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Short communication

Estimating solar radiation on slopes of arbitrary aspect

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Abstract

Solar radiation is a major environmental factor, controlling for example, plant growth, inactivation of fecal microbial contaminants, and soil property evolution. Modeling the magnitude and complexity of changes in solar radiation under cloudy conditions is a challenge. In this paper, we present a simple approach for estimating daily global solar radiation on any sloping surface with arbitrary aspect given global radiation data for nearby flat terrain. The system is a further development of existing models and consists of two parts. The first part estimates direct and diffuse components of solar radiation from measured global radiation for a horizontal surface. The second part uses separate estimates of direct and diffuse radiation to calculate the daily global radiation throughout the year on slopes of different steepness and aspect. The performance of the system was verified with solar radiation data on various slopes and aspects at latitude 37.8°S in New Zealand. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Solar radiation; Trigonometric equations; Slope orientation; Spatial modelling

1. Introduction

Accurate information on daily global solar radiation on land surfaces is important to agricultural and ecological systems modeling. Although climate stations provide reliable solar radiation records, they usually record radiation only for horizontal surfaces and data are not available with sufficient coverage or resolution for many applications (Iqbal, 1983). Solar radiation on undulating terrain is further complicated by the high variability of slope orientations relative to the sun, which can vary radically over short distances.

Therefore, a means for estimating solar radiation for an arbitrary surface orientation would be useful in several agricultural and environmental fields.

The amount of solar radiation actually arriving at a particular point is called global (or total) solar radiation and it depends mainly on cloudiness, the time of the year, latitude, and surface geometry (Iqbal, 1983). Global radiation for horizontal surfaces is the sum of the direct irradiation in the solar beam and the diffuse solar radiation scattered in the direction of the monitoring point from the solar beam as it passes through the atmosphere. Direct radiation may be a small fraction of the global radiation on cloudy days. Radiation reflected from the ground may be important in mountainous regions. Therefore, the global radiation received on surfaces of arbitrary slope and aspect

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is largely controlled by atmospheric and topographic conditions.

A number of models for estimating topographical solar radiation have been developed in recent years (Bland and Clayton, 1994; Cooter and Dhakhwa, 1995; Dubayah and Rich, 1995; McKenney et al., 1999; Thornton et al., 2000). However, these models use theoretical approaches for local cloudiness without reference to meteorological data. Predicting transmittance of solar radiation requires data on the type and optical properties of the atmosphere and clouds, and cloud cover, height, thickness and number of layers. Such theoretical determination of direct and diffuse irradiance might be appropriate in continental climates. However, in New Zealand's maritime climate, cloudy conditions are common and highly variable in character, such that parameterizing radiation models is difficult (McAneney and Noble, 1976; Benseman and Cook, 1969).

Because radiation is important to our modeling requirements, we decided to develop a model for predicting the solar radiation on slopes using the data from a nearby climate station. Revfeim (1978) proposed a useful model to predict daily radiation as a function of slope and aspect with reference to the irradiation on horizontal surfaces. However, application of this model is limited by its requirement of separate direct and diffuse radiation as inputs, because these components of global radiation are not normally measured separately at climate stations. Fortunately, there are ways for estimating the direct and diffuse components from the global radiation.

Liu and Jordan (1960) showed that the ratio of the daily total radiation to the extraterrestrial daily insolation on a horizontal surface (K_t) is a good index of the sky cloudiness. Because the diffuse proportion of daily radiation depends mainly on cloudiness, this proportion can be estimated from K_t .

This paper presents a system for estimating radiation on a slope of any aspect in relation to global radiation. Our approach combines the models of Liu and Jordan (1960), and Revfeim (1978). We separated direct and diffuse components of total radiation for horizontal surfaces using measured global radiation, G_m . We then used predictions of the diffuse and direct radiation in the model of Revfeim (1978) to predict daily radiation on slopes. The approach was verified using simultaneous measurements of

global solar radiation received by sensors with different aspect and slope at latitude 37.8°S in New Zealand. The approach should be useful for estimating daily global radiation on slopes for a range of applications.

2. Model development

Trigonometric equations for sun positions relative to the land surface are an important component of the modeling approach, and are given in appendices.

2.1. Ratio of diffuse to total solar radiation

The fraction of global radiation that is diffuse can be estimated from the ratio of measured to potential global solar radiation (or extraterrestrial radiation) following Liu and Jordan (1960). Here we develop an empirical model for calculating this fraction for any atmospheric conditions. The semi-empirical relationship is

$$K_r = \frac{D}{G} = f \frac{G}{H_0} = aK_t + b \quad (1)$$

where K_r is the ratio of diffuse radiation to global radiation for a horizontal surface, D the diffuse radiation for a horizontal surface (MJ m^{-2} per day), and G the total radiation for a horizontal surface (MJ m^{-2} per day), $K_t = G/H_0$ the ratio of global to extraterrestrial radiation (an inverse index of cloudiness), and H_0 is the extraterrestrial radiation on a horizontal surface (MJ m^{-2} per day) (Appendix A). The a and b are empirical coefficients for different climates.

2.2. Global topographic solar radiation

The solar radiation actually received on a surface varies significantly with its geometry (slope and aspect). Revfeim (1978) provided a method to estimate solar radiation on a horizontal surface given the diffuse and direct components of global radiation (Eq. (2)). The direct and diffuse components must be modeled separately because of their very different geometry

$$G_a = G_m[R_d(1 - K_r) + f_\beta K_r + 0.2(1 - f_\beta)] \quad (2)$$

where, G_a (MJ m^{-2} per day) is the global radiation received on a surface with arbitrary orientation, β the slope measured from a horizontal surface, and the “slope reduction factor”, f_β , is the proportion of the hemisphere above the slope surface that is blocked by the horizontal (infinite) plane and is estimated as $f_\beta = (1 - \beta/180)$. The ratio, R_d , of direct radiation on the slope to direct radiation on a horizontal surface, can be expressed as a function of latitude, declination, slope and aspect (Appendix A).

The term $Q = G_m R_d (1 - K_r)$ in Eq. (2) is irradiance received by the surface directly from the sun. The term $D = G_m f_\beta K_r$ is the diffuse radiation component obtained by integrating the sky radiation which is assumed isotropic over the hemisphere. The diffuse component decreases, as the surface becomes steeper, because of blocking of part of the hemisphere by the land. The term $A = G_m 0.2(1 - f_\beta)$ accounts for the reflection from the blocking land surface, with albedo assumed to be about 20% (in grassland). This albedo component is zero for a horizontal surface, which receives no surface-reflected diffuse radiation (in the absence of blocking) and increases, as the slope increases.

3. Methods

3.1. Collection of data for model validation

Global solar radiation at climate stations is typically collected only for horizontal surfaces and so could not be used for model validation. Therefore, an array of seven light sensors was assembled to measure solar radiation on surfaces with different slopes and aspects. Two of these sensors were positioned on slopes of 10° facing to the west and to the south. Three sensors were deployed in the direction of north at slopes of 10° , 20° and 30° . The remaining two sensors measured the global and diffuse radiation on a horizontal surface. The diffuse radiation was measured by blocking the direct radiation using a shadow band. The instrument array was deployed at Ruakura Research Centre at latitude 37.8°S in Hamilton, New Zealand in a flat area of grassland remote from shading hills or trees. Radiation was monitored for about 13 days (from 16th to 28th May 2000) during a variety of weather conditions (wet, cloudy, foggy and clear sun).

4. Results and discussion

4.1. Estimation of proportions of direct and diffuse radiation

The New Zealand landmass is narrow but spans about 12° of latitude north to south. Regular daily measurements of direct, diffuse and global radiation were available from two climate stations: Kaitaia (latitude 35°S in the far north of NZ) and Invercargill (latitude 46°S in the far south). Without loss of generality, the coefficients a and b in the Eq. (1) were calibrated using climate data from Kaitaia station in 1990 for the purposes of analysis. The coefficients were also re-calibrated for the validation using the data at Hamilton (latitude 37.8°S).

The model estimates of the direct and diffuse solar radiation for horizontal surfaces for the two sites were compared with the measurements in Fig. 1. The model is able to explain more than 62% of variation of the Kaitaia data. The estimation was improved up to 97% for the measurement at Hamilton. The better result for the Hamilton site is due to the high quality of the data. The estimation is satisfactory considering the complexities of sky cloudiness.

4.2. Sensitivity analysis

4.2.1. Effect of slope

We analyzed the model predictions of direct solar radiation received on north-facing slopes relative to that received on a horizontal surface (R_d) at latitude 35°S . During mid-winter (June), R_d increases with increasing slope steepness because elevation of the sun is relatively low, and the angle between solar beam and slope surface normal decreases with increasing slope steepness. R_d reaches a maximum around a slope of 70° , for which the solar beam is parallel to the slope surface normal. In contrast, the angle between the solar beam and the horizontal surface normal is at its minimum (maximum R_d) in the summer.

The angle between slope surface normal and solar beam mainly controls the amount of received direct radiation, which is usually a larger proportion of global radiation than diffuse radiation. This is the reason that received solar radiation on land surfaces is more sensitive to slopes during winter and summer than during spring and autumn.

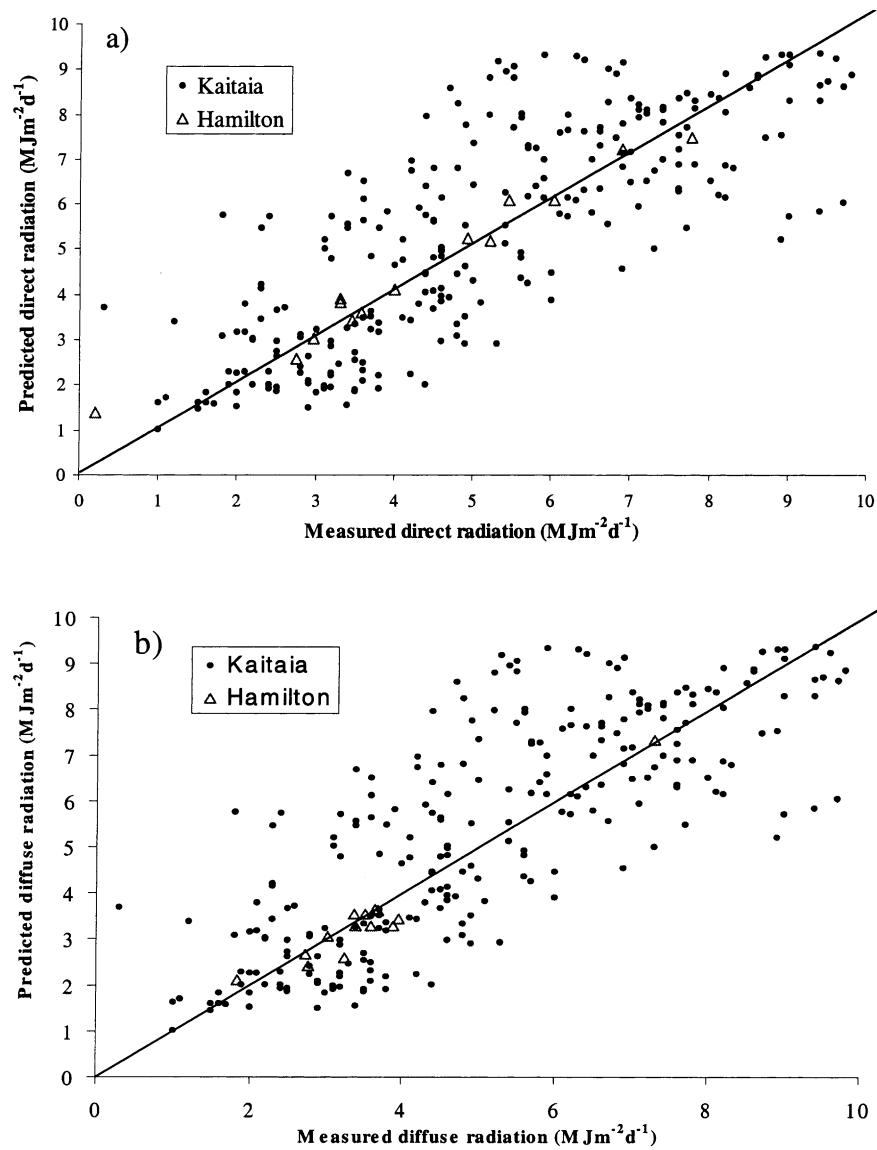


Fig. 1. Predicted proportions of direct and diffuse radiation in global radiation vs. measurements at Kaitaia and Hamilton climate stations. (a) Direct radiation; (b) diffuse radiation.

4.2.2. Effect of aspect

Effect of aspect is substantial during winter, and least during summer, since radiation on south-facing slopes is blocked more by hills during the winter due to the low elevation of the sun. Direct radiation is similar on west/east facing slopes is similar to that on horizontal surfaces. At a given latitude and slope, there

is always an aspect for which R_d is the same throughout the year. This “null point” is slightly toward north (south in north hemisphere) of the west/east meridian for latitude 35° and slopes of 15° . The null point moves toward north as latitude increases, because seasonal variation of R_d increases with latitudes (Fig. 2c).

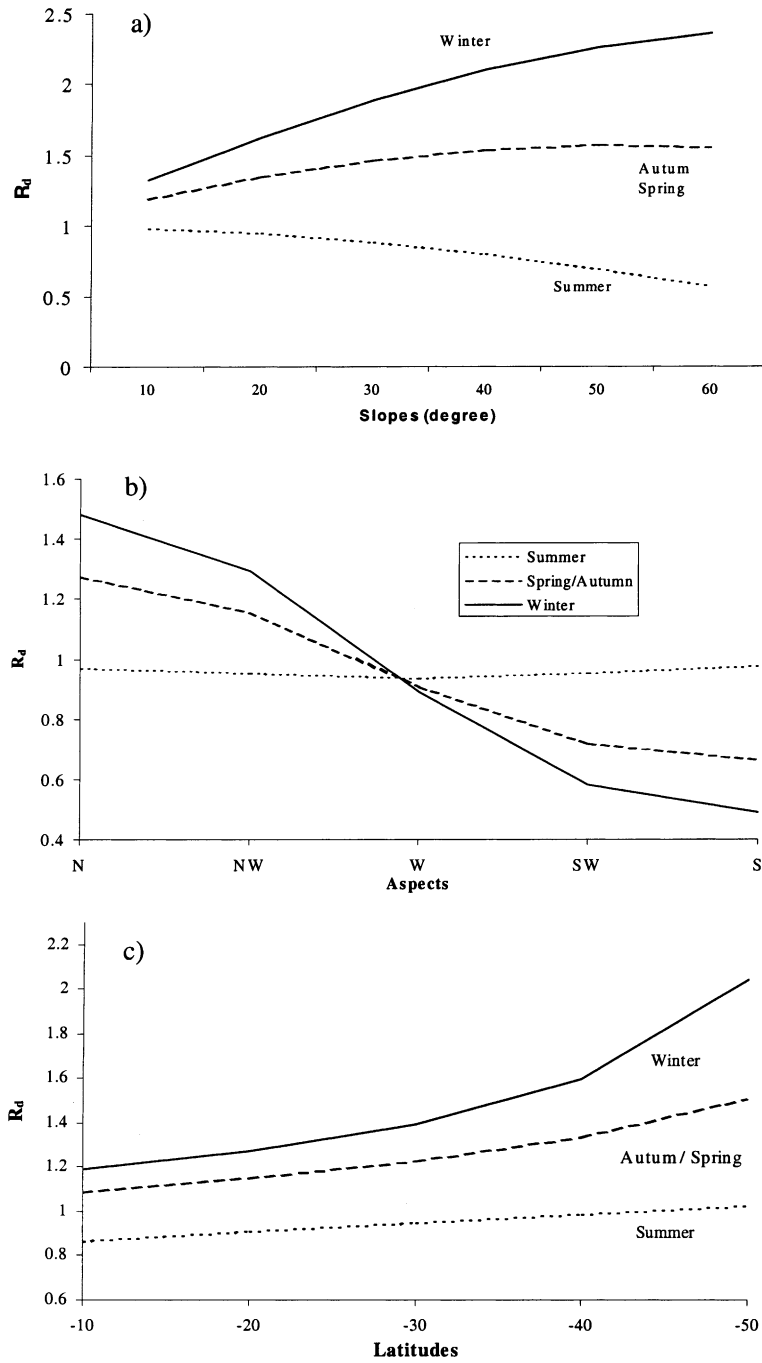


Fig. 2. Sensitivity of the model estimated R_d on slopes, aspects and latitudes: (a) for slopes (N-facing and latitude = -35°S); (b) for aspects (slope = 15° and latitude = -35°S) and (c) for latitudes (N-facing and slope = 15°).

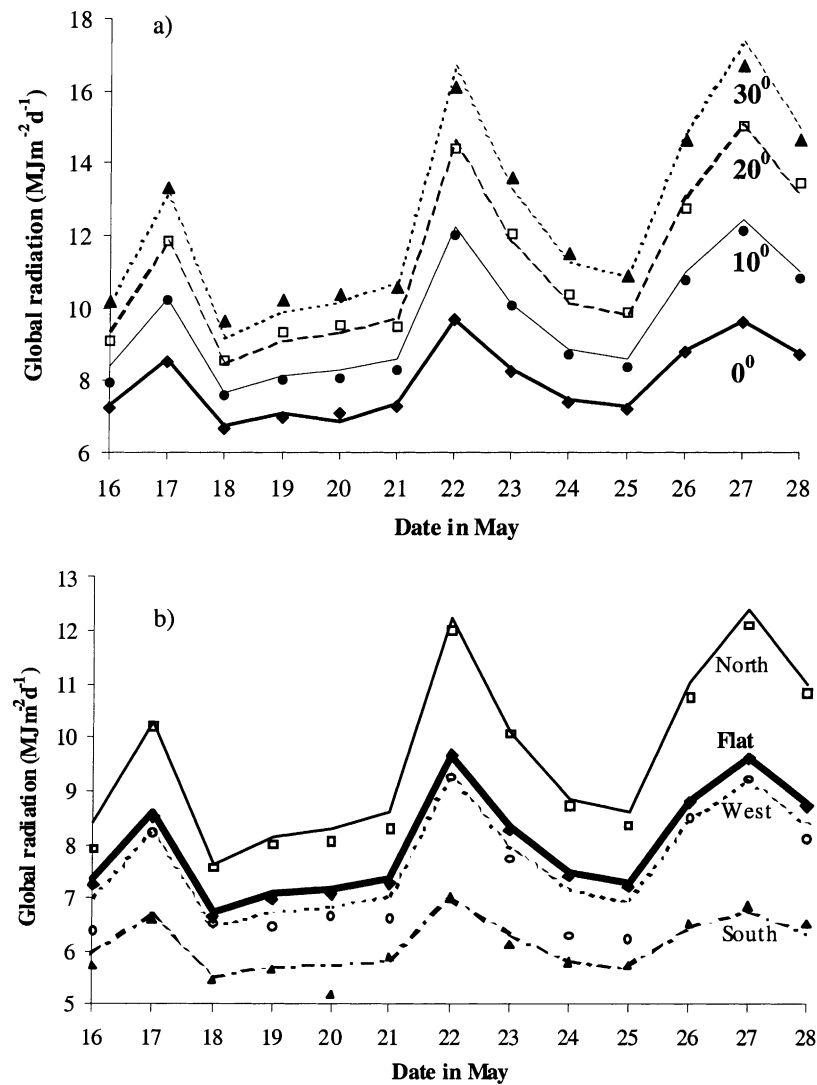


Fig. 3. Comparisons of monitored and model predicted global radiation: (a) global radiation variation on northerly slopes (10, 20 and 30°); (b) global radiation variation on 10° slopes with different aspects. The lines are for the model predictions and the scattered points are for the field data.

4.3. Model validation: effects of slope and aspect

We validated the model using the total daily radiation measured in Hamilton and the results are displayed in Fig. 3. Overall, model estimates agree very well in pattern and magnitude with field data over a period of very variable atmospheric conditions. There was a high correlation ($R^2 > 0.95$) between

model estimates and experimental data for effects of both slopes (Fig. 3a) and aspects (Fig. 3b).

5. Conclusion

We have presented an approach for estimating daily global solar radiation on any slope surface with

arbitrary aspect, given global radiation data measured nearby. The model partitions global radiation into direct and diffuse components in order to predict total radiation on slopes. The simulated global radiation for different slope and aspect agrees well with measured data. A valuable feature of the approach is a practical method for handling the magnitude and complexity of changes in solar radiation under variable atmospheric conditions. The model may be useful for providing input to agricultural and ecosystem models that are dependent on solar radiation, such as those for pasture production and carbon cycling. Other applications may include estimating disinfection (mainly by ultraviolet wavelengths in sunlight) of animal wastes on farmland, and evaporation for modeling of water quality and nutrient runoff.

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Appendix A

The extraterrestrial daily radiation, received on a horizontal surface, is computed from the following equations (Iqbal, 1983):

$$H_0 = \left(\frac{24}{\pi}\right) I_{sc} E_0 \cos(\phi) \cos(\delta) \left[\left(\frac{\pi}{180}\right) \omega_s - \tan \omega_s \right] \quad (A.1)$$

where I_{sc} is the solar constant in energy units ($I_{sc} = 4.921 \text{ MJ m}^{-2} \text{ h}^{-1}$), E_0 the eccentricity of the earth's orbit.

$$E_0 = \left(\frac{r_0}{r}\right)^2 = 1 + 0.033 \cos\left(\frac{2d_n}{365}\right) \quad (A.2)$$

Where d_n is the day number of the year, r_0 the mean sun–earth distance ($1.496 \times 10^8 \text{ km}$), r actual sun–earth distance, ϕ the geographic latitude (negative for southern hemisphere), δ the declination (the

angular position of the sun at solar noon with respect to the plane of the equator), ω_s local sunrise hour angle for a horizontal surface.

The declination was calculated from the following empirical equation (Iqbal, 1983):

$$\begin{aligned} \delta = & (0.006918 - 0.399912 \cos(\Gamma) + 0.070257 \sin(\Gamma) \\ & - 0.002697 \cos(2\Gamma) + 0.000907 \sin(2\Gamma) \\ & - 0.002697 \cos(3\Gamma) + 0.00148 \sin(3\Gamma)) \\ & \times \left(\frac{180}{\pi}\right) \end{aligned} \quad (A.3)$$

where Γ (rad), called the day angle, is represented by

$$\Gamma = \frac{1}{365} (2\pi(d_n - 1)) \quad (A.4)$$

The sunrise hour angle for a horizontal surface, ω_s , is assumed equal to the sunset hour angle (except for the sign difference), and is given below by

$$\omega_s = \arccos(-\tan \phi \tan \delta) \quad (A.5)$$

The following equations for calculating R_d were introduced in Revfeim (1978).

$$\begin{aligned} R_d(\phi, \delta, \beta, b) \\ = \frac{(\sin \phi^* / \sin \phi)(d - \sin d \cos e \cos g / \cos \omega^*)}{\omega_s - \tan \omega_s} \end{aligned} \quad (A.6)$$

$$\phi^* = \sin^{-1}(\sin \phi \cos \beta - \cos \phi \sin \beta \cos b) \quad (A.7)$$

$$d = \frac{1}{2}(h_1 - h_0); \quad e = \frac{1}{2}(h_1 - h_0) \quad (A.8)$$

$$g = \sin^{-1}(\sin \beta \sin b \sec \phi^*); \quad (A.9)$$

$$\omega^* = \cos^{-1}(-\tan \phi^* \tan \delta) \quad (A.10)$$

where b is the aspect; (south = 0° , north = 180° and east/west = 90°), h_0 the sunrise hour angle for an arbitrary slope surface. For a horizontal plane, it is given by ω_s , (Eq. (A.5)), otherwise is $\max(-\omega_s, g - \omega^*)$, h_1 is the sunset hour angle for an arbitrary slope surface. For a horizontal plane, it is given by $-\omega_s$, otherwise is $\min(\omega_s, g + \omega^*)$.

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