Ecological Modelling, 14 (1981) 1–19
Elsevier Scientific Publishing Company, Amsterdam—Printed in The Netherlands

CALCULATING SOLAR RADIATION FOR ECOLOGICAL STUDIES

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(Accepted for publication 26 May 1981)

ABSTRACT

Brock, T.D., 1981. Calculating solar radiation for ecological studies. *Ecol. Modelling*, 14: 1–19.

Several approaches are presented which permit calculation by computer or hand calculator of solar radiation for any place on Earth. Some of the approaches require input of certain measured data for the location of interest, but others permit an approximation even without actual data. Calculation of solar radiation is especially useful in aquatic ecology studies on primary production, and may also be useful in ecological modeling work when solar radiation is being used as an independent variable.

INTRODUCTION

Solar radiation is an important ecological parameter, and is often the key driving force in ecological processes. Although solar radiation measurements are often available from meteorological sources, for many purposes it is desirable to be able to calculate solar radiation for a given location. Calculation is desirable for modeling purposes because it simplifies programming and obviates the necessity of keying in vast amounts of solar radiation data with the program. Also, real data may not be available for a particular location where a modeling study is desirable.

Although there is vast literature in the solar energy field dealing with calculation of solar radiation values (Duffie and Beckman, 1980), much of this literature has, as yet, not had much impact on ecological studies. As part of a broader study modeling solar radiation for aquatic productivity, I examined a significant part of this solar energy literature for approaches to calculating solar radiation. The purpose of the present paper is to provide background for ecologists on the meteorological and astronomical principles involved in calculating solar radiation, and to present several practical approaches. One attractive aspect is that it is possible to include for ecological modeling purposes the effects of such variables as latitude, time of year, altitude, atmospheric turbidity, ozone concentration and other meteorological factors. This makes possible a simple assessment, by computer modeling, of the influence of major meteorological conditions on ecosystem function.

SOLAR ENERGY

Energy units

A variety of energy units are used in the ecological and solar energy fields, and it is especially important that proper interconversions be done. Note that we are dealing here only with absolute energy units, and photometric units such as foot-candles and lux, which depend on the sensitivity of the eye, must be used with caution. The fundamental quantity of energy is the joule (J) which is equivalent to 10^7 erg (Forsythe, 1969)— 1 Watt (W) is equal to 1 J s^{-1}. Another commonly used unit is the gram-calorie, which is equal to 4.18400 J. For solar energy purposes, it is essential to express the energy value per unit area, and both cm^2 and m^2 are used. A unit frequently used in ecology is the langley, which is 1 cal cm^{-2}. Because different wavelengths of electromagnetic radiation have different energy values, it is convenient for many purposes to express energy in terms of the quantum, but to convert quanta into energy units it is necessary to state the wavelength of radiation involved. For ecological purposes, where the main concern is

TABLE I

Energy units and interconversions (Forsythe, 1969) *

Energy
1 cal = 4.18400 J
1 E = 6.02288×10^{23} quanta (photons)
1 E at 600 nm radiation = 1.993306×10^5 J
Power (energy per unit time)
1 W = 1 J s^{-1}
Radiant flux (power per unit area)
1 E s^{-1} m^{-2} = 6.02288×10^{23} quanta s^{-1} m^{-2}
1 W m^{-2} = 4.6 μE s^{-1} m^{-2} (400–700 nm)
1000 lx = 19.53 μE s^{-1} m^{-2} (400–700 nm)
1 langley = 1 cal cm^{-2}

* See text for details. Integral radiant flux over the 400–700 nm range approximated by McCree (1972). Conversion from lx (photometric unit) to μE assumes a value of 683 lumens W^{-1} at 555 nm.

absorption of solar radiation by living organisms, the spectral distribution of radiation should be considered (McCree, 1972). One quantum at 600 nm is equivalent to 3.3096×10^{-19} J, or 7.91021×10^{-20} cal (Forsythe, 1969). Another unit is the einstein (E), which is 1 mol of quanta (more accurately, the einstein is the number of quanta necessary to induce a photochemical reaction in 1 mol of chemical substance). To calculate the amount of energy available over a portion or all of the visible spectrum, it is necessary to obtain the integral of the function relating energy to wavelength. Although complex, an approximation to this integral has been given by McCree (1972) for sunlight for the photosynthetically active range of 400–700 nm as follows: $1\ \mathrm{W\ m^{-2}} = 4.6\ \mu\mathrm{E\ s^{-1}\ m^{-2}}$. For some purposes, a wavelength in the middle of the visible spectrum can be used, such as 550 or 600 nm. Some of these energy unit interconversions are summarized in Table I.

The solar constant

The 'solar constant' is the energy received per unit time, at the Earth's mean distance from the Sun, outside the atmosphere. Recent high-altitude measurements have provided a fairly precise estimate of the solar constant, and a standard value, which has been accepted by the U.S. National Aeronautical and Space Administration and the American Society for Testing Materials, has been given by Duffie and Beckman (1980) as $1353\ \mathrm{W\ m^{-2}}$ or $1.940\ \mathrm{cal\ cm^{-2}\ min^{-1}}$ or $428\ \mathrm{BTU\ ft.^{-2}\ h^{-1}}$ or $4.871\ \mathrm{MJ\ m^{-2}\ h^{-1}}$.

MOTIONS OF THE EARTH IN RELATION TO THE SUN

Solar radiation at any location on Earth is influenced by the motions which the Earth makes in relation to the Sun. The Earth is tilted 23.45° from the plane of the Earth's orbit. At the autumn and spring equinoxes (21 March and 22 September) the Sun shines equally in both hemispheres. At the summer solstice (22 June in the Northern Hemisphere) the Sun is directly overhead at 23.45°N and at the winter solstice (22 December) the Sun is directly overhead at 23.45°S.

Declination

The declination of the Earth is the angular distance at solar noon between the Sun and the Equator, north-positive. Declination depends only on the day of the year, and will be opposite in the Southern Hemisphere. The declination is obtained precisely from ephemeris tables (The American Ephemeris and Nautical Almanac, U.S. Government Publishing Office, Washington, DC, published annually; see also List, 1971), but can be calculated close enough for all practical purposes from the equation given by Cooper (1969)

$$D1\ (\text{declination}) = 23.45 \sin[360\ (284 + N)/365] \qquad (1)$$

where N is the number of days after 1 January (the Julian Day). For a list of abbreviations, see Appendix.

Radius vector

During its revolution around the Sun, the Earth's distance varies with time of year by 3.0%, due to the Earth's eccentric orbit. This eccentricity influences in a minor way the amount of solar radiation impinging on the Earth's surface. The radius vector of the Earth, $R1$, expresses this ellipticity and can be calculated approximately from the following equation (Nicholls and Child, 1979)

$$\mathbf{R}1 = 1/\{1 + [0.033 \cos(360N/365)]\}^{1/2} \qquad (2)$$

where the number whose cosine is being taken is in degrees. To correct the solar constant for ellipticity of the Earth's orbit, the value is divided by $\mathbf{R}1^2$. Over an annual cycle, **R**1 varies from 0.98324 to 1.01671, so that this correction is not great.

Hour-angle and daylength

The other major motion is the daily rotation of the Earth around itself. The Earth moves 15° per hour and the sunset (or sunrise) hour-angle, $W1$, is the angle between the setting Sun and the south point. The value $W1$ can be calculated if the latitude (L) and declination are known (Milankovitch, 1930)

$$W1 = \arccos\{-[\tan(L) \tan(D1)]\} \qquad (3)$$

In this equation, if L and $D1$ are in degrees then $W1$ will be given in degrees. From $W1$, the daylength in hours, $L1$, can be calculated from the following equation

$$L1\ (\text{daylength}) = (W1/15)2\ (W1 \text{ in degrees}) \qquad (4)$$

If daylength is known, then the time of sunrise and sunset can be calculated from the equations

$$\text{Sunrise} = 12 - 1/2L1$$

$$\text{Sunset} = 12 + 1/2L1$$

The hour-angle at any given time can be calculated from one of the following equations

$$W2 = (T - 12)15 \tag{5}$$

$$W2 = 0.25(\text{minutes}) \tag{6}$$

where T is the time (h) from midnight and minutes is the number of minutes from solar noon.

Note that angles are often expressed or calculated in radians instead of degrees and it is essential to maintain consistency. To convert degrees to radians, multiply by $\pi/180$.

Latitude

The other parameter needed to define the geometric relationships between any position on Earth and the Sun is the latitude, which is the angular location north or south of the Equator, north positive. The latitude of a location is obtained from geographic maps and for present purposes need not be expressed more accurately than three significant figures.

Zenith angle

The zenith is the point vertically above the position in question. The zenith angle is the angle defined by the zenith and the position of the Sun and can be calculated from trigonometric relationships for any point on Earth for any time of day (see below). It is the zenith angle which is related to the intensity of solar radiation on a flat surface.

CALCULATING SOLAR RADIATION AT THE TOP OF THE ATMOSPHERE

If the solar constant is known, the solar radiation available at the top of the atmosphere can be determined from trigonometric relationships, with the following formula

$$I1 = \mathrm{I}\Theta / R1^2 \cos(Z) \tag{7}$$

where I0 is the solar constant, $R1$ is the radius vector and Z is the zenith angle.

As first developed by Milankovitch (1930), the zenith angle, Z, can be calculated if the declination, $D1$, the latitude, $L1$, and the hour-angle, $W2$, are known. The formula relating these parameters to zenith angle is the following

$$\cos(Z) = \sin(D1)\sin(L) + \cos(D1)\cos(L)\cos(W2) \tag{8}$$

Since $D1$ and $W2$ can be calculated from time of day and day of year (see above), and L is given for the location of interest, the zenith angle can be calculated. A simple computer program can be written that permits calculation of solar radiation at the top of the atmosphere for any time of day and day of year at any latitude. For many ecological purposes, this calculation may be sufficient, although it is likely that the effects of the atmosphere will be needed for most ecological studies (see below). Note that because eq. 8 has incorporated into it time of day and day of year (through $D1$ and $W2$), it can be used to define precisely the diurnal curve of solar radiation for any point on Earth.

The equation of Vollenweider (1965), frequently used in aquatic ecology literature, is similar to the above

$$I4 = I5/2\{1 + \cos[2P1(T-12)/L1]\} \tag{9}$$

where $I5$ is the solar radiation per unit area for the hour around solar noon (calculated from eqs. 5, 7, and 8), T is the time of day in hours, $L1$ is the day length in hours, calculated as described above, and $I4$ is the solar radiation per unit area for any hour. Because $L1$ is calculated from the same trigonometric relationships, Vollenweider's equation reduces to that of Milankovitch but may have some use in calculating diurnal curves without a computer.

Another relationship that has been used occasionally is that of Ikushima (1967)

$$\text{solar radiation at time } T = \text{maximum hourly solar radiation} \sin^3[(P1/L1)T]$$

which was empirically derived from meteorological data in Japan. Under many conditions, Ikushima's relationship is similar to that presented above, but has no real advantage.

DAILY INTEGRATED VALUES OF SOLAR RADIATION

For many purposes, it is not the diurnal curve of solar radiation that is needed, but an integrated value for the whole day. Milankovitch (1930) integrated eq. 7 and obtained the following equation, which has been widely used in the solar energy field (Liu and Jordan, 1960; Duffie and Beckman, 1980)

$$\begin{aligned} I6 = {} & (24/P1)(\mathrm{I0}/R1^2)[W1P1/180 \sin(L) \sin(D1) \\ & + \sin(W1) \cos(L) \cos(D1)] \end{aligned} \tag{10}$$

where $W1$ is the sunset hour-angle and I0 is the solar constant in energy units per hour. As shown by Milankovitch (1930), this equation will not work for latitudes where the Sun does not come up at all in the winter. For such locations, the following equation can be used

$$I6 = 24I0/R1^2 \sin(L) \sin(D1) \tag{11}$$

To calculate IH, the radiation for a given time period (useful in primary productivity studies), values of the hour angle $W2$ are calculated for the beginning ($W2$B) and end ($W2$E) of the interval of interest (using eq. 5 or 6) and eq. 11A below is then used (Duffie and Beckman, 1980)

$$IH = I2/P1I0/R1^2\{\cos(D1)\cos(L)\left[\sin(W2\mathrm{E}) - \sin(W2\mathrm{B})\right] + P1/180(W2\mathrm{E} - W2\mathrm{B})\sin(D1)\sin(L)\} \tag{11A}$$

AVERAGE SOLAR RADIATION FOR THE MONTH

For many purposes, it is convenient to calculate a solar radiation value which would be an average for a given month. This can be done by selecting an average day for the month and using this day in the calculations described above. Table II gives values recommended for solar energy purposes for each month of the year. The average day given is that day which has the extraterrestrial radiation closest to the average for the month (Klein, 1976).

TABLE II

Recommended average day for each month and values of Julian day by month (from Duffie and Beckman, 1980) *

Month	Date	Julian day	Declination (°)
January	17	17	−20.9
February	16	47	−13.0
March	16	75	−2.4
April	15	105	9.4
May	15	135	18.8
June	11	162	23.1
July	17	198	21.2
August	16	228	13.5
September	15	258	2.2
October	15	288	−9.6
November	14	318	−18.9
December	10	334	−23.0

* The average day is that day which has the extraterrestrial radiation closest to the average day for the month. Values do not account for leap year; correct by adding 1 to months from March onward. Declination values will also shift slightly.

CALCULATING RADIATION AT THE SURFACE OF THE EARTH

Radiation at the surface of the Earth is composed of two components, direct and diffuse. Together, these comprise the global radiation received at a location

$$\text{Global radiation} = \text{Direct} + \text{Diffuse} \tag{12}$$

A vast amount of literature has developed regarding the calculation or measurement of direct and diffuse solar radiation at the Earth's surface. The principles involved are given in considerable detail by Robinson (1966), who also gives a number of practical examples and nomographs.

Direct solar radiation is that arising from direct penetration of the Sun's rays to the surface of the Earth, whereas diffuse solar radiation reaches the Earth's surface as a result of scattering from atmospheric particles. Diffuse radiation is of greater significance when atmospheric particles are present in larger amounts, and is of less significance in clear air.

Attenuation of direct solar radiation by the atmosphere can be divided into several components: (1) scattering, which is due primarily to small particles and has a pronounced wavelength dependence; (2) absorption, primarily by gases such as ozone, water vapor and carbon dioxide; and (3) turbidity, which is due to attenuation by larger particles (dust, etc.). Turbidity is handled differently from the other two components and will be discussed later. As discussed by Robinson (1966), scattering occurs in accordance with Rayleigh's theory, which indicates that the scattering coefficient varies approximately as $(\text{wavelength})^{-4}$. However, water-vapor scattering, which is the major component, depends on the amount of precipitable water in the air, and because of the difficulty of obtaining water-vapor data, calculation of the Rayleigh scattering coefficient is of limited practical utility (Duffie and Beckman, 1980).

Absorption of radiation is due primarily to ozone in the ultraviolet region and water vapor in the infrared. At wavelengths above 350 nm there is essentially no absorption by ozone, so that ozone absorption can probably be ignored for ecological studies unless ultraviolet effects are being studied. Again, since water vapor absorbs only in the infrared, absorption by this component can be ignored for ecological studies.

The extent of atmospheric extinction due to absorption and scattering is a function of the relative air mass, which is the ratio of the air mass at the current angle of the Sun to the air mass when the Sun is at the zenith. At sea level, the relative air mass is equivalent to the secant of the zenith angle (1/cosine of the zenith angle). This relationship is valid for zenith angles less than about 85° (Robinson, 1966). Correction for altitude above sea level can be obtained by multiplying the relative air mass by the ratio of atmospheric pressure at altitude to that at sea level.

Turbidity

Although the depletion of direct solar radiation by absorption or scattering is relatively minor and can be calculated from the relative air mass, the effect of atmospheric turbidity can be a major factor. Turbidity is due to water vapor, dust, haze or other aerosol particles. Turbidity is obviously site-dependent, and, except for general relationships, cannot be calculated. It must be measured, and from such measurements, site-specific relationships may be found. This has been discussed in considerable detail by Robinson (1966), who should be referred to for details on the above relationships.

For many ecological purposes, calculation of solar radiation at the Earth's surface with the atmospheric attenuation parameters described above is not necessary. As discussed by Duffie and Beckman (1980), large amounts of solar radiation data for many parts of the world are available which can be used to estimate average incident radiation. From such averages, it is possible to calculate daily, monthly, or diurnal values of solar radiation which will provide useful input to ecological models. The rest of this paper will deal with procedures for such calculations.

ESTIMATION OF AVERAGE SOLAR RADIATION

A simplified and very useful discussion of procedures for using weather bureau data to estimate solar radiation is given by Duffie and Beckman (1980), and other discussions can be found in Liu and Jordan (1960), Lof et al. (1966), deJong (1973), and many recent papers in the journal Solar Energy. If reliable solar radiation measurements are available for a site of interest, it is then possible to use these data to estimate average incident radiation, and then to use this estimate in modeling studies. If such data are not available, the empirical relationship between radiation and hours of sunshine or cloudiness can be used. This relationship, first used by Angstrom (1924), involves the input of actual data to obtain coefficients for a regression equation of the following type

$$I2/I6 = A + [B(N1/L1)] \tag{13}$$

where $I6$ is the radiation outside the atmosphere, calculated as described in eq. 10 or 11, $I2$ is the daily radiation measured for the average day of the month, A and B are empirical constants (not measured, but obtained by regression), $N1$ is the measured daily hours of bright sunshine, $L1$ is the maximum possible daily hours of bright sunshine (equivalent to the day length of the average day of the month, as calculated by eq. 4). Since $I6$ and $L1$ can be calculated (eqs. 4 and 10), and A, B and $N1$ can be measured, it is possible to estimate $I2$. The ratio $I2/I6$ is termed the clearness index in the solar energy literature, and is frequently used in calculations. The climatic constants needed for this equation have been determined for a large number of areas of the world, as given by Lof et al. (1966) and deJong (1973). Some representative values are given in Table III.

TABLE III

Climatic constants for eq. 13 (from Lof et al., 1966) *

Location	Climate	Vegetation	*N*1		*A*	*B*
			Range	Average		
Albuquerque, NM	BS–BW	E	68–85	78	0.41	0.37
Atlanta, GA	Cf	M	45–71	59	0.38	0.26
Blue Hill, MA	Df	D	42–60	52	0.22	0.50
Brownsville, TX	BS	GDsp	47–80	62	0.35	0.31
Buenos Aires, Argentina	Cf	G	47–68	59	0.26	0.50
Charleston, SC	Cf	E	60–75	67	0.48	0.09
Darien, Manchuria	Dw	D	55–81	67	0.36	0.23
El Paso, TX	BW	Dsi	78–88	84	0.54	0.20
Ely, NV	BW	Bzi	61–89	77	0.54	0.18
Hamburg, Germany	Cf	D	11–49	36	0.22	0.57
Honolulu, Hawaii	Af	G	57–77	65	0.14	0.73
Madison, WI	Df	M	40–72	58	0.30	0.34
Malange, Angola	Aw–BS	GD	41–84	58	0.34	0.34
Miami, FL	Aw	E–GD	56–71	65	0.42	0.22
Nice, France	Cs	SE	49–76	61	0.17	0.63
Poona, India	Am	S	25–49	37	0.30	0.51
(monsoon, dry)			65–89	81	0.41	0.34
Stanleyville, Congo	Af	B	34–56	48	0.28	0.39
Tamanrasset, Algeria	BW	Dsp	76–88	83	0.30	0.43

* Climatic classification based on Trewartha. Vegetation classification based on Kuchler: (Af) tropical forest climate, constantly moist; rainfall all through the year; (Am) tropical forest climate, monsoon rain; short dry season, but total rainfall sufficient to support rain forest; (Aw) tropical forest climate, dry season in winter; (BS) steppe or semiarid climate; (BW) desert or arid climate; (Cf) mesothermal forest climate—constantly moist, rainfall all through the year; (Cs) mesothermal forest climate, dry season in winter; (Df) microthermal snow forest climate, constantly moist, rainfall all through the year; (Dw) microthermal snow forest climate, dry season in winter; (B) broadleaf evergreen trees; (Bzi) broadleaf evergreen, shrubform, minimum height 3 ft., growth singly or in groups or patches; (D) broadleaf deciduous trees; (Dsi) broadleaf deciduous, shrubform, minimum height 3 ft., plants sufficiently far apart that they frequently do not touch; (Dsp) broadleaf deciduous, shrubform, minimum height 3 ft., growth singly or in groups or patches; (E) needle-leaf evergreen trees; (G) grass and other herbaceous plants; (GD) grass and other herbaceous plants, broadleaf deciduous trees; (GDsp) grass and other herbaceous plants, broadleaf deciduous, shrubforms, minimum height 3 ft., growth singly or in groups or patches; (M) mixed broadleaf deciduous and needle-leaf evergreen trees; (S) semideciduous broadleaf evergreen and broadleaf deciduous trees; (SE) semideciduous, broadleaf evergreen and broadleaf deciduous trees, needle-leaf evergreen trees.

Equation 13 and the data in Table III provide a useful practical procedure for estimating the daily solar radiation for the average day each month. The procedure has been explained in detail by Duffie and Beckman (1980) and will be outlined here. From Table III, the coefficients A, B and $N1/L1$ for a given location are selected. For $N1/L1$, which is the sunshine hours as a fraction of the possible, a yearly average can be used, or a value for a given month, if this is available. From eq. 10 or 11 the value of $I6$ is calculated, and from eq. 4 the value of $L1$ is calculated, using the Julian day appropriate for that month (see Table II). Upon substitution of these values in eq. 13, the value of $I2$ can be calculated. The value of $I2$ obtained would then be the total daily solar radiation for the average day in that month. Because solar radiation changes very slowly as the year progresses, it is acceptable for most purposes to use a single value of $I2$ for each day of a whole month. Table IV provides a comparison of values of $I2$ calculated by eq. 13 with measured

TABLE IV

Comparison of calculated and measured values of total daily radiation for Madison, WI (latitude 43°) *

Month	Day	Day-length (h)	Sunshine (h)	Radiation ($MJ\ m^2$) Top of atmosphere	Calculated Earth's surface	Measured Earth's surface
January	17	9.2	4.5	12.8	6.0	5.9
February	47	10.3	5.7	18.2	8.9	9.1
March	75	11.7	6.9	25.5	12.8	12.9
April	105	13.2	7.5	33.7	16.6	15.9
May	135	14.5	9.1	39.9	20.5	19.8
June	162	15.1	10.1	42.6	22.4	22.1
July	198	14.8	9.8	41.4	21.7	22.0
August	228	13.7	9.9	36.4	19.9	19.4
September	258	12.3	8.6	28.7	15.4	14.8
October	288	10.8	7.2	20.5	10.8	10.3
November	318	9.5	4.2	14.1	6.4	5.7
December	334	9.1	4.0	12.1	5.5	4.4

* Calculations using the average day of each month (see Table II). Daylength calculated using eqs. 3 and 4. Sunshine hours input from average values provided by Duffie and Beckman (1980). Radiation at the top of the atmosphere calculated from eqs. 7 and 8. Radiation at the Earth's surface calculated from eq. 13, using average values of A and B of 0.30 and 0.34 (see Table III). Measured values from the U.S. Weather Bureau as provided by Duffie and Beckman (1980).

values for each month of the year for a single site, Madison, WI. The correspondence between calculated and measured values is fairly close.

Because climatic constants A, B and $N1/L1$ are available for various parts of the world, the procedure outlined in this section is probably preferable for most ecological purposes. A detailed listing of climatic constants for many locations is given in Lof et al. (1966) and deJong (1973).

Note that in the procedure used in this section, empirical data are used which include, within them, corrections for all of the major atmospheric attenuation factors, scattering, absorption and turbidity. For some purposes it may be desirable to estimate clear-sky radiation for a given location (that is, the maximum possible solar radiation at a given location), and for this purpose estimates of atmospheric attenuation coefficients are necessary. The use of such attenuation coefficients in calculating clear-sky radiation is discussed in a later section.

ESTIMATION OF THE DIURNAL SOLAR RADIATION CYCLE FROM DAILY AVERAGES

Daily average solar radiation values for a given site can either be obtained by calculation using measured coefficients in eq. 13, as described in the previous section, or can be obtained for a given day and year from measured data (from meteorological sources). For many purposes, it is desirable to use hourly values rather than daily ones. This is discussed in some detail by Liu and Jordan (1960) and Duffie and Beckman (1980). The ratio of solar radiation for an hour to total daily irradiation, $R3$, is approximated from the following equation

$$R3 = (P1/24)[A1 + B1\cos(W2) - \cos(W1)]/ \\ [\sin(W1) - W1(P1/180)\cos(W1)] \tag{14}$$

where $W1$ is the sunset hour-angle in degrees, calculated from eq. 3, and $W2$ is the hour-angle at time T, calculated from eq. 5. The coefficients $A1$ and $B1$ are calculated from the following equations (Collares-Pereira and Rabl, 1979)

$$A1 = 0.409 + 0.5016\sin(W1 - 60) \tag{15}$$

$$B1 = 0.6609 - 0.4767\sin(W1 - 60) \tag{16}$$

To calculate solar radiation for any hour, determine the value of $R3$ for that hour and then use the following equation

$$I4 = R3I2 \tag{17}$$

Another approach makes use of eq. 9 (Vollenweider, 1965), which permits calculation of hourly values from the maximum daily values. To obtain the latter, $W2$ is set to 0 (solar noon), and the value of $R3$ obtained in eq. 14 is then the maximum ratio of hourly to total daily irradiation for the day. If this value of $R3$ is multiplied by the total daily radiation at the Earth's surface, $I2$, then the maximum daily intensity, $I5$, is obtained

$$I5 = R3I2 \tag{18}$$

The value of $I5$ can then be inserted into Vollenweider's equation (eq. 9) to permit calculation of solar radiation for any time of day.

By either procedure, it is possible to draw a diurnal solar radiation curve for any site of interest for which total daily radiation is known. Such diurnals curves are of considerable utility in primary production calculations in aquatic systems (Fee, 1977).

Lof et al. (1966) and Kondratyev (1969) have compiled values of total daily radiation for the average day of each month for a large number of locations on Earth. These values can be used as variable $I2$ in eq. 17.

ESTIMATION OF CLEAR-SKY RADIATION

The procedure described in the previous section permits estimation from climatic data of total daily solar radiation for any day of the year, and hourly values for any hour of the day. The values obtained will be reasonably close to actual values measured with instruments, and will be very useful in primary production calculations and other procedures in which an average daily or hourly value is needed. However, the values obtained will be averages, and on clear days values higher than the average will obtain. For many purposes it is useful to use clear-sky radiation values. Theoretical discussion of this problem can be found in Robinson (1966), and, as outlined earlier, requires a knowledge of absorption, scattering and turbidity components. It should be obvious that 'clear sky' is not a constant throughout the world, and climatic factors will influence the clarity of the air. Duffie and Beckman (1980) present several practical approaches to estimation of clear-sky radiation which are suitable for ecological purposes.

The simplest approach is to define a standard atmosphere and use data from this standard for calculations. Such a standard atmosphere, based on measurements made in Minnesota, has been defined by the American Society for Heating, Refrigerating and Air-conditioning Engineers (ASHRAE), and Farber and Morrison (1977) provide data of total radiation as a function of zenith angle for this standard atmosphere. The data of Farber and Morrison (1977) fit the following regression equation

$$\text{Radiation (W m}^{-2}) = 1121[\cos(Z) - 0.08251] \tag{18A}$$

Since zenith angle can be calculated for any location from eq. 8, it is possible to use this equation to estimate clear-sky radiation. For a given day, hourly estimates can be made using the mid-points of the hours. From such hourly estimates over the whole day, a daily total can be calculated.

A more general approach takes into consideration climatic factors causing atmospheric attenuation (Hottel, 1976). A transmission factor (TR) for radiation from the following equation

$$TR = \mathrm{AC} + \mathrm{AB}\ \exp[-\mathrm{K}/\cos(Z)] \tag{19}$$

where AC, AB and K are empirical constants which are related to altitude and climate. These constants for a standard atmosphere with 23 km visibility are found from the following equations

$$\mathrm{AC} = 0.4237 - 0.00821(6 - AT)^2 \tag{20}$$

$$\mathrm{AB} = 0.5055 + 0.00595(6.5 - AT)^2 \tag{21}$$

$$\mathrm{K} = 0.2711 + 0.01858(2.5 - AT)^2 \tag{22}$$

where AT is the altitude in kilometers. The constants obtained are corrected for climatic types by use of Table V, where RC is the correction factor for AC, RB is the correction factor for AB, and RK is the correction factor for K. From these equations, a transmission factor for a standard atmosphere can be calculated which can then be used in calculation of clear-air radiation at any time, using the following formula

$$IC = IH\ TR \cos(Z) \tag{23}$$

which gives the value of direct radiation for any hour for which Z is calculated. In this equation, IH is a value for an appropriate area and time calculated as $I1$ in eq. 7. To this must be added the transmission coefficient (TD) for clear-sky diffuse radiation, which can be estimated from the empirical relationship developed by Liu and Jordan (1960) between the

TABLE V

Correction factors for climate types (from Hottel, 1976) *

Climate type	RC	RB	RK
Tropical	0.95	0.98	1.02
Mid-latitude summer	0.97	0.99	1.02
Subarctic summer	0.99	0.99	1.01
Mid-latitude winter	1.03	1.01	1.00

* Correction factors to convert values calculated in eqs. 20, 21 and 22 to values appropriate for eq. 19. Multiply the values in eqs. 20, 21 and 22 by the above factors. RC is the factor for AC, RB for AB, and RK for K.

transmission coefficient for direct and diffuse radiation

$$TD = 0.2710 - 0.2939TR \tag{24}$$

where TD is the ratio of diffuse radiation to extraterrestrial radiation. Then to calculate diffuse radiation, the following equation is used

$$ID = IH\ TD\cos(Z) \tag{25}$$

To obtain total clear-sky radiation, IT, the direct and diffuse components are added:

$$IT = IC + ID \tag{26}$$

For further discussion of the complications regarding the calculation of diffuse radiation see Robinson (1966) and Duffie and Beckman (1980). Also note that for some ecological purposes, the differential scattering or absorption of some wavelengths of photosynthetically active radiation may need to be considered.

A SIMPLER APPROACH TO CALCULATION OF TOTAL CLEAR-SKY RADIATION

The American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) has presented a simpler approach to calculation of clear-sky solar radiation (both direct and diffuse) (ASHRAE, 1977). The following formula is used to calculate direct solar radiation

$$I3 = I7/\exp[B2/\cos(Z)] \tag{27}$$

where $I7$ is the apparent solar radiation at air mass = 0, $B2$ is an empirical

TABLE VI

Data for calculation of solar radiation using eqs. 27 and 28 (from ASHRAE, 1977)

Date	A (MJ m^{-2} h^{-1})	$B2$ (air mass^{-1})	C (dimensionless)
21 January	4.438	0.142	0.058
21 February	4.382	0.144	0.060
21 March	4.279	0.156	0.071
21 April	4.097	0.180	0.097
21 May	3.983	0.196	0.121
21 June	3.926	0.205	0.134
21 July	3.915	0.207	0.136
21 August	3.995	0.201	0.122
21 September	4.154	0.177	0.092
21 October	4.302	0.160	0.073
21 November	4.404	0.149	0.063
21 December	4.450	0.142	0.057

atmospheric extinction coefficient (both obtained from Table VI), and $\cos(Z)$ is calculated from eq. 8. To obtain the diffuse component, the following equation is used

$$ID = C\, I3 \tag{28}$$

where C is an empirical constant obtained from Table VI. Although considerably more simplified, this approach may be useful for quick calculations or when a computer is not available.

RECOMMENDATIONS

A number of procedures for calculating solar radiation have been presented which eliminate the need for inputing raw data. Although all of these procedures lead to approximations of the 'true' solar radiation, they may be more accurate for many purposes than real data which, for many parts of the world, are of uncertain reliability. Some of the procedures require input of 'average' values for the locality of interest, whereas others permit calculation strictly from geographical and astronomical factors.

(1) If extensive amounts of local data are available, so that average values for the month can be calculated (or have already been published), then it is possible to generate a diurnal curve for any day of the year, using eqs. 14–17.

(2) If average monthly values are not available, Angstrom coefficients may have been determined. Much data of this type can be obtained from Lof et al. (1966) and deJong (1973). Angstrom coefficients can be used in eq. 13 to obtain a value for total daily radiation for any day of the year, and from this value a diurnal curve can be generated using eq. 14.

(3) If Angstrom coefficients are not available, it may be possible to get or record real solar radiation measurements for the whole year (or season of interest) and use these measurements to calculate an average daily value for each month, and then use eqs. 14–17 to generate diurnal curves, as discussed in (1) above.

(4) If only clear-sky radiation values are needed, then the standard curve of Farber and Morrison (1977), the regression equations of Hottel (1976) or the standard atmosphere of ASHRAE can be used to calculate clear-sky radiation for any hour of interest, using the calculated zenith angle of the sun.

(5) If clear-sky values are calculated as in (4), another approach to get values for an average day is to obtain or measure solar radiation for the locality for a year and calculate an average attenuation factor by which the local values are reduced from the clear-sky values. This attenuation factor can then be multiplied by the value obtained by calculation by the procedure of (4).

(6) It should be emphasized that none of the procedures outlined will permit calculation for a specific day of a specific year. Obviously, a day that is cloudy one year may not be cloudy the next. If data for a specific day are needed, then the only way these data can be obtained is by measurement.

ACKNOWLEDGMENTS

This work was supported by the College of Agricultural and Life Sciences, and by a research grant from the National Science Foundation (DEB-7906030).

APPENDIX

Symbols used

A Coefficient in eq. 13, dimensionless.
AB Empirical constant in eq. 19, calculated in eq. 21.
AC Empirical constant in eq. 19, calculated in eq. 20.
AT Altitude above sea level, kilometers.
*A*1 Coefficients used in eq. 14, dimensionless; eq. 15.
B Coefficient in eq. 13, dimensionless.
*B*1 Coefficient used in eq. 14, dimensionless; calculated in eq. 16.
*B*2 ASHRAE empirical extinction coefficient in eq. 27, air mass^{-1}.
C Empirical constant in eq. 28.
*D*1 Declination of the Earth, degrees; eq. 1.
IC Clear air radiation at the Earth's surface per unit area for any interval of time; eq. 23.
ID Total diffuse radiation; eqs. 25 and 28.
IH Solar radiation per unit area for any interval of time, top of the atmosphere; eq. 11A.
IT Total clear sky radiation; eq. 26.
I0 Solar constant, 1.94 cal cm^{-2} or 1353 W m^{-2} or 4.871 MJ m^{-2} h^{-1}.
*I*1 Solar radiation per unit area per unit time, top of the atmosphere; eq. 7.
*I*2 Total solar radiation per unit area for the day, Earth's surface; eq. 13.
*I*3 Solar direct radiation per unit area per hour, Earth's surface; eq. 27.
*I*4 Solar radiation per unit area for any hour; eqs. 9 and 17.
*I*5 Solar radiation per unit area for an hour around solar noon; eqs. 9 and 18.
*I*6 Total solar radiation per unit area per day, top of the atmosphere; eq. 10 or 11.
*I*7 Average clear-sky radiation in Table VI, per unit area per hour.
K Empirical constant in eq. 19 (calculated in eq. 22).

L Latitude (°).
$L1$ Day length (h); eq. 4.
N Julian day (number of days after 1 January).
$N1$ Measured sunshine hours for given day.
$P1$ Value of π, 3.14159.
RB Correction factor for AB in eq. 21; see Table V.
RC Correction factor for AC in eq. 20; see Table V.
RK Correction factor for K in eq. 22; see Table V.
R1 Radius vector of the Earth, dimensionless; eq. 2.
$R3$ Ratio of hourly to total daily radiation for any hour; eq. 14.
T Time (h).
TD Transmission coefficient, diffuse radiation; eq. 24.
TR Transmission factor, dimensionless; eq. 19.
$W1$ Sunset hour-angle (°); eq. 3.
$W2$ Hour-angle at any hour (°); eq. 5.
Z Zenith angle (°); eq. 8.

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