$

$$\text{vectorial part}\left(\tilde{F}\right) = \text{vectorial part}\left(i \cdot \frac{P}{c}, \vec{F}\right) \tag{40}$$

At equation (40) we assume only real force components, we equate the imaginary ones with zero. This can be used to express $\vec{B}$ as the function of $\vec{E}$. Then we get the following equation:

$$\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E} \tag{41}$$

$$\vec{F} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B} - \Lambda \cdot \vec{v}\right) \tag{42}$$

The following authors reached a similar result:

- (Arbab, Extended electrodynamics and its consequences, 2017)
- (Arbab & Satti, On the Generalized Maxwell Equations and Their Prediction of ElectroscalarWave, 2009)
- (van Vlaenderen & Waser, 2001)
- (Waser, 2015)

A force component $-q \cdot \Lambda \cdot \vec{v}$ appears, which is velocity dependent and has a negative sign. This force component is characteristics of viscous fluids, bit it exclusively the work of Λ and can also occur in a vacuum The liquid (ether) had once been introduced into which electromagnetic radiation could have spread, and was later discarded (Arbab, Do we need to modify Maxwell's equations?, 2020). According to the current experimental results, if a point charge moves with speed $\vec{v}$ in an external magnetic field $\vec{B}$ and the velocity vector and the magnetic induction are perpendicular to each other, then its trajectory will be circular. Let’s start from equation (42). Let’s neglect the electric field strength $\vec{E}$ and let the velocity and the magnetic induction vector be perpendicular to each other. The force on the point charge should be equal to the centripetal force. We write their magnitudes in place of vectorial quantities. Then we arrive the following equation:

$$m \cdot \frac{v^2}{r} = q \cdot v \cdot B - q \cdot \Lambda \cdot v \tag{43}$$

Simplifying the above equation to the speed and rearranging it to the radius (r) of the circular path, we arrive at the following relationship:

$$r = \frac{m}{q \cdot (B - \Lambda)} \cdot v \tag{44}$$

We conclude that the radius of the circular orbit will be larger in the presence of the magneto scalar field Λ than without it.

5.3. The continuity equation

One consequence of the Maxwell equations is the charge conservation law (continuity equation). The quaternion variant of this law can be written with the product of two quaternion quantities: the $\tilde{J}$ current density quaternion and the $\tilde{\nabla}$ Nabla quaternion. We will see that two additional equations are added to the continuity equation.

$$\tilde{\nabla}\cdot\tilde{J}=\left(\frac{i}{c}\cdot\frac{\partial}{\partial t},\vec{\nabla}\right)\cdot\left(i\cdot c\cdot\rho,\vec{J}\right)= \\ \left(-\left(\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\vec{J}\right),\frac{i}{c}\cdot\left(\frac{\partial\vec{J}}{\partial t}+\vec{\nabla}\rho\cdot c^{2}\right)+i\cdot c\cdot\rho\cdot\left(\vec{\nabla}\times\vec{J}\right)\right) \tag{45}$$

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\vec{J}=0 \tag{46}$$

$$\frac{\partial\vec{J}}{\partial t}+\vec{\nabla}\rho\cdot c^{2}=0 \tag{47}$$

$$\vec{\nabla}\times\vec{J}=0 \tag{48}$$

According to equation (46), if the charge leaves an infinitesimal volume through a given surface, then the amount of charge in the volume decreases. Applying Stokes's theorem to equation (48) we get that the line integral of the current density along a closed curve is considered to be zero. The result obtained is analogous to Kirchhoff's law. The following authors reached a similar result:

- (Arbab & Satti, On the Generalized Maxwell Equations and Their Prediction of ElectroscalarWave, 2009)
- (Waser, 2015)

6. The static magneto scalar field

In this chapter, I examine the solution of the modified Maxwell equations, where the charge density does not change over time, and the presence of constant stationary currents is also allowed. Then the modified Maxwell equations will obviously have a solution where the spatial quantities are constant in time. Take equations (17), (18), (19), then substitute them into equation (29) and use equation (24). Then we get one of the familiar differential equations of magnetostatics.

$$-\nabla^2\vec{A} = \mu_0 \cdot \vec{J} \tag{49}$$

Note that the Coulomb gauge was not used. In other words, the neglect of the Lorenz gauge has a much deeper meaning than previously thought. Following the derivation of the Biot-Savart law, we already know the shape of the $\vec{A}$ vector potential:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4 \cdot \pi} \int_V \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV' = \frac{\mu_0 \cdot I}{4 \cdot \pi} \oint_C \frac{d\vec{l}'}{|\vec{r} - \vec{r}'|} \tag{50}$$

Substitute equation (19) into equation (29). Since the physical quantities do not change in time, we obtain the following equation:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}\vec{\nabla} \cdot \vec{A} - \nabla^2\vec{A} = \mu_0 \cdot \vec{J} + \vec{\nabla}\Lambda \tag{51}$$

Using equation (49) the term $\mu_0 \cdot \vec{J}$ can be dropped. Integrating both sides of the above equation and choosing the integration constant as 0, we arrive at the following relationship:

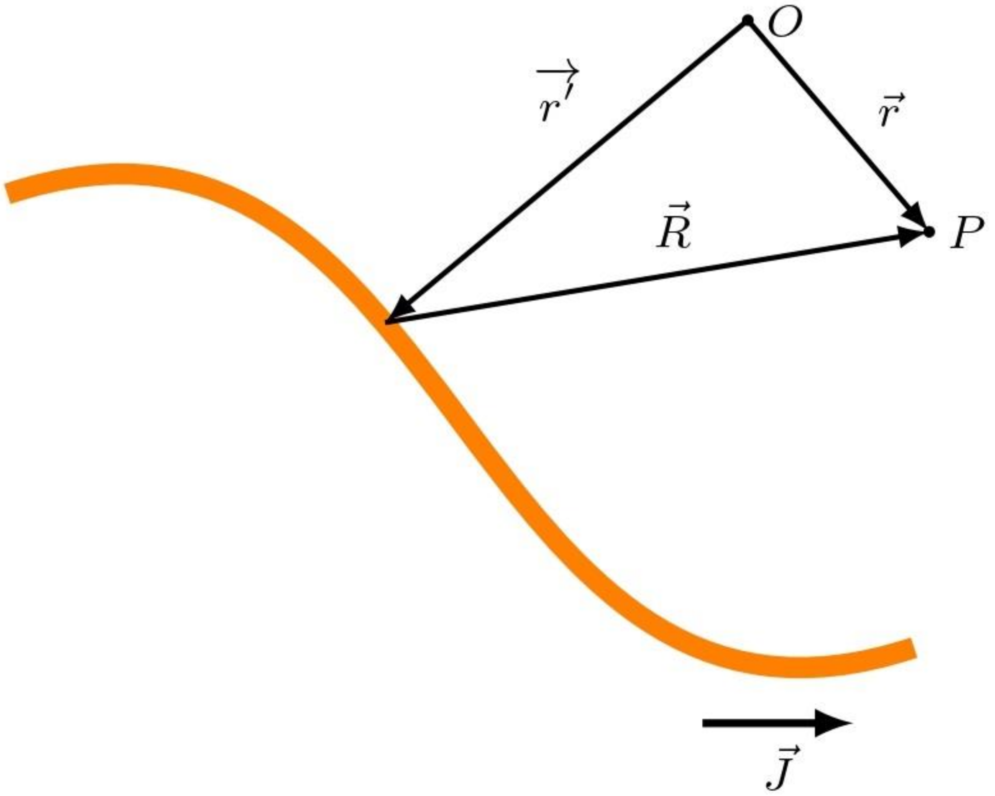
$$\Lambda = \vec{\nabla} \cdot \vec{A} \tag{52}$$

According to equation (52), the divergence of the vector potential $\vec{A}$ must now be taken to obtain the scalar potential Λ. We can live with the freedom to put the differential operator behind the integration. Notice that we are trying to derive a scalar and a vector function. We can use the following identity for this:

$$\vec{\nabla} \cdot (\varphi \cdot \vec{a}) = \vec{a} \cdot \vec{\nabla}\varphi + \varphi\vec{\nabla} \cdot \vec{a} \tag{53}$$

$$\Lambda(\vec{r}) = \frac{\mu_0}{4 \cdot \pi} \int_V \frac{\vec{\nabla} \cdot \vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV' - \frac{\mu_0}{4 \cdot \pi} \int_V \vec{J}(\vec{r}') \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} dV' \tag{54}$$

$$\vec{R} = \vec{r} - \vec{r}' \quad R = |\vec{r} - \vec{r}'| \tag{55}$$



1. Figure

In addition to the magnetic and electric fields, a third field (called the magneto scalar field) appears, which together form the electro-magnetic-scalar field. That can be said about $\vec{\nabla} \cdot \vec{J}$, that is different from zero if and only if $\vec{J}$ is non stationary. Since we are now examining stationary $\vec{J}$ current density, $\vec{\nabla} \cdot \vec{J}$ can be ruled out. We may need to calculate the magneto scalar field of a thin straight current-carrying conductor. Then vector $\vec{\mathrm{R}}$ and its absolute value change little when integrated over the cross section of the conductor located along the curve C (Figure 1). After the appropriate transformations, the surface integral of $\vec{\mathrm{J}}$ gives the current I flowing in the conductor. In this case, we get a line integral for Λ.

$$\Lambda = -\frac{\mu_0}{4\pi}\int_S \vec{\mathrm{J}}_S \cdot \frac{\vec{\mathrm{R}}}{R^3} d\vec{\mathrm{S}}' = -\frac{\mu_0 I}{4\pi}\oint_C \frac{\vec{\mathrm{R}}}{R^3} d\vec{\mathrm{l}}' = -\frac{\mu_0 I}{4\pi}\oint_C \frac{\hat{R}}{R^2} d\vec{\mathrm{l}}' \tag{56}$$

, where $\vec{\mathrm{J}}_S$ the surface current density vector, I the current, $\hat{R} = \vec{\mathrm{R}}/\mathrm{R}$ the normal vector. Consider a straight conductor of length "a", in which current I flows in the direction of the z-axis. Determine the magneto scalar field it creates in cylindrical coordinates.

$$d\vec{l}' = \vec{k} \cdot dz' \tag{57}$$

$$\vec{r} = \vec{\imath} \cdot x + \vec{\jmath} \cdot y + \vec{k} \cdot z = \vec{e}_\rho \cdot \rho \cdot cos\varphi + \vec{e}_\varphi \cdot \rho \cdot sin\varphi + \vec{k} \cdot z \tag{58}$$

$$\vec{r}' = \vec{k}z' \tag{59}$$

$$R=|\vec{r} - \vec{r}'|=\sqrt{\rho^2 \cdot cos^2\varphi + \rho^2 \cdot sin^2\varphi + (z - z')^2} = \sqrt{\rho^2 + (z - z')^2} \tag{60}$$

$$\begin{gathered}\Lambda = -\frac{\mu_0}{4\pi}\int_S \vec{J}_S \frac{\vec{R}}{R^3} d\vec{S}' = \\ -\frac{\mu_0 I}{4\pi}\int_0^a \frac{\vec{k} \cdot \left(\vec{e}_\rho \cdot \rho \cdot cos\varphi + \vec{e}_\varphi \cdot \rho \cdot sin\varphi + \vec{k} \cdot (z - z')\right)}{\left(\rho^2 + (z - z')^2\right)^{\frac{3}{2}}} dz' \\ = -\frac{\mu_0 I}{4\pi}\int_0^a \frac{(z - z')}{\left(\rho^2 + (z - z')^2\right)^{\frac{3}{2}}} dz' = \frac{\mu_0 I}{4\pi}\left(\frac{1}{\rho} - \frac{1}{\sqrt{\rho^2 + a^2}}\right)\end{gathered} \tag{61}$$

Equation (50) is suitable for determining the potential, but in the case of a complex $\vec{J}$ function, due to both practical and illustrative reasons, it is customary to use an approximation procedure (multipolar series decomposition). The nature of electromagnetic radiation depends on the system of charges or currents that create it. In case of a complex system, it's direct definition is difficult and sometimes impossible. Since the Maxwell equations are linear, the electric and magnetic fields depend linearly on the distribution of their source. This linearity offers us the possibility to decompose the radiation (more precisely the structure of the source) into the sum of moments of increasing complexity (using the principle of superposition). Because the electromagnetic field is more strongly dependent on lower-order moments than on higher-order ones, the electromagnetic field can be approximated without a detailed knowledge of it's source. Suppose that the corresponding $\vec{J}$ currents are localized, that is, they disappear outside a finite volume. Take the origin within this range. To determine each member of the series, Let's take the Taylor series of the following function around the of location of $\vec{r}' = 0$ (i.e. r=$|\vec{r}| \gg$ r'=$|\vec{r}'|$):

$$\begin{gathered}\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{\left(r^2 - 2\vec{r} \cdot \vec{r}' + r'^2\right)^{\frac{1}{2}}} = \frac{1}{r\left(1 + \frac{r'^2}{r^2} - 2 \cdot \frac{r'}{r} \cdot cos\vartheta'\right)^{\frac{1}{2}}} = \\ \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(cos\vartheta')\end{gathered} \tag{62}$$

$$cos\vartheta' = \frac{\vec{r}}{|\vec{r}|} \cdot \frac{\vec{r}'}{|\vec{r}'|} = \hat{r} \cdot \hat{r}' \tag{63}$$

, where $P_n(cos\vartheta')$ are the Legendre polynomials. Far enough from the localized currents $\vec{J}$ we can get a good approximation to $\vec{A}$ by taking the first (or the first two) non-disappearing term. To do this, use the equation (50). Then we obtain the magnetic monopole, dipole (and quadrupole) moments for the vector potential $\vec{A}$.

$$\vec{A}=\frac{\mu_0}{4\pi}\cdot\frac{1}{r}\int_V \vec{J}(\vec{r}')dV'$$
$$+\frac{\mu_0}{4\pi}\cdot\frac{1}{r^2}\int_V \vec{J}(\vec{r}')\cdot r'\cdot cos\vartheta' dV'$$
$$+\frac{1}{r^3}\int_V \vec{J}(\vec{r}')\cdot r'^2\left(\frac{3}{2}\cdot cos^2\vartheta' - \frac{1}{2}\right)dV' + \cdots \tag{64}$$
$$=\frac{\mu_0}{4\pi}\cdot\frac{1}{r}\int_V \vec{J}(\vec{r}')dV' +$$
$$\frac{\mu_0}{4\pi}\cdot\frac{\hat{r}}{r^2}\int_V \vec{J}(\vec{r}')\cdot\vec{r}'dV +$$
$$\frac{\mu_0}{4\pi}\cdot\frac{1}{r^3}\int_V \vec{J}(\vec{r}')\cdot\left(\frac{3}{2}(\hat{r}\cdot\vec{r}')^2-\frac{1}{2}\cdot r'^2\right)dV' + \cdots$$

Since there are no magnetic monopoles, we can expect this term to fall out. The dipole term will then dominant, which we will use in our calculations in the future. After some derivation we get the following expression:

$$\vec{A}_D(\vec{r}) = -\frac{\mu_0}{8\cdot\pi}\cdot\frac{\hat{r}}{r^2}\times\int_V \vec{r}'\times\vec{J}(\vec{r}')dV' = \frac{\mu_0}{4\cdot\pi}\cdot\frac{\vec{m}\times\hat{r}}{r^2} \tag{65}$$

, where $\vec{m}$ is the magnetic dipole momentum vector. To obtain the magneto scalar potential of the dipole, all we have to do is take the divergence of the vector potential $\vec{A}_D$ according to equation (52).

$$\Lambda_D(\vec{r}) = \vec{\nabla}\cdot\vec{A}_D = \frac{\mu_0}{4\cdot\pi}\cdot\frac{(\vec{\nabla}\times\vec{m})\cdot\hat{r}}{r^2} = \frac{\mu_0}{4\cdot\pi}\int_V \vec{J}(\vec{r})_m\cdot\frac{\vec{r}}{r^3}dV \tag{66}$$

By definition, the magnetic moment vector $\vec{m}$ is the volume integral of the magnetization vector $\vec{M}$. The curl of the magnetization vector gives the magnetization current density $\vec{J}_M$. Using these two definitions, we arrive at relation (66). Let’s compute $\vec{\nabla}\times\left(\vec{r}'\times\vec{J}(\vec{r}')\right)$:

$$\vec{\nabla}\times\left(\vec{r}'\times\vec{J}(\vec{r}')\right) = \left(\vec{\nabla}\cdot\vec{J}(\vec{r}')\right)\cdot\vec{r}' - \left[\vec{r}'\cdot\vec{\nabla}\right]\cdot\vec{J}(\vec{r}') - \left(\vec{\nabla}\cdot\vec{r}'\right)\cdot\vec{J}(\vec{r}') + \left[\vec{J}(\vec{r}')\cdot\vec{\nabla}\right]\cdot\vec{r}' \tag{67}$$

- The first and the second terms represents the $\Lambda_{non-stationary}(\vec{r})$ case.
- The third and the fourth terms represents the $\Lambda_{stationary}(\vec{r})$ case.
- The stationary case in terms of static magneto scalar field means $\vec{\nabla}\cdot\vec{J}=0$ and $\partial\rho/\partial t=0$.
- When we have a charge source/sink in case of static magneto scalar field means $\vec{\nabla}\cdot\vec{J}\neq 0$, but $\partial\rho/\partial t=0$.

7. The magneto scalar waves

In this chapter, in the modified Maxwell equations, I take into account the change of field quantities over time, and I show that it follows from the modified Maxwell theory that the electromagnetic and magneto scalar fields propagate in the form of waves. The task is thus given: to derive the inhomogeneous wave equations for $\vec{E}$, $\vec{B}$ and Λ. Let's start from connections (24), (25) and (29). Derive equation (25) with respect to time, form the divergence of both sides of equation (26) and use equation (24). We add relations (25) and (29) and obtain the inhomogeneous wave equation for Λ.

$$\frac{1}{c^2}\cdot\frac{\partial^2 \Lambda}{\partial t^2}-\vec{\nabla}\cdot\vec{\nabla}\Lambda=\mu_0\cdot\left(\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\vec{J}\right) \tag{68}$$

The above equation has a double meaning. On the one hand, charges can be sources and sinks of magneto scalar waves, and on the other hand, magneto scalar waves can be sources and sinks of charges. According to the equation, the charge is a locally non-conserving quantity.

Let's continue the derivation with the inhomogeneous wave equation for $\vec{E}$. Let's start from equations (24), (25), (28) and (29). We form the time derivative of equation (29) Let's form the rotation of equation (28) Substitute equation 28) into equation (29). We form the gradient of both sides of equation (25). Equation (25) is added to equation (29). Let's use relation (24).

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{E}}{\partial t^2}-\nabla^2\vec{E}=-\frac{1}{\varepsilon_0}\cdot\left(\frac{1}{c^2}\cdot\frac{\partial\vec{J}}{\partial t}+\vec{\nabla}\rho\right) \tag{69}$$

Finally, the derivation of the inhomogeneous wave equation for $\vec{B}$ should follow. Let's start from equations (24), (26), (28) and (29). We form the rotation of equation (29). We form the time derivative of equation (28) Form the gradient of equation (26) Equation (28) is multiplied by equation (24). Subtract equation (26) from equation (29). Equations (28) and (29) are added together.

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{B}}{\partial t^2}-\nabla^2\vec{B}=\mu_0\cdot\left(\vec{\nabla}\times\vec{J}\right) \tag{70}$$

If we take the right side of the three equations as 0 (assuming no charges and currents), we get back the generalized continuity equations, along with the homogeneous wave equations. Many solutions to homogeneous wave equations are known, let us confine ourselves to the solution of the plane wave for the time being.

Longitudinal waves can occur elastic media. The analogies between electromagnetism and elastomechanics have been intensely discussed, and it was a great surprise when it was discovered that electromagnetic waves could not be longitudinal, only transverse.

This result is due to the fact that electromagnetic waves must satisfy not only the wave equation (like elastomechanical waves) but also the maxwell equations. The following plane and sphere waves are solutions to the homogenous wave equations:

$$\vec{E} = \vec{E}_0 \cdot e^{i\cdot(\omega\cdot t - \vec{k}\cdot\vec{r})} \quad \vec{E} = \frac{\vec{E}_0 \cdot e^{i\cdot(\omega\cdot t - \vec{k}\cdot\vec{r})}}{r} \tag{71}$$

$$\vec{B} = \vec{B}_0 \cdot e^{i\cdot(\omega\cdot t - \vec{k}\cdot\vec{r})} \quad \vec{B} = \frac{\vec{B}_0 \cdot e^{i\cdot(\omega\cdot t - \vec{k}\cdot\vec{r})}}{r} \tag{72}$$

$$\Lambda = \Lambda_0 \cdot e^{i\left(\omega\cdot t - \vec{k}\cdot\vec{r}\right)} \quad \Lambda = \frac{\Lambda_0 \cdot e^{i\left(\omega\cdot t - \vec{k}\cdot\vec{r}\right)}}{r} \tag{73}$$

$$\vec{k} = \frac{\omega}{c} \cdot \hat{k} \tag{74}$$

, where

- $\vec{E}_0$, $\vec{B}_0$, $\Lambda_0$the wave amplitudes
- ω the circular frequency
- $\vec{k}$ the wavenumber vector (points in the direction of the normal of the plane wave)
- $\hat{k}$ the plane/sphere wave normal vector

In the next step, I derive the relationship between the electric field strength and the magneto scalar potential. Let's start from equation (25) and assume a vacuum, and let $u = \omega \cdot t - \vec{k} \cdot \vec{r}$ the phase of the plane wave.

$$\vec{\nabla} \cdot \vec{E} = \sum_{j=1}^{3} \frac{\partial E_j}{\partial x_j} = \sum_{j=1}^{3} \frac{dE_j}{du} \cdot \frac{\partial u}{\partial x_j} = -\frac{\omega}{c} \sum_{j=1}^{3} \frac{dE_j}{\omega dt} \cdot \hat{k}_j = -\frac{1}{c} \cdot \frac{\partial}{\partial t}\left(\vec{E} \cdot \hat{k}\right) \tag{75}$$

Then we arrive at the following relation:

$$\frac{1}{c} \cdot \hat{k} \cdot \vec{E} = \Lambda \tag{76}$$

In this case, the electrical waves can be considered longitudinal. The possible existence of such a wave may be subject to experimentation, so the theory can be tested (van Vlaenderen & Waser, Generalisation of classical electrodynamics to admit a scalar field and longitudinal waves, 2001). The electric field strength vector included in the modified Maxwell equations is in some cases transversely (T) and in other cases longitudinally (L) polarized. This leads to inconsistency in notation. In the future, I will call attention to this and distinguish between the two.

In the next step, let's see what shape the time-dependent scalar and vector potential will have. Take equations (17) and (18) and substitute them into equation (25). Finally, we form the (-1) times of equation (25). Then we get the following familiar differential equation for φ:

$$\frac{1}{c^2} \cdot \frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi = \frac{\rho}{\varepsilon_0} \tag{77}$$

Following classical electrodynamics, we already know the shape of φ scalar potential:

$$\varphi(\vec{r},t) = \frac{1}{4\cdot\pi\cdot\varepsilon_0}\int_V \frac{\rho(\vec{r}',t')}{|\vec{r}-\vec{r}'|}dV' \tag{78}$$

Take equations (17), (18) and (19), then substitute them into equation (29). Finally, let's form the (-1) times of equation (29). Then we get the following familiar differential equation for $\vec{A}$:

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{A}}{\partial t^2} - \nabla^2\vec{A} = \mu_0\cdot\vec{J} \tag{79}$$

Following classical electrodynamics, we already know the shape of $\vec{A}$ vector potential:

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\cdot\pi}\int_V \frac{\vec{J}(\vec{r}',t')}{|\vec{r}-\vec{r}'|}dV' \tag{80}$$

The retarded time coordinate t' can be expressed as follows:

$$t' = t - \frac{|\vec{r}-\vec{r}'|}{c} \tag{81}$$

Note that I did not use the Lorenz gauge.

Now let's turn to the time-dependent magneto scalar potential, which can be defined in the form of equation (17). For this we can use the definition (78) of the time-dependent scalar potential and the definition (80) of the time-dependent vector potential.

$$\begin{gathered}\Lambda(\vec{r},t) = \\ \frac{\mu_0}{4\cdot\pi}\int_V \frac{\vec{\nabla}\cdot\vec{J}(\vec{r}',t')}{|\vec{r}-\vec{r}'|}dV' - \frac{\mu_0}{4\cdot\pi}\int_V \vec{J}(\vec{r}',t')\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}dV' \\ + \frac{\mu_0}{4\cdot\pi}\int_V \frac{\frac{\partial\rho(\vec{r}',t')}{\partial t}}{|\vec{r}-\vec{r}'|}dV'\end{gathered} \tag{82}$$

Summarising the results:

1. The inhomogeneous wave equation for Λ expresses that the charges can transform into a magneto scalar field and vice versa. That is, the charges are withdrawn from the conductor.
2. Examining equation (82) alone, we can draw the conclusion that the magneto scalar field consists not only of the removal of charges, but also of their flow.
3. If the left side of the inhomogeneous wave equation for Λ is set equal to 0, then charge conservation will be fulfilled.
4. If the right-hand side of the inhomogeneous wave equation for Λ is set equal to 0, we will obtain the freely propagating magneto scalar waves without a source.
5. When we have a charge source/sink in case of time-dependent magneto scalar field means $\vec{\nabla}\cdot\vec{J} \neq 0$ and $\partial\rho/\partial t \neq 0$.

In a system here the charge density and the current density change over time, the method of separating the time-dependent and the space-dependent parts is often used. Assumed a periodic time dependence:

$$\rho(\vec{r},t) = \rho(\vec{r}) \cdot e^{-i\omega t} \tag{83}$$

$$\vec{J}(\vec{r},t) = \vec{J}(\vec{r}) \cdot e^{-i\omega t} \tag{84}$$

Let us assume that potentials and fields have similar time dependences. Then the space-dependent part of the scalar potential changes to the following form:

$$\varphi(\vec{r}) = \frac{1}{4 \cdot \pi \cdot \varepsilon_0} \int_V \rho(\vec{r}') \cdot \frac{e^{i \cdot k \cdot |\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV' \tag{85}$$

The space-dependent part of the vector potential changes to the following shape:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4 \cdot \pi} \int_V \vec{J}(\vec{r}') \cdot \frac{e^{i \cdot k \cdot |\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV' \tag{86}$$

After reviewing the time-dependent scalar and vector potential, in this section, in a special but very important approximation, I specifically define the multipole radiation created by the monopole and the dipole moments. The multiple radiation is a theoretical toolkit that perfectly characterizes the electromagnetic of the gravitational radiation from time-varying sources. The techniques used to study the multipole radiation are somewhat similar to those used to study static sources. However, there are significant differences in the details of the analysis because time-dependent fields behave differently than static ones. The field from the multipolar momentum depends on both the distance from the origin and the angular direction of an evaluation point relative to the coordinate system. Depending on the size of the source, the wavelength of the radiation, and the distance from the origin, three zones are distinguished: near field, middle field and far field. In the near field, the distance from the source is much smaller than the wavelength (i.e. $\lambda \gg r$). Then the field behaves quasi-statically and $k \cdot r \ll 1$. In the middle field, the distance from the source is proportional to the wavelength (i.e. $\lambda \approx r$). We don't talk much about the middle field, because it is very complex. In the far field, the distance from the source is much larger than the wavelength. ($i.e. \lambda \ll r$). Let's stick to the far field radiation zone. Then we can use the following approximation:

$$\frac{e^{i \cdot k \cdot |\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \approx \frac{e^{i \cdot k \cdot r}}{r} \cdot \left(1 + \left(\frac{1}{r} - ik\right) \cdot \hat{r} \cdot \vec{r}'\right) \tag{87}$$

In classical electrodynamics, the amount of charge cannot change over time, the field created by the monopole member must be static. However, equation (68) clearly states that the charges transform into a magneto scalar field and vice versa. So the first time-varying moment must be concentrated on the monopole, i.e. the first member of the bracketed part.

$$\varphi_M(\vec{r}) = \frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{e^{i\cdot k\cdot r}}{r}\int_V \rho(\vec{r}')dV' = \frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{e^{i\cdot k\cdot r}}{r}\cdot q \tag{88}$$

Here q is the electric charge. In order to obtain the time-dependent magneto scalar potential of the monopole, all we have to do is proceed according to equation (17). Since according to equation (26) there are no magnetic monopoles, therefore $\vec{\nabla}\cdot\vec{A}_M(\vec{r},t)=0$. With this, we obtained the time-dependent magneto scalar potential Λ in the monopole approximation.

$$\vec{\nabla}\cdot\vec{A}_M(\vec{r},t)=0 \tag{89}$$

$$\frac{\partial\varphi_M(\vec{r},t)}{\partial t} = -i\cdot\omega\cdot\frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{e^{i\cdot k\cdot r-i\omega t}}{r}\cdot q \tag{90}$$

$$\Lambda_M(\vec{r},t) = \vec{\nabla}\cdot\vec{A}_M(\vec{r},t) + \frac{1}{c^2}\cdot\frac{\partial\varphi_M(\vec{r},t)}{\partial t} = -i\cdot\omega\cdot\frac{\mu_0}{4\cdot\pi}\cdot\frac{e^{i\cdot k\cdot r-i\omega t}}{r}\cdot q \tag{91}$$

In the dipole approximation, I would be satisfied with the second term of the series expansion.

$$\vec{\mathrm{A}}_D = \frac{\mu_0}{4\pi}\cdot e^{-i\cdot\omega\cdot t}\cdot\frac{e^{i\cdot k\cdot r}}{r}\cdot\left(\frac{1}{r}-ik\right)\cdot\vec{m}\times\hat{r} \tag{92}$$

$$\begin{aligned}\varphi_D(\vec{r}) &= \frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{e^{i\cdot k\cdot r}}{r}\cdot\left(\frac{1}{r}-ik\right)\cdot\hat{r}\int_V \rho(\vec{r}')\cdot\vec{r}^{\,'}dV' = \\ &\frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{e^{i\cdot k\cdot r}}{r}\cdot\left(\frac{1}{r}-ik\right)\cdot\hat{r}\cdot\vec{p}\end{aligned} \tag{93}$$

, where $\vec{p}$ is the electric dipole moment vector. In order to obtain the time-dependent magneto scalar potential of the dipole, all we have to do is proceed according to equation (17).

$$\vec{\nabla}\cdot\vec{\mathrm{A}}_D = \frac{\mu_0}{4\pi}\cdot e^{-i\cdot\omega\cdot t}\cdot\frac{e^{i\cdot k\cdot r}}{r}\cdot\left(\frac{1}{r}-ik\right)\cdot(\vec{\nabla}\times\vec{\mathrm{m}})\cdot\hat{r} \tag{94}$$

$$\frac{\partial\varphi_D(\vec{r},t)}{\partial t} = -i\cdot\omega\cdot\frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\frac{e^{i\cdot k\cdot r-i\omega t}}{r}\cdot\left(\frac{1}{r}-ik\right)\cdot\hat{r}\cdot\vec{p} \tag{95}$$

$$\begin{aligned}\Lambda_D(\vec{r},t) &= \vec{\nabla}\cdot\vec{A}_D(\vec{r},t) + \frac{1}{c^2}\cdot\frac{\partial\varphi_D(\vec{r},t)}{\partial t} = \\ &\frac{\mu_0}{4\pi}\cdot\frac{e^{i\cdot k\cdot r-i\omega t}}{r}\cdot\left(\frac{1}{r}-ik\right)\cdot\left[(\vec{\nabla}\times\vec{\mathrm{m}})-i\cdot\omega\cdot\vec{p}\right]\cdot\hat{r}\end{aligned} \tag{96}$$

8. Poynting theorem

In the field of electrodynamics Poynting's theorem expresses the energy conservation of an electromagnetic field developed by British physicist John Henry Poynting. Poynting's theorem is analogous to the work of classical mechanics and is mathematically similar to the continuity equation. Let's start with Newton's second axiom and suppose that all the forces acting on a given point charge are exactly the Lorentz force. Let's introduce the material density ρ_m and the charge density ρ. Use the equation $\vec{J} = \rho \cdot \vec{v}$ to create a relationship between current density and density. Multiply both sides of the equation by $\vec{v}$. Since $\left(\vec{v} \times \vec{B}\right) \cdot \vec{v} = 0$, we get the following equation:

$$\frac{d}{dt}\left(\frac{1}{2} \cdot \rho_m \cdot \vec{v}^2\right) = \vec{J} \cdot \vec{E} \tag{97}$$

The total work performed by the fields for a continuous charge and current distribution in a finite volume:

$$\int_V \vec{J} \cdot \vec{E} dV \tag{98}$$

This power characterizes the conversion of electromagnetic energy into mechanical or thermal energy. In the other pan in the balance must therefore have the same rate of decrease in the energy of the electromagnetic field. In order to write the explicit form of the law, we express it in another form based on the Maxwell equations. $\vec{J}$ can be eliminated using equation (29).

$$\begin{gathered}\int_V -\vec{J} \cdot \vec{E} dV = \\ \int_V \left(-\frac{1}{\mu_0} \cdot \vec{E} \cdot \vec{\nabla} \times \vec{B} + \frac{1}{\mu_0} \cdot \vec{E} \cdot \vec{\nabla} \Lambda + \varepsilon_0 \cdot \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}\right) dV \\ = \int_V \left(-\frac{1}{\mu_0} \cdot \vec{E} \cdot \vec{\nabla} \times \vec{B} + \frac{1}{\mu_0} \cdot \vec{E} \cdot \vec{\nabla} \Lambda + \frac{d}{dt}\left(\frac{1}{2} \cdot \varepsilon_0 \cdot \vec{E}^2\right)\right) dV\end{gathered} \tag{99}$$

Multiply equation (25) by Λ/μ_0 and take the volumetric integral on both sides.

$$\int_V (\Lambda \cdot \rho \cdot c^2) dV = \int_V \left(\frac{1}{\mu_0} \cdot \Lambda \cdot \vec{\nabla} \cdot \vec{E} + \frac{d}{dt}\left(\frac{1}{2} \cdot \frac{1}{\mu_0} \cdot \Lambda^2\right)\right) dV \tag{100}$$

Multiply equation (28) by $\vec{B}/\mu_0$ and take the volumetric integral on both sides.

$$-\int_V \left(\frac{d}{dt}\left(\frac{1}{2} \cdot \frac{1}{\mu_0} \cdot \vec{B}^2\right)\right) dV = \int_V \left(\frac{1}{\mu_0} \cdot \vec{B} \cdot \vec{\nabla} \times \vec{E}\right) dV \tag{101}$$

Add equations (99), (100), (101), substitute equation (97) and use the identities below:

$\vec{\nabla}\cdot\left(\vec{E}\times\vec{B}\right)=\vec{B}\cdot(\vec{\nabla}\times\vec{E})-\vec{E}\cdot(\vec{\nabla}\times\vec{B})$	(102)
$\vec{\nabla}\cdot\left(\Lambda\cdot\vec{E}\right)=\left(\vec{\nabla}\Lambda\right)\cdot\vec{E}+\Lambda\cdot\vec{\nabla}\cdot\vec{E}$	(103)

Then we come to the following relation:

$$\int_V \left(\frac{\vec{\nabla}\cdot\left(\vec{E}\times\vec{B}+\Lambda\cdot\vec{E}\right)}{\mu_0}\right)dV+\int_V\left(+\frac{d}{dt}\left(\frac{1}{2}\cdot\rho_m\cdot\vec{v}^2+\frac{1}{2}\cdot\varepsilon_0\cdot\vec{E}^2+\frac{1}{2}\cdot\frac{1}{\mu_0}\cdot\vec{B}^2+\frac{1}{2}\cdot\frac{1}{\mu_0}\Lambda^2\right)\right)dV=\int_V(\Lambda\cdot\rho\cdot c^2)dV \tag{104}$$

In terms of the result (Arbab, Modified electrodynamics for London's superconductivity, 2017), came to a similar conclusion with the difference that I did not use the generalized gauge transformation he invented. Let u_{EMS} be the total energy density, $\vec{S}_{EMS}$the Poynting vector describing the energy flow, Ω the rate at which the field does the work on the charged particles. Since the volume V can be chosen as desired, the above relation can be transformed into a differential equation:

$$\vec{\nabla}\cdot\vec{S}_{EMS}+\frac{du_{EMS}}{dt}=\Omega \tag{105}$$

The Poynting vector is the directed energy flow of the electromagnetic field (the energy transfer is per unit area and time). It can be seen that the direction of energy propagation does not coincide with the direction of propagation of the waves. This phenomenon is typical of anisotropic media, however, it is solely the work of Λ and can also occur in a vacuum. Notice that Λ has an energy density, i.e. it can be considered as a real physical object. Furthermore, if Λ=0, we get back the well-known energy conservation for electromagnetic radiation. The physical meaning of the integral and differential pointing theorems is as follows: the rate of the energy transfer (per unit volume) from a region of space is equal to the rate of work done on the charge distribution, plus the energy flow leaving the region. A few lines above I came to the conclusion (here), that the direction of propagation of energy is not the same as the direction of propagation of waves. $\vec{E}$ belonging to the second term of the Poynting vector is longitudinally polarized, for which equation (76) can be used. If considered

$$\vec{S}_{EMS}=\frac{\vec{E}\times\vec{B}}{\mu_0}+\frac{1}{\mu_0}\cdot\frac{1}{c}\cdot\hat{k}\cdot\vec{E}\cdot\vec{E} \tag{106}$$

I would like to point out again that this designation of electric field strength vector is inconsistent. The electric field strength in the first term of the Poynting vector is transverse. It will become clear later (here) that the second term is longitudinal. We use the appropriate notations (T and L). The relationship between $\vec{B}$ and $\vec{E}$ can be deduced from equation (28):

$$\vec{B}_T = \frac{1}{c} \cdot \hat{k} \times \vec{E}_T \tag{107}$$

Using equations (107) and (76), the following conclusion can be reached:

$$\begin{aligned} \vec{S}_{EMS} &= \frac{1}{\mu_0} \cdot \frac{1}{c} \cdot \vec{E}_T \times \left(\hat{k} \times \vec{E}_T\right) + \frac{1}{\mu_0} \cdot \frac{1}{c} \cdot \hat{k} \cdot \vec{E}_L \cdot \vec{E}_L \\ &= \frac{1}{\mu_0} \cdot \frac{1}{c} \cdot \hat{k} \cdot \left(\left|\vec{E}_T\right|^2 + \left|\vec{E}_L\right|^2\right) \end{aligned} \tag{108}$$

I do not make an equal sign between $\left|\vec{E}_T\right|^2$ and $\left|\vec{E}_L\right|^2$ for now.

9. The constitutive relations in vacuum

To obtain Maxwell's constitutive equations, the relationships between $\vec{D}$ and $\vec{E}$, furthermore $\vec{B}$ and $\vec{H}$ must be determined. In addition, a relationship between Λ and it's pair can be determined by examining the dimensions of the physical quantities. First, we declare the units of measurement of the physical quantities that occurs.

Notation	Physical quantity	SI unit of measure
$\vec{J}$	Current density	$\frac{A}{m^2}$
ρ	Electrical charge density	$\frac{C}{m^3}$
Q	Electrical charge	$A \cdot s = C$
$\vec{B}$	Magnetic induction	$\frac{V \cdot s}{m^2} = T$
$\vec{H}$	Magnetic field strength	$\frac{A}{m}$
$\vec{D}$	Displacement field strength	$\frac{A \cdot s}{m^2} = \frac{Q}{m^2}$
$\vec{E}$	Electric field strength	$\frac{V}{m}$

Table 1

The next step is to declare the (original) Maxwell equations containing $\vec{D}$ and $\vec{H}$.

$$\vec{\nabla} \cdot \vec{D} = \rho \tag{109}$$
$$\vec{\nabla} \cdot \vec{B} = 0 \tag{110}$$
$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0 \tag{111}$$
$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{112}$$

With the constitutive equations we can determine the material's response to the electromagnetic radiation. In real life, they are rarely easy to declare, on the other hand, for small energy values, the relationship is linear with a good approximation. Furthermore, note that the unit of Λ, like $\vec{B}$, is Tesla [T]. By comparing the original and the modified Maxwell equations, we can obtain the constitutive equations. Since vacuum is now assumed and there is no polarization or magnetization, the relative permittivity and permeability will not play a role.

$$\vec{D} = \varepsilon_0 \cdot \vec{E} \tag{113}$$
$$\vec{B} = \mu_0 \cdot \vec{H} \tag{114}$$
$$\Lambda = \mu_0 \cdot \zeta \tag{115}$$

According to equation (115), the magneto scalar radiation interacts with the material. Therefore a receiver antenna can be built in order to intercept this kind of radiation.

10. The Lagrangian density

In this chapter, I derive the modified Maxwell equations from the Lagrangian within the framework of covariant electrodynamics with the restriction that I do not use the Lorenz gauge.

$$x^\alpha = (c \cdot t, \vec{r}) \ \& \ x_\alpha = (c \cdot t, -\vec{r}) \tag{116}$$

$$\partial^\alpha = \frac{\partial}{\partial x_\alpha} = \left(\frac{1}{c} \cdot \frac{\partial}{\partial t}, -\vec{\nabla}\right) \ \& \ \partial_\alpha = \frac{\partial}{\partial x^\alpha} = \left(\frac{1}{c} \cdot \frac{\partial}{\partial t}, \vec{\nabla}\right) \tag{117}$$

$$J^\alpha = (c \cdot \rho, \vec{J}) \tag{118}$$

$$A^\alpha = \left(\frac{\varphi}{c}, \vec{A}\right) \ \& \ A_\alpha = \left(\frac{\varphi}{c}, -\vec{A}\right) \tag{119}$$

$$\eta^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{120}$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{121}$$

$$F^{\alpha\beta} = \eta^{\alpha\mu} \cdot \eta^{\alpha\upsilon} \cdot F_{\mu\upsilon} \tag{122}$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & -B_z & B_y \\ -\frac{E_y}{c} & B_z & 0 & -B_x \\ -\frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix} \tag{123}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix} \tag{124}$$

$$\begin{gathered} L = -\frac{1}{4 \cdot \mu_0} \cdot F_{\alpha\beta} \cdot F^{\alpha\beta} + J^\alpha \cdot A_\alpha - \frac{1}{2 \cdot \mu_0} \cdot (\partial_\alpha A^\alpha)^2 = \\ -\frac{1}{4 \cdot \mu_0} \cdot (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \cdot (\partial^\alpha A^\beta - \partial^\beta A^\alpha) + J^\alpha \cdot A_\alpha - \frac{1}{\mu_0} \cdot \partial^\beta A_\beta \partial_\alpha A^\alpha \\ -\frac{1}{4 \cdot \mu_0} \cdot \left(\partial_\alpha A_\beta \partial^\alpha A^\beta - \partial_\alpha A_\beta \partial^\beta A^\alpha - \partial_\beta A_\alpha \partial^\alpha A^\beta + \partial_\beta A_\beta \partial^\beta A^\alpha\right) \\ \\ +J^\alpha \cdot A_\alpha - \frac{1}{\mu_0} \cdot \partial^\beta A_\beta \partial_\alpha A^\alpha = \end{gathered} \tag{125}$$

$$= -\frac{1}{2 \cdot \mu_0} \cdot \left(\partial_\alpha A_\beta \partial^\alpha A^\beta - \partial_\beta A_\alpha \partial^\alpha A^\beta\right) + J^\alpha \cdot A_\alpha - \frac{1}{\mu_0} \cdot \partial^\beta A_\beta \partial_\alpha A^\alpha$$

, where:

- x^α and x_α are the contravariant and covariant form of the four-vector
- ∂^α and ∂_α are the contravariant and covariant form of the four-partial
- A^α and A_α are the contravariant and covariant form of the four-potential
- $\eta^{\alpha\beta}$ is the metric tensor
- $F^{\alpha\beta}$ and $F_{\alpha\beta}$ are the contravariant and covariant form of the electromagnetic tensor
- L is the Lagrangian

By minimizing the action, we obtain the Euler-Lagrange equation, from which we can obtain the modified versions of Maxwell's equations (109) and (112).

Let $J^\alpha \cdot A_\alpha \to J^\beta \cdot A_\beta$ and $\partial_\alpha A^\alpha \to \partial_\mu A^\mu$.

$$\partial_\alpha \left(\frac{\partial L}{\partial(\partial_\alpha A_\beta)}\right) - \frac{\partial L}{\partial A_\beta} = 0 \tag{126}$$

$$\frac{\partial L}{\partial(\partial_\alpha A_\beta)} = -\frac{1}{\mu_0} \cdot \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right) = -\frac{1}{\mu_0} \cdot F^{\alpha\beta} \tag{127}$$

$$\partial_\alpha \left(\frac{\partial L}{\partial(\partial_\alpha A_\beta)}\right) = -\frac{1}{\mu_0} \cdot \partial_\alpha F^{\alpha\beta} = \frac{1}{\mu_0} \cdot \partial_\alpha F^{\beta\alpha} \tag{128}$$

$$\frac{\partial L}{\partial A_\beta} = J^\beta - \frac{1}{\mu_0} \cdot \partial^\beta \partial_\mu A^\mu \tag{129}$$

$$\partial_\alpha F^{\beta\alpha} = \mu_0 \cdot J^\beta - \partial^\beta \partial_\mu A^\mu \tag{130}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \frac{\partial \Lambda}{\partial t} \tag{131}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{J} + \frac{1}{c^2} \cdot \frac{\partial \vec{E}}{\partial t} + \vec{\nabla} \Lambda \tag{132}$$

It can be seen that the Λ contributions resulting from the abandonment of the Lorenz gauge is the same in quaternion and covariant electrodynamics. The other two Maxwell equations can be derived through the Bianchi identity, I leave the proof to the reader.

$$\partial_\sigma F^{\alpha\beta} + \partial_\alpha F^{\beta\sigma} + \partial_\beta F^{\sigma\alpha} = 0 \tag{133}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{134}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0 \tag{135}$$

The Lagrangian density must meet several criteria to adequately describe the properties of the electromagnetic interaction.

The first such criterion is the number of polarization degrees of freedom. For the full Lagrangian density of Stueckelberg, if γ=k=0 is chosen, it describes the well-known transverse electromagnetic interaction. The electric field strength vector $\vec{E}$ and the magnetic induction $\vec{B}$ basically have 3 polarization degrees of freedom each. So far, there are a total of 6 polarization degrees of freedom. We know that $\vec{k} \cdot \vec{E} = 0$ and $\vec{k} \cdot \vec{B} = 0$, which allows 1-1 polarization degrees of freedom for $\vec{E}$ and $\vec{B}$- separately. We also know that $\vec{E} \cdot \vec{B} = 0$, but this does not change the number of polarization degrees of freedom. The time component in the contravariant and covariant forms of the four vector is not considered a degree of polarization freedom. So the transverse electromagnetic interaction has 2 polarization degrees of freedom. Experimental results confirm this finding. As the next step, we examine what the situation is in the case of γ=1 and k=0. The electric field strength vector $\vec{E}$ and the magnetic induction $\vec{B}$ basically have 3 polarization degrees of freedom each. Since Λ is a scalar function, it has 1 degree of polarization freedom. So far, there are a total of 7 polarization degrees of freedom. We know that $\vec{k} \cdot \vec{E} = 0$, $\vec{k} \cdot \vec{B} = 0$ and $\vec{k} \cdot \vec{E} = \Lambda$, which allows 1 polarization degree of freedom for $\vec{B}$ and 2 for $\vec{E}$. I would like to point out again that this designation of electric field strength vector is inconsistent. The electric field strength vector cannot be transverse and longitudinal at the same time. Λ still has 1 degree of polarization freedom. So the interaction described by the modified Maxwell equations has 4 polarization degrees of freedom.

The second criterion is the range of interaction. The range of the transverse electromagnetic interaction is infinite, because the particles mediating the interaction have no mass. Experimental results confirm this finding. The range of the interaction described by the modified Maxwell equations is also infinite, since the mass will automatically be equal to zero if γ=1 and k=0 is chosen.

The third criterion is invariance to the Lorenz measure. This criterion is fulfilled in the case of the transverse electromagnetic interaction, but not in the case of the interaction described by the modified Maxwell equations, since we chose to omit the Lorenz gauge.

The fourth criterion is invariance to the Lorentz transformation. The Lagrangian density must be invariant to the Lorentz transformation. If the Lagrangian density consists of several components, the components must be Lorentz invariant individually, as well as their sum. If the Lorentz density is invariant to the Lorentz transformation, then the equations of motion derived from them must also be invariant. This statement is also true in reverse.

It can be shown that the Lorentz invariant quantities of the first component of the total Lagrangian density are: $2 \cdot \left(\vec{B}^2 - \frac{\vec{E}^2}{c^2}\right)$ and $-\frac{4}{c} \cdot \vec{B} \cdot \vec{E}$. It can be shown that the Lorentz invariant quantity of the second component of the total Lagrangian density is: Λ^2. Thus, the transverse electromagnetic interaction and the interaction constructed from the modified Maxwell equations are also invariant to the Lorentz transformation.

11. The Lorentz force

In this chapter, I derive the Lorentz force. I start from the Lagrangian density outlined in (here) and check whether it will be the same as in (here) with quaternions.

$$L = L(x_\mu, \dot{x}_\mu) = -\frac{\rho_m \cdot c^2}{\gamma} + J^\upsilon \cdot A_\upsilon - \frac{1}{2 \cdot \mu_0} \cdot (\partial_\upsilon A^\upsilon)^2$$
$$= -\frac{\rho_m \cdot c^2}{\gamma} + \rho_q \cdot \dot{x}^\upsilon \cdot A_\upsilon - \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2 \tag{136}$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{c}{\sqrt{-\dot{x}_\mu \cdot \dot{x}^\mu}} \tag{137}$$

$$\dot{x}^\mu = (c, \vec{v}) \ \& \ \dot{x}_\mu = (c, -\vec{v}) \tag{138}$$

$$J^\upsilon = \rho_q \cdot \dot{x}^\upsilon \tag{139}$$

, where:

- L is the Lagrangian density
- $\dot{x}^\mu$ and $\dot{x}_\alpha$contravariant and covariant form of the four-speed
- γ is the Lorentz factor
- $\rho_m$is the mass density
- ρ_q is the charge density
- J^υ is the contravariant form of the four-current

By minimizing the action, we get the Euler-Lagrange equation, from which we can get the Lorentz force density.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^\mu} - \frac{\partial L}{\partial x^\mu} = 0 \tag{140}$$

$$\frac{\partial L}{\partial x^\mu} = \rho_q \cdot \dot{x}^\upsilon \cdot \partial_\mu A_\upsilon - \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2 \tag{141}$$

$$\frac{\partial L}{\partial \dot{x}^\mu} = -\rho_m \cdot c^2 \cdot \frac{\partial}{\partial \dot{x}^\mu}\left(\frac{1}{\gamma}\right) + \rho_q \cdot \frac{\partial \dot{x}^\upsilon}{\partial \dot{x}^\mu} \cdot A_\upsilon = \gamma \cdot \rho_m \cdot \dot{x}_\mu + \rho_q \cdot A_\mu \tag{142}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^\mu} = \dot{p}_\mu + \rho_q \cdot \dot{A}_\mu \tag{143}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^\mu} - \frac{\partial L}{\partial x^\mu} = \dot{p}_\mu + \rho_q \cdot \dot{A}_\mu - \rho_q \cdot \dot{x}^\upsilon \cdot \partial_\mu A_\upsilon + \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2$$
$$= \dot{p}_\mu + \rho_q \cdot \dot{x}^\upsilon \cdot \partial_\upsilon A_\mu - \rho_q \cdot \dot{x}^\upsilon \cdot \partial_\mu A_\upsilon + \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2$$
$$= \dot{p}_\mu + \rho_q \cdot \dot{x}^\upsilon \cdot (\partial_\upsilon A_\mu - \partial_\mu A_\upsilon) + \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2$$
$$= \dot{p}_\mu + J^\upsilon \cdot F_{\upsilon\mu} + \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2 = 0 \tag{144}$$

$$-f_\mu = J^\upsilon \cdot F_{\upsilon\mu} + \frac{1}{2 \cdot \mu_0} \cdot \partial_\mu (\partial_\upsilon A^\upsilon)^2 \tag{145}$$

As the next step, express the spatial components of the above equation and take the volume integral of both sides of the equation.

$$\vec{F}_L = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} + \frac{1}{2 \cdot \mu_0} \int_V \vec{\nabla}(\Lambda^2) dV = \\ q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} + \frac{1}{\mu_0} \int_V \Lambda \cdot \vec{\nabla} \Lambda dV \tag{146}$$

Let's use the (29) modified Maxwell equation.

$$\vec{F}_L = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} - \frac{1}{\mu_0} \int_V \Lambda \cdot \left(\mu_0 \cdot \vec{J} + \frac{1}{c^2} \cdot \frac{\partial \vec{E}}{\partial t} - \vec{\nabla} \times \vec{B} \right) dV \\ = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} - q \cdot \vec{v} \cdot \Lambda - \varepsilon_0 \cdot \int_V \frac{\partial \vec{E}}{\partial t} \cdot \Lambda dV + \frac{1}{\mu_0} \int_V \Lambda \cdot \vec{\nabla} \times \vec{B} dV \tag{147}$$

, where:

- p_μ is the covariant form of the momentum density
- q is the charge
- $\dot{p}_\mu$ or f_μ the covariant form of the force density
- $\vec{F}_L$ is the Lorentz force

The first three terms of the above equation describe the field-charge, and the last two terms describe the field-field interaction.

If the external field $\vec{E}$ is constant in time and the field $\vec{B}$ is curl-free, then the equation (42) obtained with quaternions is obtained.

$$\vec{F}_L = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} - q \cdot \vec{v} \cdot \Lambda \tag{148}$$

If we were to integrate this force, then by definition work would not be a quantity remaining in time, i.e. energy would not be time-shift invariant.

12. The curl free vector potential and its consequences

In this chapter, I present the potential and consequences of the curl-free vector. The Helmholtz decomposition uniquely decomposes any three-vector into longitudinal (L) and transverse (T) components (Griffiths, 2007). For example, the $\vec{J}$ current density takes the following general form:

$$\vec{J} = \vec{J}_L + \vec{J}_T = \vec{\nabla}\delta + \vec{\nabla} \times \vec{j} \tag{149}$$

, where $\vec{\nabla}\delta$ is the longitudinal component, $\vec{\nabla} \times \vec{j}$ is the transversal component. The $\vec{B}$ magnetic induction takes the following general form:

$$\vec{B} = \vec{B}_L + \vec{B}_T = \vec{\nabla} \times \left(\vec{A}_L + \vec{A}_T\right) = \vec{\nabla} \times \vec{\nabla}\sigma + \vec{\nabla} \times \vec{\nabla} \times \vec{a} \tag{150}$$

The rotation of the longitudinal member is automatically eliminated, the rotation of the transverse member is as follows:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{a} = \vec{\nabla}\vec{\nabla} \cdot \vec{a} - \nabla^2\vec{a}. \tag{151}$$

This means that the magnetic induction $\vec{B}$ has only a transverse component, ie: $\vec{B} = \vec{B}_T$. C. Monstein and J.P. Wesley, as well as Lee M. Hively and Andrew S. Loebl assume in their experiments that the vector potential is curl-free. This means that $\vec{B} = \vec{\nabla} \times \vec{A} = 0$. Then the vector potential can be defined as follows: $\vec{A} = \vec{\nabla}\alpha$. This means that the vector potential and its longitudinal component can be written in the same way as the gradient of a potential: $\vec{A} \rightarrow \vec{A}_L$. Here, the arrow means that the vector potential is transformed. Since the definition of the vector potential has changed, the electric field strength vector $\vec{E}$ built from it and the definition of the magneto scalar potential Λ will also change: $\vec{E} \rightarrow \vec{E}_L$ and $\Lambda \rightarrow \Lambda_L$.. We will see later that the definition of the current density vector J ⃗ will also change as follows: $\vec{J} \rightarrow \vec{J}_L$. If all this is true, not only the modified Maxwell equations need to be modified, but perhaps everything else as well. This does not mean that electrodynamics is bad, but that the consequences of the curl-free vector potential were taken into account and had to be adapted. Hereinafter, the Maxwell equations constructed from longitudinally transformed quantities are called longitudinal Maxwell equations. The interaction it creates is called the electro-scalar interaction.

12.1. The longitudinal Maxwell equations

As a first step, I introduce the longitudinal Maxwell equations.

$$\vec{\nabla}\cdot\vec{E}_L=\frac{\rho}{\varepsilon_0}-\frac{\partial\Lambda_L}{\partial t} \tag{152}$$

$$\vec{\nabla}\times\vec{E}_L=0 \tag{153}$$

$$\mu_0\cdot\vec{J}_L+\frac{1}{c^2}\cdot\frac{\partial\vec{E}_L}{\partial t}+\vec{\nabla}\Lambda_L=0 \tag{154}$$

$$\Lambda_L=\frac{1}{c^2}\cdot\frac{\partial\varphi}{\partial t}+\vec{\nabla}\cdot\vec{A}_L=\frac{1}{c^2}\cdot\frac{\partial\varphi}{\partial t}+\vec{\nabla}\cdot\vec{\nabla}\alpha=\frac{1}{c^2}\cdot\frac{\partial\varphi}{\partial t}+\nabla^2\alpha \tag{155}$$

$$\vec{E}_L=-\frac{\partial\vec{A}_L}{\partial t}-\vec{\nabla}\varphi=-\frac{\partial\vec{\nabla}\alpha}{\partial t}-\vec{\nabla}\varphi \tag{156}$$

This gives us the longitudinal Maxwell equations. Since $\vec{\nabla}\times\vec{E}_L=0$, therefore $\vec{E}_L$ can take the following form: $\vec{E}_L=-\vec{\nabla}\beta$. Then the scalar potentials can be written in the following form:

$$-\vec{\nabla}\beta=-\frac{\partial\vec{\nabla}\alpha}{\partial t}-\vec{\nabla}\varphi \tag{157}$$

Let's integrate both sides of the above equation and let the integration constant be zero.

$$-\beta=-\frac{\partial\alpha}{\partial t}-\varphi \tag{158}$$

It can be seen that both the vector potential and the electric field strength vector can be written as the gradient of a potential. So far, I have dealt with the differential form of the longitudinal Maxwell equations. In the next step, I define their integral form. Let's start from equations (152), (153), (154).

Take the volume integral of both sides of equation (152). We apply the Gauss theorem. Then we arrive at the following equation.

$$\oint_S\vec{E}_L\cdot\vec{n}dS=\frac{1}{\varepsilon_0}\int_V\rho dV-\frac{\partial}{\partial t}\int_V\Lambda_L dV \tag{159}$$

Take the surface integral of both sides of equation (154). We apply Stokes' theorem. Then we arrive at the following equation.

$$\oint_C\vec{E}_L d\vec{r}=0 \tag{160}$$

Since $\vec{E}_L$ can be written as the gradient of a potential (multiplied by (-1)) according to equation 156.) and the integral of $\vec{E}_L$over the closed curve is equal to zero, it can be said that $\vec{E}_L$is conservative. Only the start and end points matter. The condition also includes the fact that $\vec{E}_L$ must be uniquely connected. In my opinion, this is accomplished.

Take the curve integral of both sides of equation (154). We apply the gradient theorem. Then we arrive at the following equation.

$$\mu_0 \int_C \vec{J}_L \vec{n} \mathrm{d}r + \frac{1}{c^2} \cdot \frac{\partial}{\partial t} \int_C \vec{E}_L \cdot \vec{n} \mathrm{d}r + \oint_t (\vec{\nabla} \Lambda_L) \cdot \vec{v} dt = 0 \tag{161}$$

With this, I derived the integral form of the longitudinal Maxwell equations.

Let's continue the investigation with equation (154). Let's take the integral along a closed curve of both sides of equation (154). Let's move from left to right. In the case of the first term, we can apply the Stokes theorem and the statement made in (here). I derived the current density. It turns out that the first term will be zero. For the second term, we can apply equation (160), according to which the second term will also be zero. Only the third member remains. Multiply both sides by (-1).

$$-\oint_C \vec{\nabla} \Lambda_L d\vec{r} = 0 \tag{162}$$

Let $\vec{G} = -\vec{\nabla} \Lambda_L$. Since the vector potential $\vec{G}$ can be written as the gradient of a scalar potential Λ_L (multiplied by (-1)) and the integral over the closed curve of the vector potential $\vec{G}$ is equal to zero, it can be said that $\vec{G}$is conservative. Only the start and end points matter. The condition also includes the fact that $\vec{G}$ must be uniquely connected. In my opinion, this is accomplished.

12.2. The longitudinal continuity equation

In this chapter, I derive the longitudinal continuity equation. (here) I start from the longitudinal Maxwell equations and check whether the result will be the same as in (here) with quaternions. Also, don't forget about the curl-free potential.

Derive equation (152) with respect to time, form the divergence of both sides of equation (154) and use equation (24). Finally, relations (152) and (154) are added together.

$$\frac{1}{c^2} \cdot \frac{\partial^2 \Lambda_L}{\partial t^2} - \nabla^2 \Lambda_L = \mu_0 \cdot \left(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}_L \right) \tag{163}$$

Equation (163) has a double meaning. On the one hand, charges can be sources and sinks of magneto scalar waves, and on the other hand, magneto scalar waves can be sources and sinks of charges. According to the equation, the charge is a locally non-conserving quantity.

We form the time derivative of equation (154). We form the rotation of equation (153). We form the gradient of both sides of the equation (152). Substitute equation (153) into equation (152). Add equation (152) to equation (154). Finally, we use relation (24).

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{E}_L}{\partial t^2}-\nabla^2\vec{E}_L=-\frac{1}{\varepsilon_0}\cdot\left(\frac{1}{c^2}\cdot\frac{\partial\vec{J}_L}{\partial t}+\vec{\nabla}\rho\right) \tag{164}$$

Assume that equations (163) and (164) have no source. Then we get back the conservation equations (46) and (47) obtained by the quaternions.

Finally, let the third equation follow. Divide both sides of equation (154) by μ_0. Form the rotation of both sides of equation (154) Using equation (148) and the identity $\vec{\nabla}\times\vec{\nabla}\Lambda_L=0$, we will get that the longitudinal current density is curl-free.

$$\vec{\nabla}\times\vec{J}_L=0 \tag{165}$$

This equation is the same as equation (48) obtained with quaternions. This means that $\vec{J}_L$ can be written as the gradient of a potential: $\vec{J}_L=\vec{\nabla}\kappa$.

12.3. The static magneto scalar field

When examining the static magneto scalar field (taking Helmholtz theory into account), as a first step, if we omit the time derivative component of equation (154), integrate both sides of the equation and omit the integration constant, we arrive at the following relationship:

$$\Lambda(\vec{r})=-\mu_0\cdot\int\vec{J}_L(\vec{r})d\mathrm{r} \tag{166}$$

Since $\vec{J}_L=\vec{\nabla}\kappa$, therefore, equation (166) takes the following form:

$$\Lambda=-\mu_0\cdot\kappa \tag{167}$$

Since the spatial quantities remain constant in time, we omit the time derivative components of equations (154) and (155) and form the gradient of both sides of equation (155) and use the $\vec{\nabla}\times\vec{\nabla}\times\vec{A}_L=\vec{\nabla}\vec{\nabla}\cdot\vec{A}_L-\nabla^2\vec{A}_L$ identity. Since $\vec{\nabla}\times\vec{A}_L=0$, therefore $\vec{\nabla}\vec{\nabla}\cdot\vec{A}_L=\nabla^2\vec{A}_L$. Substituting this into equation (154), we arrive at the following relationship:

$$\vec{\nabla}\Lambda=\vec{\nabla}\vec{\nabla}\cdot\vec{A}_L=\nabla^2\vec{A}_L=-\mu_0\cdot\vec{J}_L \tag{168}$$

Then we get equation (49) for the longitudinal quantities. Since $\vec{J}_L=\vec{\nabla}\kappa$ and $\vec{A}_L=\vec{\nabla}\alpha$, substituting these into (168), integrating both sides and choosing the integration constant as 0, we arrive at the following relationship:

$$-\nabla^2\alpha = \mu_0 \cdot \kappa \tag{169}$$

The shape of the scalar potential α takes the following form:

$$\alpha(\vec{r}) = \frac{\mu_0}{4\cdot\pi}\int_V \frac{\kappa(\vec{r}')}{|\vec{r}-\vec{r}'|}dV' \tag{170}$$

As a next step, I prove that $\vec{A}_L$ indeed curl-free. Let's take the gradient of equation (170)

$$\vec{\nabla}\alpha(\vec{r}) = \frac{\mu_0}{4\cdot\pi}\int_V \frac{\vec{\nabla}\kappa(\vec{r}')}{|\vec{r}-\vec{r}'|}dV' + \frac{\mu_0}{4\cdot\pi}\int_V \kappa(\vec{r}')\cdot\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)dV' \tag{171}$$

Let's take the rotation of equation (171).

$$\begin{aligned} &\vec{\nabla}\times\vec{\nabla}\alpha(\vec{r}) = \\ &\frac{\mu_0}{4\cdot\pi}\int_V \frac{\vec{\nabla}\times\left(\vec{\nabla}\kappa(\vec{r}')\right)}{\left|\vec{r}-\vec{r}^{'}\right|}dV^{'} \\ &-\frac{\mu_0}{4\cdot\pi}\int_V \vec{\nabla}\kappa(\vec{r}')\times\vec{\nabla}\left(\frac{1}{\left|\vec{r}-\vec{r}^{'}\right|}\right)dV' \\ &+\frac{\mu_0}{4\cdot\pi}\int_V \kappa(\vec{r}')\cdot\vec{\nabla}\times\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)dV' \end{aligned} \tag{172}$$

$$-\frac{\mu_0}{4\cdot\pi}\int_V \vec{\nabla}\left(\frac{1}{\left|\vec{r}-\vec{r}^{'}\right|}\right)\times\vec{\nabla}\kappa(\vec{r}')dV' = 0$$

12.4. The magneto scalar waves

In the case of magneto scalar waves (taking into account the curl-free vector potential), the first step is to derive the inhomogeneous wave equation for Λ_L. Derive equation (152) with respect to time, form the divergence of both sides of equation (154) and use equation (24). Finally, relations (152) and (154) are added together.

$$\frac{1}{c^2}\cdot\frac{\partial^2\Lambda_L}{\partial t^2} - \nabla^2\Lambda_L = \mu_0\cdot\left(\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J}_L\right) \tag{173}$$

As the next step, I derive the inhomogeneous wave equation for $\vec{E}_L$.

We form the time derivative of equation (154). We form the rotation of equation (153). We form the gradient of both sides of the equation (152). Substitute equation (153) into equation (152). Add equation (152) to equation (154). Finally, we use relation (24).

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{E}_L}{\partial t^2} - \nabla^2\vec{E}_L = -\frac{1}{\varepsilon_0}\cdot\left(\frac{1}{c^2}\cdot\frac{\partial\vec{J}_L}{\partial t} + \vec{\nabla}\rho\right) \tag{174}$$

As the next step, I derive the inhomogeneous wave equation for φ . Take equations (155) and (156) and substitute them into equation (152). Finally, we form the equation (-1) times (152).

$$\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2}-\nabla^2\varphi=\frac{\rho}{\varepsilon_0} \tag{175}$$

As the next step, I derive the inhomogeneous wave equation for $\vec{A}_L$. Let's take equations (155), (156) and then substitute them into equation (154). Finally, we form the equation (-1) times (154).

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{A}_L}{\partial t^2}-\nabla^2\vec{A}_L=\mu_0\cdot\vec{J}_L \tag{176}$$

Since $\vec{J}_L=\vec{\nabla}\kappa$ and $\vec{A}_L=\vec{\nabla}\alpha$, substituting these into equation (176), integrating both sides and choosing the integration constant as 0, we arrive at the following relationship:

$$\frac{1}{c^2}\cdot\frac{\partial^2\alpha}{\partial t^2}-\nabla^2\alpha=\mu_0\cdot\kappa \tag{177}$$

The shape of the scalar potential α takes the following form:

$$\alpha(\vec{r},t)=\frac{\mu_0}{4\cdot\pi}\int_V\frac{\kappa(\vec{r}',t')}{|\vec{r}-\vec{r}'|}dV' \tag{178}$$

12.5. The longitudinal Poynting theorem

In the case of examining Poynting's theorem, I announce without derivation that, taking into account the curl-free vector potential, I reached a different result than before.

$$\begin{gathered}\int_V\left(\frac{\vec{\nabla}\cdot(\Lambda_L\cdot\vec{E}_L)}{\mu_0}\right)dV+\\ \int_V\left(+\frac{d}{dt}\left(\frac{1}{2}\cdot\varepsilon_0\cdot\vec{E}_L^2+\frac{1}{2}\cdot\frac{1}{\mu_0}\Lambda_L{}^2\right)\right)dV=\\ \int_V\left(\Lambda_L\cdot\rho\cdot c^2-\vec{J}_L\cdot\vec{E}_L\right)dV\end{gathered} \tag{179}$$

It can be seen that the term $\vec{E}\times\vec{B}$ has disappeared, as has the energy density of the magnetic induction $\vec{B}$. If we take equation (76) into account, then through $\hat{k}$ (which is the normal vector of the plane/sphere wave) the energy propagation and the propagation direction of the waves will coincide.

$$\vec{S}_{ES}=\frac{\Lambda_L\cdot\vec{E}_L}{\mu_0}=\frac{1}{\mu_0}\cdot\frac{1}{c}\cdot\hat{k}\cdot\vec{E}_L\cdot\vec{E}_L=\frac{1}{\mu_0}\cdot\frac{1}{c}\cdot\hat{k}\cdot\left|\vec{E}_L\right|^2 \tag{180}$$

For now, I do not equate the time-averaged power (intensity) of electromagnetic and electro-scalar radiation to a unit surface. And the surface integral of the intensity will be the power.

$$I_{EM} = \langle \vec{S}_{EM} \rangle = \frac{1}{2} \cdot Re\left(\frac{\vec{E}_{0_T} \times \vec{B}_{0_T}{}^{*}}{\mu_0}\right) = \frac{1}{2} \cdot \frac{1}{Z_0} \cdot \hat{k} \cdot \left|\vec{E}_{0_T}\right|^2 \tag{181}$$

$$I_{ES} = \langle \vec{S}_{ES} \rangle = \frac{1}{2} \cdot Re\left(\frac{\Lambda_{0_L}{}^{*} \cdot \vec{E}_{0_L}}{\mu_0}\right) = \frac{1}{2} \cdot \frac{1}{Z_0} \cdot \hat{k} \cdot \left|\vec{E}_{0_L}\right|^2 \tag{182}$$

$$I_{EM} \,?=\, I_{ES} \tag{183}$$

, where:

- $*$ is the complex conjugate
- I_{EM} is the intensity of the electromagnetic radiation
- I_{ES} is the intensity of the electro-scalar radiation
- Z_0 is the wave impedance of the vacuum ($Z_0 \approx 377\ \Omega$)

12.6. The longitudinal Lagrangian density

In this chapter, I derive the longitudinal Lagrangian density.

$$L = L(\vartheta_i, \dot{\vartheta}_i, \vec{\nabla}\vartheta_i) \tag{184}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial(\dot{\vartheta}_i)}\right) + \vec{\nabla} \cdot \left(\frac{\partial L}{\partial(\vec{\nabla}\vartheta_i)}\right) - \frac{\partial L}{\partial \vartheta_i} = 0 \tag{185}$$

$$\begin{array}{lll} \vartheta_1 = A_1(x_1 x_2, x_3, t) & \dot{\vartheta}_1 = \dot{A}_1 & \vec{\nabla}\vartheta_1 = \vec{\nabla}A_1 \\ \vartheta_2 = A_2(x_1 x_2, x_3, t) & \dot{\vartheta}_2 = \dot{A}_2 & \vec{\nabla}\vartheta_2 = \vec{\nabla}A_2 \\ \vartheta_3 = A_3(x_1 x_2, x_3, t) & \dot{\vartheta}_3 = \dot{A}_3 & \vec{\nabla}\vartheta_3 = \vec{\nabla}A_3 \\ \vartheta_4 = \varphi(x_1 x_2, x_3, t) & \dot{\vartheta}_4 = \dot{\varphi} & \vec{\nabla}\vartheta_4 = \vec{\nabla}\varphi \end{array} \tag{186}$$

Substitute equations (155) and (156) into equation (152). Multiply both sides of the equation by (-1).

$$\frac{\partial}{\partial t}\left(-\frac{1}{c^2} \cdot \frac{\partial \varphi}{\partial t}\right) + \vec{\nabla} \cdot (\vec{\nabla}\varphi) - \left(-\frac{\rho}{\varepsilon_0}\right) = 0 \tag{187}$$

Let i=4 and $\vartheta_4 = \varphi$. Substitute it into the Euler-Lagrange equation (185). Let's compare it with equation (187). Then we get the following derivatives.

$$\frac{\partial L}{\partial(\dot{\varphi})} = -\frac{1}{c^2} \cdot \frac{\partial \varphi}{\partial t} \qquad \frac{\partial L}{\partial(\vec{\nabla}\varphi)} = \vec{\nabla}\varphi \qquad \frac{\partial L}{\partial \varphi} = -\frac{\rho}{\varepsilon_0} \tag{188}$$

The components of the Lagrangian density of the equation (188) are determined by integration from left to right according to $\dot{\varphi}$, $\vec{\nabla}\varphi$ and φ.

$$L_{a_1} = -\frac{1}{2}\cdot\frac{1}{c^2}\cdot(\dot{\varphi})^2 \qquad L_{a_2} = \frac{1}{2}\cdot\left(\vec{\nabla}\varphi\right)^2 \qquad L_{a_3} = -\frac{\rho\cdot\varphi}{\varepsilon_0} \tag{189}$$

The individual Lagrangian density components are added together.

$$L_a = L_{a_1} + L_{a_2} + L_{a_2} = -\frac{1}{2}\cdot\frac{1}{c^2}\cdot(\dot{\varphi})^2+\frac{1}{2}\cdot\left(\vec{\nabla}\varphi\right)^2 - \frac{\rho\cdot\varphi}{\varepsilon_0} \tag{190}$$

Substitute equations (155) and (156) into equation (154). Multiply both sides of the equation by (-1) and c^2.

190.)

$$\frac{\partial}{\partial t}\left(\frac{\partial \vec{A}_L}{\partial t}\right) + \vec{\nabla}\left(-c^2\cdot\vec{\nabla}\cdot\vec{A}_L\right) - \frac{\vec{J}_L}{\varepsilon_0} = 0 \tag{191}$$

Let i=j=1,2,3 and $\vartheta_j = A_j$. Substitute it into the Euler-Lagrange equation (185). Let's compare it with equation (191). Then we get the following derivatives.

$$\frac{\partial L}{\partial(\dot{A}_j)} = \frac{\partial A_j}{\partial t} \qquad \frac{\partial L}{\partial(\vec{\nabla}A_j)} = -c^2\cdot\vec{\nabla}A_j \qquad \frac{\partial L}{\partial A_j} = \frac{J_j}{\varepsilon_0} \tag{192}$$

The components of the Lagrangian density of the equation (192) are determined by integration from left to right according to $\dot{A}_j$, $\vec{\nabla}A_j$ and A_j.

$$L_{b_1} = \frac{1}{2}\cdot\left(\dot{A}_j\right)^2 \qquad L_{b_2} = -\frac{1}{2}\cdot c^2\left(\vec{\nabla}A_j\right)^2 \qquad L_{b_3} = \frac{A_j\cdot J_j}{\varepsilon_0} \tag{193}$$

The individual Lagrangian density components are added together.

$$L_b = L_{b_1} + L_b + L_{b_2} = \frac{1}{2}\cdot\left(\dot{A}_j\right)^2 - \frac{1}{2}\cdot c^2\left(\vec{\nabla}A_j\right)^2 + \frac{A_j\cdot J_j}{\varepsilon_0} \tag{194}$$

Add the Lagrange density components (190) and (194) and rearrange the equation. Compress the A_j components into $\vec{A}_L$.

$$\begin{gathered} L = L_a + L_b = \\ -\frac{1}{2}\cdot\frac{1}{c^2}\cdot(\dot{\varphi})^2+\frac{1}{2}\cdot\left(\vec{\nabla}\varphi\right)^2 - \frac{\rho\cdot\varphi}{\varepsilon_0} \\ +\frac{1}{2}\cdot\left(\dot{\vec{A}}_L\right)^2 - \frac{1}{2}\cdot c^2\cdot\left(\vec{\nabla}\cdot\vec{A}_L\right)^2 + \frac{\vec{A}_L\cdot\vec{J}_L}{\varepsilon_0} = \\ \frac{1}{2}\cdot\left(-\vec{\nabla}\varphi - \dot{\vec{A}}_L\right)^2 - \vec{\nabla}\varphi\cdot\dot{\vec{A}}_L - \frac{1}{2}\cdot c^2\cdot\left(\frac{1}{c^2}\cdot\dot{\varphi} + \vec{\nabla}\cdot\vec{A}_L\right)^2 + \dot{\varphi}\cdot\vec{\nabla}\cdot\vec{A}_L \\ + \frac{\vec{J}_L\cdot\vec{A}_L}{\varepsilon_0} - \frac{\rho\cdot\varphi}{\varepsilon_0} \end{gathered} \tag{195}$$

Since the Lagrangian density must have an energy density dimension, L must be multiplied by ε_0.

$$\begin{gathered} L_{wave} = \varepsilon_0\cdot L = \frac{1}{2}\cdot\varepsilon_0\cdot\vec{E}_L^{\,2} - \frac{1}{2}\cdot\frac{1}{\mu_0}\cdot\Lambda_L^{\,2} \\ +\varepsilon_0\cdot\left(\dot{\varphi}\cdot\vec{\nabla}\cdot\vec{A}_L - \vec{\nabla}\varphi\cdot\dot{\vec{A}}_L\right) + \left(\vec{J}_L\cdot\vec{A}_L - \rho\cdot\varphi\right) \end{gathered} \tag{196}$$

In equation (196), the first two terms form the Lagrangian density of free field, the other two terms form the Lagrangian density of the interaction. When I substitute the longitudinal electric field strength and the magneto scalar potential into the longitudinal Maxwell equations by definition, we get the wave equations for the scalar and vector potential. This Lagrangian density does not give us the longitudinal Maxwell equations, only the wave equations.

I announce without derivation that, if we subtract the first new interaction term from the L_{wave} and substitute it into the Euler-Lagrange equation (185), we will get the longitudinal Maxwell equations (152) and (154) separately for φ and $\vec{A}_L$.

$$L_{longitudinal} = L_{wave} - \varepsilon_0 \cdot \left(\dot{\varphi} \cdot \vec{\nabla} \cdot \vec{A}_L - \vec{\nabla}\varphi \cdot \dot{\vec{A}}_L\right) \tag{197}$$

This gives us the longitudinal Lagrangian density.

As a next step, I define the longitudinal Lagrangian density using a covariant formalism. The magnetic induction components of the electromagnetic tensor can be replaced by zeros, since the $\vec{A}_L$ vector potential is curl-free. The definition of the electromagnetic tensor remains the same.

$$F_{\alpha\beta} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & 0 & 0 \\ -\frac{E_y}{c} & 0 & 0 & 0 \\ -\frac{E_z}{c} & 0 & 0 & 0 \end{pmatrix} \tag{198}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & 0 & 0 \\ \frac{E_y}{c} & 0 & 0 & 0 \\ \frac{E_z}{c} & 0 & 0 & 0 \end{pmatrix} \tag{199}$$

After that, write down the longitudinal Lagrange density corresponding to equation (125). Without further ado, I announce that by minimizing the effect, we get the Euler-Lagrange equation (126), from which we get back the longitudinal Maxwell equations (152) and (154). The longitudinal Maxwell equation (153) can be derived by means of the Bianchi identity (133) I leave the proof to the reader.

As the next step, we examine how many polarization degrees of freedom are allowed by the interaction described by the longitudinal Maxwell equations. The longitudinal electric field strength vector $\vec{E}_L$ has 3 polarization degrees of freedom. Since Λ_L is a scalar function, it has 1 degree of polarization freedom. So far, there are a total of 4 polarization degrees of freedom. We know that $\vec{k} \cdot \vec{E}_L = \Lambda_L$, which allows 1 degree of polarization freedom for $\vec{E}_L$. Λ still has 1 degree of polarization freedom. So the interaction described by the longitudinal Maxwell equations has 2 polarization degrees of freedom.

As the next step, we examine the range of the interaction described by the longitudinal Maxwell equations. The range of the interaction described by the longitudinal Maxwell equations is infinite, since the mass will automatically be equal to zero if γ=1 and k=0 are chosen.

As the next step, we examine whether the longitudinal Maxwell equations are invariant to the Lorenz gauge. In the case of the interaction described by the modified Maxwell equations, it is not fulfilled since we have chosen to leave out the Lorenz gauge.

As the next step, we examine whether the longitudinal Maxwell equations are invariant to the Lorentz transformation. It can be shown that the Lorentz invariant quantity of the first component of the longitudinal Lagrange density is as follows: $-\frac{\vec{E}_L{}^2}{c^2}$. It can be shown that the Lorentz invariant quantity of the second component of the longitudinal Lagrange density is as follows: $\Lambda_L{}^2$. Therefore, the interaction constructed from the longitudinal Maxwell equations is also invariant to the Lorentz transformation.

12.7. The longitudinal Lorentz force

In the case of the Lorentz force investigation, I announce without derivation that, taking into account the curl-free vector potential, I reached a different result than before.

$$\begin{aligned} \vec{F}_{LL} = q \cdot \vec{E}_L + \frac{1}{2 \cdot \mu_0} \int_V \vec{\nabla}(\Lambda_L{}^2) dV = \\ q \cdot \vec{E}_L + \frac{1}{\mu_0} \int_V \Lambda_L \cdot \vec{\nabla} \Lambda_L dV \end{aligned} \tag{200}$$

Let's use the (154) longitudinal Maxwell equation.

$$
\begin{aligned}
\vec{F}_{LL} &= q \cdot \vec{E}_L - \frac{1}{\mu_0}\int_V \Lambda_L \cdot \left(\mu_0 \cdot \vec{J}_L + \frac{1}{c^2} \cdot \frac{\partial \vec{E}_L}{\partial t}\right) dV \\
&= q \cdot \vec{E}_L - q \cdot \vec{v} \cdot \Lambda_L - \varepsilon_0 \cdot \int_V \frac{\partial \vec{E}_L}{\partial t} \cdot \Lambda_L dV
\end{aligned}
\tag{201}
$$

, where:

- $p_\mu$is the covariant form of the momentum density
- q is the charge
- $\dot{p}_\mu$ or $f_\mu$are the covariant form of the force density
- $\vec{F}_{LL}$ is the longitudinal Lorentz force

The first two terms of the above equation describe the field-charge, and the last term describes the field-field interaction. It can be seen that the term $q \cdot \vec{v} \times \vec{B}$ has disappeared, as has the field-field interacting part of the magnetic induction$\vec{B}$. If we were to integrate the longitudinal Lorentz force, then by definition work would not be a time-remaining quantity, i.e. energy would not be time-shift invariant. The Lorentz force known from classical electrodynamics is also velocity-dependent, but its integral is a quantity that remains constant over time.

13. Invariance to the Lorentz transformation

In this chapter, I prove that the modified Maxwell equations are invariant to the Lorentz transformation. Assume two coordinate systems: K and K'. Let's assume that the system K' moves in a straight line with speed $\vec{v}$ in the x direction with respect to K and is not acted upon by a force or field. Then the two systems form an inertial system, i.e. K and K' are mechanically indistinguishable from each other. Furthermore, we use the fact that the speed of light in a vacuum is at the end of every relevant system.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{202}$$

It can be shown that the coordinates are transformed as follows:

$$x = \gamma \cdot (x' + v \cdot t') \tag{203}$$
$$y' = y' \tag{204}$$
$$z' = z \tag{205}$$
$$t = \gamma \cdot \left(t' + \frac{v \cdot x'}{c^2}\right) \tag{206}$$

It can be shown that the partial derivatives transform as follows:

$$\frac{\partial}{\partial x} = \gamma \cdot \left(\frac{\partial}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial}{\partial t'}\right) \tag{207}$$
$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y'} \tag{208}$$
$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z'} \tag{209}$$
$$\frac{\partial}{\partial t} = \gamma \cdot \left(\frac{\partial}{\partial t'} - v \cdot \frac{\partial}{\partial x'}\right) \tag{210}$$

It can be shown that the potentials are transformed as follows:

$$\varphi = \gamma \cdot (\varphi' + A'_x \cdot v) \tag{211}$$
$$A_x = \gamma \cdot \left(\frac{v}{c^2} \cdot \varphi' + A'_x\right) \tag{212}$$
$$A_y = A'_y \tag{213}$$
$$A_z = A'_z \tag{214}$$

As a first step, we determine the transformation of the magneto scalar potential Λ using the definition according to equation (17).

$$\Lambda = \frac{1}{c^2} \cdot \frac{\partial \varphi}{\partial t} + \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{215}$$

$$\Lambda = \frac{\gamma}{c^2} \cdot \left(\frac{\partial \varphi}{\partial t'} - v \cdot \frac{\partial \varphi}{\partial x'}\right) + \cdot \gamma \left(\frac{\partial A_x}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial A_x}{\partial t'}\right) + \frac{\partial A_y}{\partial y'} + \frac{\partial A_z}{\partial z'} \tag{216}$$

$$\Lambda = \frac{1}{c^2} \cdot \frac{\partial \varphi'}{\partial t'} + \frac{\partial A'_x}{\partial x'} + \frac{\partial A'_y}{\partial y'} + \frac{\partial A'_z}{\partial z'} \tag{217}$$

$$\Lambda = \Lambda' \tag{218}$$

It can be shown that the electric field strength components $\vec{E}$ are transformed as follows:

$$E_x = E_x' \tag{219}$$
$$E_y = \gamma \cdot \left(E_y' + v \cdot B_z'\right) \tag{220}$$
$$E_z = \gamma \cdot \left(E_z' - v \cdot B_y'\right) \tag{221}$$

It can be shown that the magnetic induction components$\vec{B}$ are transformed as follows:

$$B_x = B_x' \tag{222}$$
$$B_y = \gamma \cdot \left(B_y' - \frac{v}{c^2} \cdot E_z'\right) \tag{223}$$
$$B_z = \gamma \cdot \left(B_z' + \frac{v}{c^2} \cdot E_y'\right) \tag{224}$$

It can be shown that the current density components $\vec{J}$ and the charge density ρ are transformed as follows:

$$J_x = \gamma \cdot (J_x' + \rho' \cdot v) \tag{225}$$
$$J_y = J_y' \tag{226}$$
$$J_z = J_z' \tag{227}$$
$$\rho = \gamma \cdot \left(\rho' + \frac{v}{c^2} \cdot J_x'\right) \tag{228}$$

As a second step, let's perform the transformation on equation (25):

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0} - \frac{\partial \Lambda}{\partial t} \tag{229}$$

$$\begin{gathered} \gamma \cdot \left(\frac{\partial E_x}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial E_x}{\partial t'}\right) + \frac{\partial E_y}{\partial y'} + \frac{\partial E_z}{\partial z'} = \\ \frac{1}{\varepsilon_0} \cdot \gamma \cdot \left(\rho' + \frac{v}{c^2} \cdot J'_x\right) - \gamma \cdot \left(\frac{\partial \Lambda}{\partial t'} - v \cdot \frac{\partial \Lambda}{\partial x'}\right) \end{gathered} \tag{230}$$

$$\begin{gathered} \gamma \cdot \left(\frac{\partial E_x'}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial E_x'}{\partial t'}\right) + \gamma \cdot \left(\frac{\partial E_y'}{\partial y'} + v \cdot \frac{\partial B_z'}{\partial y'}\right) \\ + \gamma \cdot \left(\frac{\partial E_z'}{\partial z'} - v \cdot \frac{\partial B_y'}{\partial z'}\right) = \\ \frac{1}{\varepsilon_0} \cdot \gamma \cdot \left(\rho' + \frac{v}{c^2} \cdot J'_x\right) - \gamma \cdot \left(\frac{\partial \Lambda'}{\partial t'} - v \cdot \frac{\partial \Lambda'}{\partial x'}\right) \end{gathered} \tag{231}$$

$$\frac{\partial E_x'}{\partial x'} + \frac{\partial E_y'}{\partial y'} + \frac{\partial E_z'}{\partial z'} = \tag{232}$$

$$= \frac{\rho'}{\varepsilon_0} - \frac{\partial \Lambda'}{\partial t'} + v \cdot \left(\frac{\partial B_y{}'}{\partial z'} - \frac{\partial B_z{}'}{\partial y'} + \frac{1}{c^2} \cdot \frac{\partial E_x{}'}{\partial t'} + \mu_0 \cdot J_x' + \frac{\partial \Lambda'}{\partial x'} \right)$$

The bracketed part of equation (232) is the x component of equation (29) set to 0. As a third step, perform the transformation on the x component of equation (29):

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0 \cdot J_x + \frac{1}{c^2} \cdot \frac{\partial E_x}{\partial t} + \frac{\partial \Lambda}{\partial x} \tag{233}$$

$$\frac{\partial B_z}{\partial y'} - \frac{\partial B_y}{\partial z'} = \mu_0 \cdot J_x + \frac{1}{c^2} \cdot \gamma \cdot \left(\frac{\partial E_x}{\partial t'} - v \cdot \frac{\partial E_x}{\partial x'} \right) + \gamma \cdot \left(\frac{\partial \Lambda}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial \Lambda}{\partial t'} \right) \tag{234}$$

$$\gamma \cdot \left(\frac{\partial B_z{}'}{\partial y'} + \frac{v}{c^2} \cdot \frac{\partial E_y{}'}{\partial y'} \right) - \gamma \cdot \left(\frac{\partial B_y{}'}{\partial z'} - \frac{v}{c^2} \cdot \frac{\partial E_z{}'}{\partial z'} \right) = \mu_0 \cdot \gamma \cdot (J_x' + \rho' \cdot v) + \frac{1}{c^2} \cdot \gamma \cdot \left(\frac{\partial E_x{}'}{\partial t'} - v \cdot \frac{\partial E_x{}'}{\partial x'} \right) + \gamma \cdot \left(\frac{\partial \Lambda'}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial \Lambda'}{\partial t'} \right) \tag{235}$$

$$\frac{\partial B_z{}'}{\partial y'} - \frac{\partial B_y{}'}{\partial z'} = \mu_0 \cdot J_x' + \frac{1}{c^2} \cdot \frac{\partial E_x{}'}{\partial t'} + + \frac{1}{c} \cdot \frac{\partial E_x{}'}{\partial x'} + v \cdot \left(-\frac{1}{c^2} \cdot \frac{\partial E_x{}'}{\partial x'} - \frac{1}{c^2} \cdot \frac{\partial E_y{}'}{\partial y'} - \frac{1}{c^2} \cdot \frac{\partial E_z{}'}{\partial z'} + \mu_0 \cdot \rho' - \frac{1}{c^2} \cdot \frac{\partial \Lambda'}{\partial t'} \right) \tag{236}$$

The bracketed part of equation (236) is the x component of equation (25) set to 0. As a fourth step, perform the transformation on the y component of equation (29):

$$\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = \mu_0 \cdot J_y + \frac{1}{c^2} \cdot \frac{\partial E_y}{\partial t} + \frac{\partial \Lambda}{\partial y} \tag{237}$$

$$\frac{\partial B_x}{\partial z'} - \gamma \cdot \left(\frac{\partial B_z}{\partial x'} - \frac{v}{c^2} \cdot \frac{\partial B_z}{\partial t'} \right) = \mu_0 \cdot J_y + \frac{1}{c^2} \cdot \gamma \cdot \left(\frac{\partial E_y}{\partial t'} - v \cdot \frac{\partial E_y}{\partial x'} \right) + \frac{\partial \Lambda}{\partial y'} \tag{238}$$

$$\frac{\partial B_x{}'}{\partial z'} - \gamma^2 \cdot \frac{\partial}{\partial x'} \left(B_z' + \frac{v}{c^2} \cdot E_y{}' \right) + \gamma^2 \cdot \frac{v}{c^2} \cdot \frac{\partial}{\partial t'} \left(B_z' + \frac{v}{c^2} \cdot E_y{}' \right) = \mu_0 \cdot J_y' + \frac{1}{c^2} \cdot \gamma^2 \cdot \frac{\partial}{\partial t'} \left(E_y' + v \cdot B_z{}' \right) - \gamma^2 \cdot \frac{v}{c^2} \cdot \frac{\partial}{\partial x'} \left(E_y{}' + v \cdot B_z' \right) + \frac{\partial \Lambda'}{\partial y'} \tag{239}$$

$$\frac{\partial B_x{}'}{\partial z'} - \frac{\partial B_z{}'}{\partial x'} \cdot \gamma^2 \cdot \left(1 - \frac{v^2}{c^2} \right) = \mu_0 \cdot J_y' + \frac{\gamma^2}{c^2} \cdot \left(1 - \frac{v^2}{c^2} \right) \cdot \frac{\partial E_y{}'}{\partial t'} + \frac{\partial \Lambda'}{\partial y'} \tag{240}$$

$$\frac{\partial B_x{}'}{\partial z'} - \frac{\partial B_z{}'}{\partial x'} = \mu_0 \cdot J_y' + \frac{1}{c^2} \cdot \frac{\partial E_y{}'}{\partial t'} + \frac{\partial \Lambda'}{\partial y'} \tag{241}$$

As a fifth step, perform the transformation on the z component of equation (29):

$$\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=\mu_0\cdot J_z+\frac{1}{c^2}\cdot\frac{\partial E_z}{\partial t}+\frac{\partial \Lambda}{\partial z} \tag{242}$$

$$\gamma\cdot\left(\frac{\partial B_y}{\partial x'}-\frac{v}{c^2}\cdot\frac{\partial B_y}{\partial t'}\right)-\frac{\partial B_x}{\partial y'}=\mu_0\cdot J_z+\frac{1}{c^2}\cdot\gamma\cdot\left(\frac{\partial E_z}{\partial t'}-v\cdot\frac{\partial E_z}{\partial x'}\right)+\frac{\partial \Lambda}{\partial z'} \tag{243}$$

$$\gamma^2\cdot\frac{\partial}{\partial x'}\left(B_y'-\frac{v}{c^2}\cdot E_z{}'\right)-\frac{v}{c^2}\cdot\gamma^2\cdot\frac{\partial}{\partial t'}\left(B_y'-\frac{v}{c^2}\cdot E_z{}'\right)-\frac{\partial B_x{}'}{\partial y'}= \mu_0\cdot J_z'+\frac{1}{c^2}\cdot\gamma^2\cdot\frac{\partial}{\partial t'}\left(E_z'-v\cdot B_y{}'\right)-\frac{v}{c^2}\cdot\gamma^2\cdot\frac{\partial}{\partial x'}\left(E_z'-v\cdot B_y{}'\right)+\frac{\partial \Lambda'}{\partial z'} \tag{244}$$

$$\frac{\partial B_y{}'}{\partial x'}\cdot\gamma^2\cdot\left(1-\frac{v^2}{c^2}\right)-\frac{\partial B_x{}'}{\partial y'}=\mu_0\cdot J_y'+\frac{\gamma^2}{c^2}\cdot\left(1-\frac{v^2}{c^2}\right)\cdot\frac{\partial E_z{}'}{\partial t'}+\frac{\partial \Lambda'}{\partial z'} \tag{245}$$

$$\frac{\partial B_y{}'}{\partial x'}-\frac{\partial B_x{}'}{\partial y'}=\mu_0\cdot J_z'+\frac{1}{c^2}\cdot\frac{\partial E_z{}'}{\partial t'}+\frac{\partial \Lambda'}{\partial z'} \tag{246}$$

It can be seen that equations (25) and (29) work in the K' system in the same way as in the K system. Equations (26) and (28) are also invariant to the Lorentz transformation, I leave the proof to the reader. Taking into account the invariance to the Lorentz transformation and the homogeneous version of equation (68), we can conclude that magneto scalar waves propagate at the speed of light relative to all reference systems. Let the plane wave normal vector $\hat{k}$ point in the x direction. Since the magnitude of $\hat{k}$ is 1, $\hat{k}$ can be $(1,0,0)$, or $(0,1,0)$, or $(0,0,1)$. It can be shown that the transformation of Maxwell's equations from one coordinate system to another selects $\hat{k}=(1,0,0)$, thus $\Lambda=E_x/c$. I would like to point out again that this notation of the electric field strength vector is inconsistent.

14. Practical considerations for an SLW antenna

In this chapter, I present a possible experimental implementation that brings us one step closer to understanding magneto scalar waves (scalar longitudinal wave, slw), (Hively & Loebl, Classical and extended electrodynamics, 2019), (Reed & Hively, 2020). In the case of electromagnetic radiation, we use the Lorenz measure. In the case of electro-scalar radiation, the curl-free vector potential is used.

Electromagnetic radiation	Electro-scalar radiation
$\vec{\nabla} \times \vec{E} \neq 0$ $\vec{B} \neq 0$ $\Lambda = 0$	$\vec{\nabla} \times \vec{E}_L = 0$ $\vec{B} = 0$ $\Lambda_L \neq 0$

Table 2

How can one build one and what requirements must it meet?

An antenna of this type consists of two spiral conductors connected in the middle of the coil, causing currents in opposite directions in adjacent turns. These conductors are indicated by solid (702) and dashed lines. The magnetic field is canceled by currents in opposite directions. In the case of time-varying current strength, classical electromagnetic radiation cannot be generated either. This creates the conditions for gradient-driven current density.

This coil (non-inductive bifilar coil) is a two-dimensional monopole antenna where

- • the inductance is zero (due to the opposite electric currents)
- the capacity is zero (adjacent threads have the same electric charge density)
- the charge density and current density show spatial periodicity along the length of the conductor
- the coil length must be less than λ/10 (United States Patent No. US9306527B1, 2016).

Another such antenna consists of two parts. The outer conductor (204) is electrically connected to the top of the skirt balun (206) with a length (λ/4) that is 206 (0 degrees) in current from the bottom (inner surface) to the top (inner surface) (206) (90 degrees) and back (204) (180 degrees). The 180 degree phase shift eliminates the return current along 204, forming a monopole antenna. Essentially all of the current goes to charge and discharge the center conductor (202) without creating any bias current. The resulting current density is curl-free, which is a necessary condition for generating magneto scalar and longitudinal electric waves. An eddy-free current density does not create a magnetic field (United States Patent No. US9306527B1, 2016).

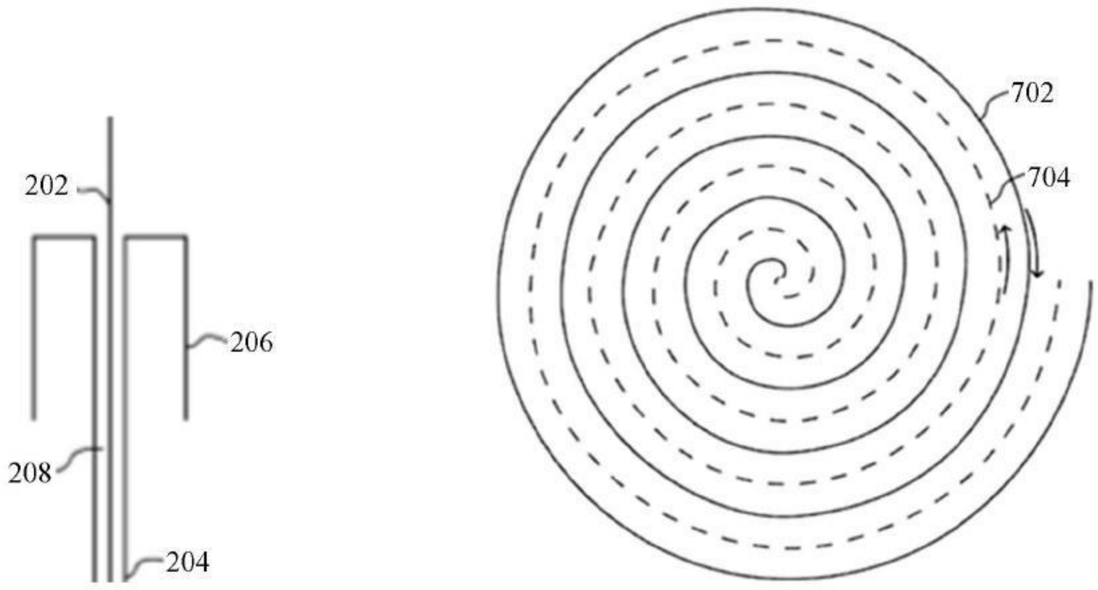


Figure 2 (Hively, 2016)

My problem with the explanation for the measurement arrangement on the right of Figure 2 is that, according to the argument, the two parallel conductors separately create the magnetic field around them. Magnetic fields cancel each other out. So, the magnetic fields are already created. Since the magnetic fields have cancelled each other out, another field cannot appear, because that would contradict the principle of conservation of energy. Since magnetic fields are created, this means that the electromagnetic interaction, which prefers conservation of energy, is making its way. The other thing is that electric and magnetic waves create each other, the two phenomena are one and the same, one cannot exist without the other. From this we can draw the conclusion that the requirement of a curl-free potential does not hold. In the case of the electro-scalar interaction, energy is a non-conservative quantity. Conditions must be created in advance for this type of interaction to make its way.

In the measurement arrangement on the left of Figure 2, if the current is reflected from the upper part of the outer conductor with a phase shift of 180 degrees and extinguishes the upward current (more precisely, a standing wave is generated). In my opinion, the inner conductor can still create a magnetic field around itself. Also, I don't understand how the displacement current would be extinguished. However, there are no measurement results on the conservation (or damage) of charge and energy, at least they do not mention it. The question may arise in us, whether we can change the nature of the electromagnetic interaction just by properly designed experimental conditions?

Based on what has been described so far, we do not have clear evidence of the existence of electro-scalar radiation.

15. Conclusion

In the early 2000s, there was a minor flare-up in the history of physics due to the article by C. Monstein and J.P. Wesley (C. Monstein, 2002), when they claimed to have experimentally supported the existence of longitudinal electromagnetic waves. Later, following K. Rębilas (Rębilas, On the origin of "longitudinal electrodynamic waves", 2008), it became clear that the experimental results can be explained within the framework of classical electrodynamics. The existence of the magneto scalar field and its consequences can only be proven or disproved by a series of well-designed experiments. Still, we cannot avoid the fact that the relations (49) and (79) derived with the modified Maxwell equations were obtained without using gauge theory. It is as if he "knows" and "selects" the right measure. The theory of modified Maxwell's equations makes predictions such as the local violation of charge conservation. In his article (Okun, 1989), Lev Borisovich Okun analyzed a total of 30 articles and came to the conclusion that this would violate the Pauli exclusion principle. So this cannot occur in nature.

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