ASTRIGUE

Raffles Institution
&
Hwa Chong Institution

Contents

Astrobiology

Astrobiology is the study of the development of life in the universe. It can be thought of a search for sentient beings out in deep space, and whether or not we can communicate with them.

The origin of life

While life may be abundant on Earth, it is in fact quite rare outside of it. In the nearly 70 years since we started searching for extraterrestrial life, we have found nothing. Which leads us to two very important questions: what makes Earth so special, and how did life begin here in the first place? There are many theories for the origin of life, such as abiogenesis, the primordial soup theory, the deep-sea vent theory, the RNA world hypothesis, and Panspermia.

Abiogenesis

Abiogenesis is the theory that life began from simple organic substances. Any cell will contain the following 4 classes of molecules in any cell: lipids, carbohydrates, amino acids and nucleic acids. Any theory of abiogenesis must explain the origins and interactions of all these 4 classes of molecules.

There is some evidence supporting this theory. In the Miller-Urey experiment and other similar experiments, it was shown that all 20 amino acids, along with carbohydrates, could be created using just lightning, water vapour, natural gas, and heat, which were all present in the Earth as it formed. As for how these molecules eventually formed cells remains an open question.

The primordial soup

Primordial soup, or prebiotic soup, can be thought of as a subset of abiogenesis. According to this theory, the organic molecules formed condense into a rich organic ocean (thus the name “soup”). Given the much closer proximity of these molecules with each other, the chance of forming more complex molecules like proteins become much higher.

In today’s environment this would have been highly unlikely since the high oxygen content of our atmosphere would have oxidised all these organic compounds. However, primaeval conditions were much friendlier to abiogenesis. Alexander Oparin and J. B. S. Haldane suggested that the atmosphere then was oxygen-poor, consisting primarily of gases like methane, ammonia and water, resulting in a chemically reducing atmosphere.

The deep-sea vent theory

A competing theory for the origin of life suggests that hydrothermal vents could have been critical to the formation of life. This is motivated by the fact that the mixture of hot, basic fluids below the seabed with the cool, acidic ocean water at deep-sea vents would create an electrochemical gradient which could, like a battery, drive redox reactions, creating a stable source of energy for the first forms of life. This is supported by the presence of numerous organisms living around deep-sea vents today, showing that such an environment is habitable for life.

The RNA-world hypothesis

The most popular replication-first theory is the RNA world hypothesis. It posits that life did not begin with proteins, but with RNA. This is because RNA can also be used to construct enzymes, called ribozymes, which can control and catalyse chemical processes. This theory provides an easy explanation for information storage and replication since RNA could have also been used as the genetic code.

Early protocells could then evolve the ability to create complexes of RNA and proteins, which are better enzymes. This could then lead to the development of the first ribosomes, resulting in the synthesis of proteins which would eventually out-compete ribozymes in catalytic ability.

Panspermia theory

Panspermia is a hypothesis proposing that microscopic life exists throughout the universe distributed in debris ejected into space after collisions between planets and small Solar System bodies that harbour life.

Extremophiles are organisms that can survive for long periods of time in extreme conditions, such as those found in space. If such bacteria were shielded from radiation in space such as in the core of meteoroids or comets, they could well survive dormant for millions of years during their travel to Earth in space.

Life could also have been transmitted by spores which have been found to be resistant to many of the adverse conditions of space, including radiation, cell wall degradation, extreme temperatures, starvation and chemical disinfectants. Spores could have remained metabolically inactive throughout their journey through space and germinate only after reaching Earth.

What makes Earth so habitable for life?

Atmosphere

Earth's atmospheric composition is critical to the survival of life today. It is primarily composed of nitrogen and oxygen. Nitrogen, which comprises 70% of Earth's atmosphere, is important in the formation of ammonia which is required by plants for protein synthesis.

Oxygen, which comprises 21% of the Earth's atmosphere, is required for aerobic respiration. This could have been produced during the beginning of life after photosynthesis was developed as a source of energy.

Carbon dioxide and other greenhouse gases maintain the temperature of Earth's atmosphere through the greenhouse effect. However, the concentration of greenhouse gases cannot be too high as it could cause a runaway greenhouse effect that would create excessively high temperatures unsuitable for life on Earth. Such a runaway greenhouse effect is postulated to have occurred on Mars. The presence of carbon dioxide is also required for photosynthesis in many organisms today.

Water

Water is essential for all forms of life on Earth, and likely essential for all forms of life. Which is why the first step in finding life, say many astrobiologists, is to find water. Water is unique in its remarkable ability to form hydrogen bonds, allowing it to stay liquid at a much larger range of temperatures than usual, providing a useful liquid medium. Furthermore, its hydrogen bonds and polarity make it an excellent solvent for polar molecules and ions. It is the best transport medium for hydrophilic substances, which is why it is such an important aspect when looking for life.

Water is also the main reason why stars have something called a "habitable zone". It is the zone of orbits around the star that allows the planet within said orbit to be situated in a temperature range that allows for liquid water.

Earth's mass

The mass of Earth affects the type and quantity of gases that it can permanently retain on its surface. Earth's mass is sufficiently large to retain all gases except hydrogen. Were Earth's mass too small, it would be unable to retain an atmosphere. Were Earth's mass too large, it would attract an undesirably thick atmosphere that would block light and heat from the Sun from reaching its surface.

The Drake Equation

The Drake Equation is an attempt by Frank Drake to estimate how many intelligent civilizations currently exist in the Milky Way and are communicating.

$$N = R \times f_p \times n_e \times f_l \times f_i \times f_c \times L,$$

where:

N = The number of broadcasting civilizations.

R = Rate of star birth

$\mathbf{f_p}$ = Fraction of stars that develop planetary systems

$\mathbf{n_e}$ = Average number of habitable planets per star

$\mathbf{f_l}$ = Fraction of habitable planets ($\mathbf{n_e}$) with life

$\mathbf{f_i}$ = Fraction of habitable planets with life that becomes intelligent

$\mathbf{f_c}$ = Fraction of planets with intelligent life capable of interstellar communication

L = Years a civilization remains detectable

Celestial Coordinates

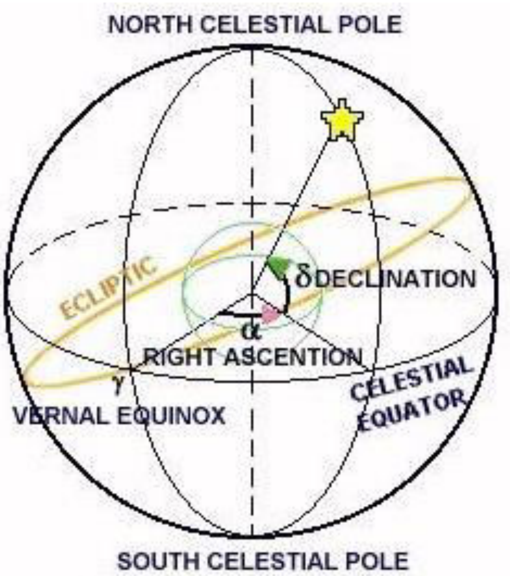
How do astronomers identify and label objects on the night sky? Well, this is done with the use of coordinate systems that can label the position of a star in space. This allows us to predict the rising and setting times of different stars, and find objects much more easily.

The Celestial Sphere is an imaginary fixed sphere, with an arbitrarily large radius that the sky can be conceived to be projected on. Two main celestial coordinate systems are used: the equatorial system and the horizontal system.

Equatorial System

The Equatorial system uses the celestial equator as the reference plane. In this system, the celestial equator and poles are just celestial analogues of Earth's equator and poles. Earth is at the center of this coordinate system and the plane in which Sun seems to "revolve" around Earth is tilted at 23.5 degrees from the celestial equator due to the obliquity.

An object's position on the celestial sphere is determined by its **right ascension (α)** and **declination (δ)**. The right ascension of an object is the angle measured east from the vernal equinox and along the celestial equator of its position on the celestial equator, but it is often expressed in hours, minutes and seconds (from 0h 0m 0s to 24h 0m 0s). The declination of the object is the angle measured from its position on the celestial equator along a (great) circle passing through the north and south celestial poles (from 0° to +90° and -90°). Do note that declination is indicated as positive in the northern direction.[1]



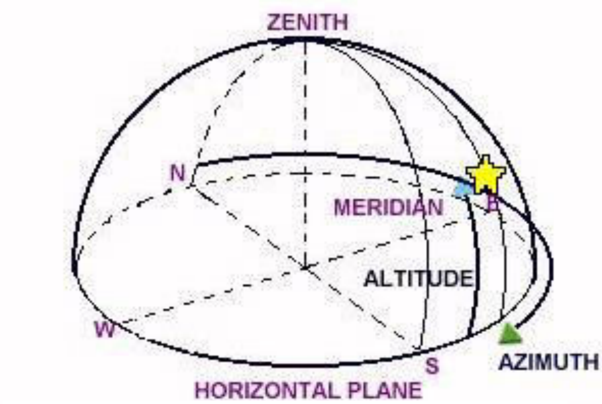
Equatorial Coordinate System defined with Right Ascension and Declination

Horizontal System

The horizon is a plane, which separates the visible and invisible hemispheres of the celestial sphere and thus the appearance and disappearance of celestial objects in the sky. In this system, the Zenith refers to the point directly above the observer and perpendicular to the ground, and the point directly opposite this is known as the Nadir. The meridian is a great circle passing through the Zenith and Nadir in the North-South direction.

[1] http://members.uia.net/tajames/astronomy/notes-coordinates.html

The system uses the altitude and the azimuth to define a position in space. The altitude is the angle the object makes with the horizon, measured in degrees (from 0° to +90° and -90°, e.g. a star that is rising will have an altitude of 0°). The azimuth is the angle from the North to the position of the object along the horizon (measured from 0° to 360°).[2]



Horizontal Coordinate System defined with Altitude and Azimuth

When an object reaches its highest altitude in the sky, it is said to have reached its upper culmination. When it reaches its lowest altitude, it is said to have reached its lower culmination. Both culminations will occur on the meridian (note that both culminations may not always be observed in the same day if the star rises and sets).

Differences and Applications

The equatorial coordinates of a celestial object is more or less fixed on the stationary celestial sphere that is infinitely far away (still affected by the precession of the Earth's axis) while horizontal coordinates of the same object is different at different positions on Earth. Thus most of the coordinates found in star catalogues are equatorial instead of horizontal ones. Since the horizontal system can give information about the observer's location, it is often useful in navigation.

In general, for an observer located at latitude α∘N, stars above a declination +(90-α)∘ will never set, while stars below a declination -(90-α)∘ will never be seen. Correspondingly, for an observer located at latitude α∘S, stars above a declination -(90-α)∘ will never set, while stars below a declination +(90-α)∘ will never be seen.

[2] http://members.uia.net/tajames/astronomy/notes-coordinates.html

Celestial mechanics

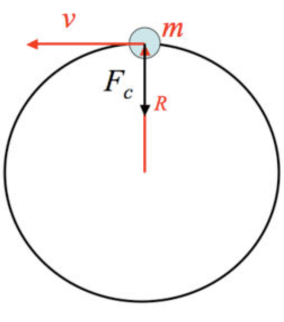
Celestial mechanics is the branch of astronomy that studies the motion of celestial objects, which include planets, stars, galaxies, etc. This area of study is extremely important in describing how the structure of the Universe arose, and as we build up our understanding on these tiny interactions between celestial bodies, we are gradually able to see the infinitesimal puzzle pieces fall into place in the grand design of the Universe.

Gravitation

The expression for the gravitational force between any 2 objects is $\frac{GM_1M_2}{r^2}$, where G is a constant, M_1 and M_2 are the 2 masses of the objects, r is the distance between them and the direction is towards the other object. This is the force that governs the motion of planets around stars, and galaxies around the centre of a galaxy cluster.

Kepler's 3 laws

Because of gravity, when we jump, we all fall back towards Earth. Yet, it doesn't seem like Earth is falling towards the Sun. Why is this so? It turns out that it's because Earth, and other planets, are orbiting around the sun. Gravity, in this case, doesn't accelerate them towards the sun. Rather, gravity provides the acceleration needed to change the direction of the planet's trajectory so that they move in ellipses around the Sun rather than in a straight line. This is extremely similar to swinging a weight in a circle that is tied to your hand. While the weight is revolving around your hand, you feel the weight constantly pulling on you. Thus, you must be constantly pulling on the weight. Yet, the weight never comes closer to your hand.



Centripetal force required to keep an object rotating is given by $F = m\omega^2 r$

So then, how much force do you have to exert? What is the force necessary to keep a planet in a circular orbit? This force required, also known as the centripetal force, is given by $F = m\omega^2 r$, where m is the mass of the revolving object, ω is the angular velocity of the object (also defined as $\frac{2\pi}{T}$, where T is the period of revolution), and r is the radius of revolution.[3] In orbits, this force is exactly equal to the gravitational force. If it weren't, then our Earth w [4] ill be falling towards the sun. Thus, $\frac{GMm}{r^2} = m\omega^2 r$. Rearranging, we get $T^2 = \frac{4\pi^2}{GM} r^3$, which is a special case of Kepler's Third Law applied to circular orbits. The general form of Kepler's Third Law is $T^2 = \frac{4\pi^2}{GM} a^3$, where a is the semi-major axis. This result can be derived from considering

[3] http://astronomy.swin.edu.au/cosmos/C/Centripetal+Force

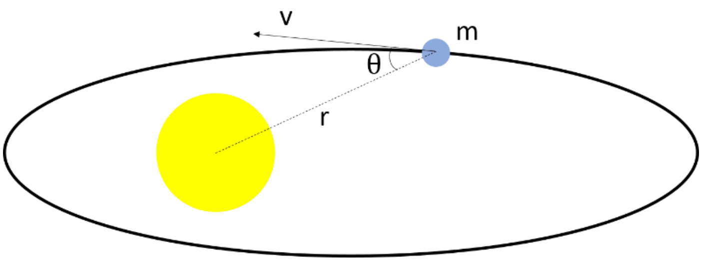
the mechanics of an object in an elliptical orbit; however, such derivation is far more mathematically rigorous.

Kepler's First Law states that all planets orbit around the Sun in an ellipse, with the Sun being at one of the foci.

Kepler's Second Law states that the line joining the Sun and a planet will sweep out an area at a constant rate. While this may seem surprising at first, it is actually a direct result of conservation of angular momentum.

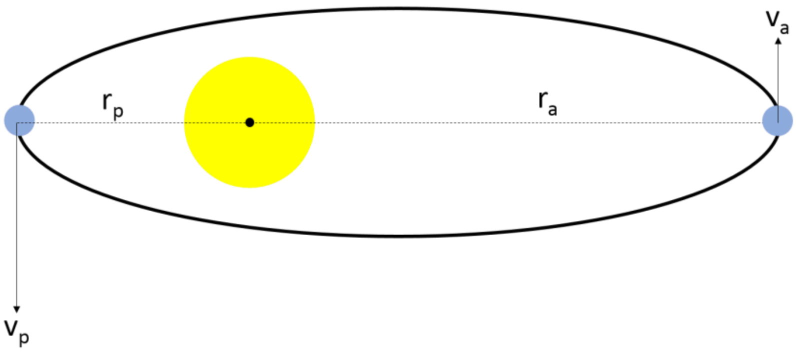
Vis-Viva equation: energy & angular momentum conservation

You can think of angular momentum as the rotational counterpart of momentum. Let us use a comet orbiting the Sun as an example. Instead of mv, the expression for linear momentum, the angular momentum of the comet orbiting about the Sun is given by $L = mvr\ sin\theta$, where m is the mass of the comet, v is the speed of the comet, r is the distance from the comet to the sun, and θ is the angle between the direction of motion and the line from the comet to the Sun.



Angular momentum of an object about a point is given by $L = mvr\ sin\theta$

So why is this quantity conserved? Similar to linear momentum, which is conserved when no external force is applied to the system, angular momentum is conserved when no external torque is applied to the system. Torque can be seen as the rotational counterpart of a force and is defined as $\tau = Fr\ sin\theta$, where F is the force applied on the comet, r is the distance between the comet and the Sun, and θ is the angle between the direction of force applied and the line joining the comet to the Sun. In orbits, the only force on the comet is the gravitational force, which always points towards the Sun. Hence, θ is always 0, and angular momentum is always conserved. This applies to all celestial objects in orbit, not just the comet. From this conclusion, we can derive the following relation



The shortest distance in an orbit is given by r_p (periapsis) while the longest distance is given by r_a apoapsis.

$$\frac{r_a}{r_p} = \frac{v_p}{v_a}$$

From conservation of energy, we know that the sum of gravitational potential energy and kinetic energy of the comet is always a constant. Easily, we can derive the following relation

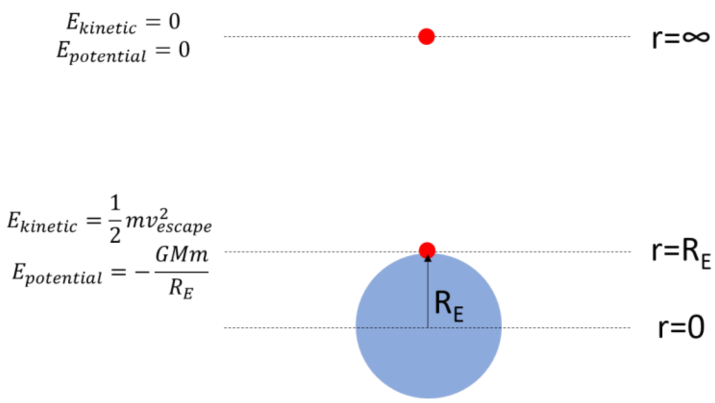
$$\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = \frac{1}{2}mv_p^2 - \frac{GMm}{r_p}$$

(Note that $-\frac{GMm}{r}$is the expression for gravitational potential energy of an object mass m gravitationally bound to a more massive object mass M. By definition, potential energy at infinity is 0, so as it gets closer its potential energy decreases, and is thus negative).

From these 2 relations, we can then derive the Vis-Viva equation $v^2 = GM(\frac{2}{r} - \frac{1}{a})$. The equation is extremely useful in determining the change in speed required for an object, say a satellite or rocket, to change from one orbit to another. As you might expect, this in itself is very crucial for space expeditions.

Escape Velocity

Have you ever wondered, what is the minimum speed you must have in order to jump away from Earth and not ever fall back down? To find this speed, it is in fact a very simple matter of manipulating the energy conservation equation. At the moment you jump, your gravitational potential energy is $-\frac{GMm}{R_E}$, while your kinetic energy is $\frac{1}{2}mv^2$. For you to not ever fall back to the Earth, you must escape its gravitational field, and that is only possible if you jumped to an infinite distance away from Earth. The minimum speed that will allow you to do so will leave you with no kinetic energy at all at infinity. Thus, both gravitational potential and kinetic energy that you possess at infinity will be 0. Writing this mathematically, it gives us $\frac{1}{2}mv^2 - \frac{GMm}{R_E} = 0$, or $v = \sqrt{\frac{2GM}{R_E}}$, which is the expression for escape velocity. Plugging Earth's mass and radius into that, it will give us v = 11.2 km/s. It does seem somewhat unrealistic for a normal human being to attain such speeds.



Kinetic and potential energy at different distances from the centre of Earth

Exercise

Table of physical constants

Gravitational constant (G)	6.67×10^{-11} $m^3kg^{-1}s^{-2}$
Mass of Earth	5.97×10^{24} kg
Radius of Earth	6.37×10^{6} m
Mass of Sun	1.99×10^{30} kg
1 Astronomical Unit (A.U.)	1.50×10^{11} m

1) Show that the gravitational acceleration on the surface of the Earth is 9.81 ms^{-2}
2) What is the gravitational acceleration of an object if it were
    a. $2R_E$ away from the centre of the Earth?
    b. $2R_E$ away from the surface of the Earth?
3) Using Kepler's 3rd law, show that the period of the Earth is 1 year.
4) Given that the semi-major axis of Jupiter is 5.2 A.U., find its orbital period in years.
5) If the Earth were orbiting about Betelgeuse (7.7 solar masses) instead of the Sun, what would its period be if it were orbiting at the same distance?

Solution

1) Since $F = ma$ and gravitational force $F = \frac{GM_E m}{R_E^2}$, $a = \frac{GM_E}{R_E^2}$. Substituting in the values for the radius and mass of the Earth, we get

$$a = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^{6})^2} = 9.81\text{ms}^{-2}$$

2) While this question can be done by using the formula $a = \frac{GM}{R^2}$, the more observant student will realise that $F \propto \frac{1}{R^2}$, and knowing the acceleration at the surface of the Earth, one simply needs to find the ratios of the squares of the distance to determine the acceleration.

   a) $a = 9.81 \times \frac{R_E^2}{(2R_E)^2} = 9.81 \times \frac{1}{4} = 2.45\text{ms}^{-2}$

   b) Since the object is $2R_E$ away from the **surface** of the Earth, it is actually $3R_E$ away from the **centre** of the Earth. Using the same technique as in (a), we find that $a = 9.81 \times \frac{R_E^2}{(3R_E)^2} = 9.81 \times \frac{1}{9} = 1.09\text{ms}^{-2}$

3) Plugging in M = mass of Sun (1.99×10^{30} kg), a = distance of Earth to Sun (1.50×10^{11} m) into $T^2 = \frac{4\pi^2}{GM}a^3$, and you will get 1 period of 1 year. In fact, if you substitute in T in terms of years and a in terms of A.U., the equation simply becomes $T^2 = a^3$.

4) From Kepler's third law $T^2 = \frac{4\pi^2}{GM}a^3$, since the mass constant, we get $T^2 \propto a^3$. Thus

$$T_{Jupiter} = T_{Earth}\sqrt{\left(\frac{a_{Jupiter}}{a_{Earth}}\right)^3} = 11.86 \text{ years}$$

5) From Kepler's third law $T^2 = \frac{4\pi^2}{GM}a^3$, since the distance of orbit is unchanged, the semi major axis remains unchanged, and we can see that $T^2 \propto \frac{1}{M}$, or $T \propto \frac{1}{\sqrt{M}}$. Therefore, $T_{new} = T_{old}\sqrt{\frac{M_{Earth}}{M_{Sun}}} = 0.360 \text{ year}$

Cosmology

How did the universe begin? How will it end? These are questions that have kept astronomers curious for decades, and while we now have a much better understanding of how it began, we are still quite uncertain how it will all end.

The Big Bang

The Big Bang is the sudden expansion of an immensely dense and hot point that occurred 13.8 billion years ago that eventually gave rise to our universe as it is today. In 1929, Edwin Hubble compared the redshift of distant Cepheid variables and their distances from Earth and discovered that these two physical quantities were linearly correlated. This meant that the further away stars were from Earth, the faster they moved away from us. Taken in context of the general relativistic Friedmann equations, this showed that the universe was expanding.
Hubble then formulated his own law, Hubble's Law:

$$v = H_0 D$$

where v represents the recessional velocity, H_0 is the Hubble's Law constant and D is the distance away from us. This described how fast the universe is expanding.
If the universe is constantly expanding, the universe must be smaller in the past, and as we go further back in time, the universe gets smaller and smaller. Eventually, it shrinks down to one point from which everything in the universe must have originated. Thus, The Big Bang Theory is formulated.

Cosmic Microwave Background Radiation (CMBR)

The CMBR is a mostly isotropic thermal radiation coming from all parts of the universe with a spectrum that precisely correlates with the blackbody spectrum, at just the right temperature suggested by the Big Bang theory. The CMBR is extremely smooth, making it difficult for it to be explained from multiple point sources, indicating that the CMBR originated from an event (that we now know as the Big Bang) that occurred throughout all space at once.

The end of the universe

While we may have determined the origins of the universe, its end is still very much an open question.

Heat Death

In this scenario, the universe expands infinitely until it approaches the absolute zero temperature, causing all thermodynamic processes to cease and the stars will no longer be able to form. As the last stars die out, the universe will grow dark. Fortunately, over an infinite period of time the Big Bang could occur again due to the Poincare recurrence theorem and thermal fluctuations.

Big Rip

In this scenario, the rate of acceleration of expansion of the universe increases. All material in the universe would eventually disintegrate into their elementary particles and radiation due to the infinitely fast expansion of the universe.

Big Crunch

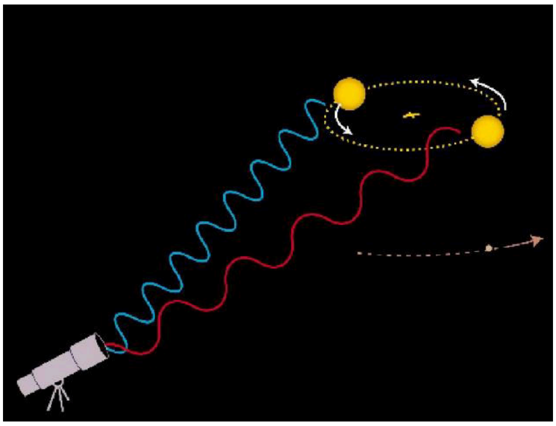
In this scenario, the universe is dense enough to stop its own expansion, and begin contracting. All the matter and space-time would collapse to a single point, a singularity. Immediately after the collapse of a universe, the Big Bang would occur again.

Exoplanet detection

Exoplanets refer to planets outside our solar system, and they possibly hold the answer to mysteries such as our planet's own history, and the possible existence of extraterrestrial life. While they might be difficult to detect, the search for such planets has been very successful. So far, there have been over 3000 confirmed planets in over 2000 systems, but how exactly do astronomers find all these exoplanets?

Doppler spectroscopy

When a star has a planet orbiting it, the star will move in its own small orbit around the common centre of mass in the system, which causes the radial velocity (i.e. how fast an object is moving towards to or away from the observer) of the star to vary. However, since the mass of a star is usually significantly larger than that of the planet, the centre of mass of the system usually lies within the star, causing the orbit to manifest as a "wobble" in the star.



Light from stars is Doppler shifted depending on its radial velocity.

When the star is moving away from Earth, the light from the star will be redshifted. Conversely, when the star is moving towards Earth, the light from the star will be blueshifted. Thus, this wobble can be detected from a shift in the spectral lines due to the Doppler effect, hence the name of the technique. An estimate of the planet's mass may also be derived using this method.[5]

Transits

Photometry

When a planet crosses in front of its star's orbit, the star will appear slightly dimmer as the planet blocks out some of the light from the star. If the star system of the planet is in the line of sight from the observer, this "shadow" can be detected. The amount of light blocked also helps determine the radius and size of the planet, and when combined with other methods such as Doppler spectroscopy, can provide information such as density.

Despite the fact that it is relatively rare for a star and an exoplanet to lie directly in the line of sight from earth, transit photometry has found over 2000 exoplanets, making it the technique most successful so far in exoplanet detection

[5] http://ugastro.berkeley.edu/infrared10/doppler/index.html

Duration / timing variation

Duration and timing variation when a planet transits in front of its star usually acts as a complement to other techniques to detect other, possibly non-transiting planets in the same star system. This technique is based on the fact that transits would have strict periodicity in the absence of external influences, and any duration and/or timing variations in these traits suggest the presence of another body of significant mass in the vicinity, which could turn out to be other exoplanets.

Pulsar timing

A pulsar is a neutron star that periodically emits radio waves as they rotate at frequent and very regular intervals. Because these emissions are so regular, slight anomalies in them can be used to track the motion of the pulsar.

Because a pulsar is still subject to gravitational influence by other bodies, a pulsar would orbit the barycentre of a star-planet similar to any other star. Thus, by tracking the motion of pulsars, the orbit parameters can be determined and exoplanets can be detected.

However, pulsars are relatively rare, and planets orbiting these pulsars are even rarer, limiting the usefulness of this technique. Nevertheless, this technique is significant as it was used in the first ever detection of exoplanets by observing the pulsar PSR 1257+12.

Direct imaging

Direct imaging is analogous to taking a photograph, albeit it includes frequencies of light beyond the visible spectrum. The light from the planet that is captured can either be light reflected off of its parent star, or from black body radiation due to the inherent temperature of the planet.

This technique is limited mainly by the fact that the parent star is usually much brighter than the planet, so any light from the exoplanets is likely to be obstructed by the light from the parent star. Thus, planets found through direct imaging are usually around brown dwarfs, which are relatively dim, and newly formed planets, possibly with protoplanetary disks intact, since they have higher temperature and thus emit more light.

Notably, multiple samples might have to be taken to ensure a brown dwarf is not mistaken as an exoplanet. Furthermore, this technique also works better with face-on planetary orbits as opposed to head-on ones since the planets can be seen at all times.

Gravitational microlensing

Gravitational microlensing is premised in general relativity, which explains that light from a star can be bent by the gravity of an object between the earth and the source star. In essence, the object acts as a lens. If the lens happens to be a star that has a planet in its orbit, the gravity from the planet will cause further bending or distortion of the light, and such variations in the can be detected.

However, this technique relies on a chance alignment between the source star, the lens star, and the observer, thus limiting its effectiveness.

Astrometry

Astrometry involves measuring a star's position in the sky accurately, and detecting how that position changes over time. Since a star with a planet will orbit around the common barycentre of the system, the star's position in the sky can be used to detect signs of this orbit. Since the further the barycentre is from the centre of the star, the more obvious its orbit, this technique is more sensitive to planets around low mass stars and planets further from its parent star.

Astrometry is notable for being the oldest technique in exoplanet detection, and was popular because it was successful in detecting binary star systems. However, a possible disadvantage is that since astrometry favours planets further from the parent star, the time taken to complete an orbit is correspondingly long, making the movements of the star even harder to detect.

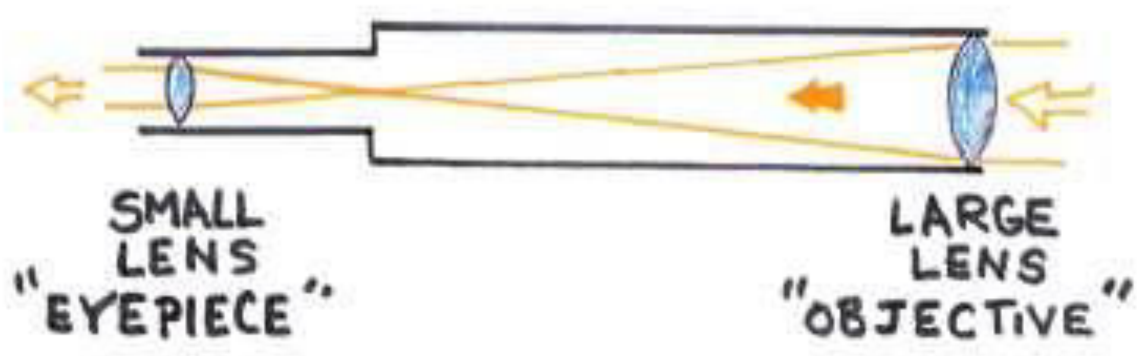
Night Sky Observation

This chapter will cover the basics of observation equipment used by amateur astronomers

Types of telescopes

There are 3 **main** types of telescopes, namely refractors, reflectors and catadioptrics. They each have their own advantages and disadvantages.

Refractors



Schematic of a typical refractor

The refractor is the simplest design of all telescopes, consisting of an objective lens to gather and refract light into focus, producing a magnified, virtual image. This allows more light to be gathered than possible by the human eye.[6]

Advantages (+)

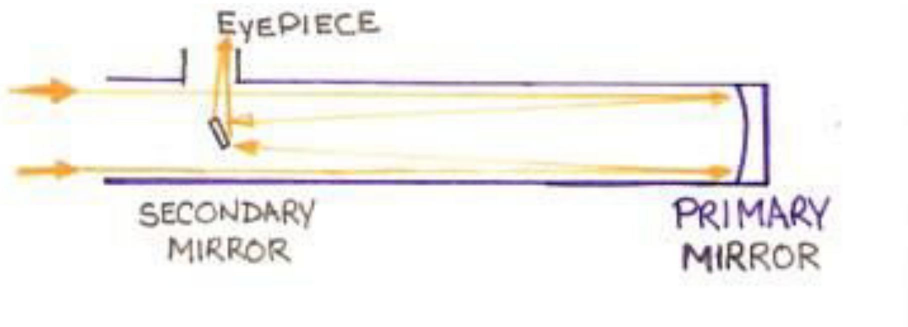
- Rugged and not prone to misalignment - less maintenance required
- Components are sealed from the atmosphere, eliminating air currents and temperature associated effects - sharper image

Disadvantages (-)

- Expensive: lenses generally cost more than mirrors
- Lenses may suffer from chromatic aberration.
- Lenses can be heavy, which may also cause them to sag under their own weight

[6] http://www.stormthecastle.com/telescopes/difference-between-a-reflector-and-a-refractor.htm

Reflectors



Schematic of a typical reflector

Reflectors, also known as Newtonians, make use of mirrors to reflect light to form an image. A reflector usually consists of a primary curved mirror that focuses light towards a secondary mirror which reflects the light to an eyepiece.[7]

(+) Advantages

- Cheaper: less precision required
- No chromatic aberration
- The curved mirror can be supported along the base, allowing for extremely large reflectors to be built (e.g. Dobsonian reflectors)

(-) Disadvantages

- Prone to coma
- As it is exposed on one end, the mirror may require frequent cleaning
- Prone to misalignments and therefore requires frequent collimation
- Central obstruction due to secondary mirror

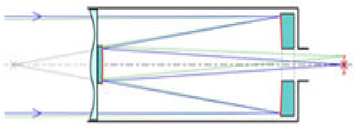
Catadioptrics

Catadioptrics are telescopes combining specifically-shaped lenses and mirrors to form an image. They tend to be folded to reduce the length of the telescope tube, making them more portable and easier to manufacture. Normally, there is a corrector lens, a primary mirror and a secondary mirror.

Schimdt-Cassegrain

[7] http://www.stormthecastle.com/telescopes/difference-between-a-reflector-and-a-refractor.htm

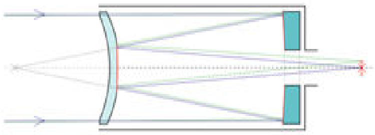
The Schmidt-Cassegrain is a Cassegrain with a spherical primary mirror and a Schimdt corrector plate to correct for spherical aberration. The secondary mirror on the correcting plate serves to flatten the field of view and relays the image through a perforation in the primary mirror to the eyepiece.[8]



Ray Diagram in a Schimdt-Cassegrain

Maksutov-Cassegrain

The Maksutov-Cassegrain is a Cassegrain with a spherical primary mirror and a spherical aluminized spot on the surface of a spherical corrector (meniscus lens) that serves to reflect light through a perforation in the primary mirror to the eyepiece.[9]



Ray Diagram in a Maksutov-Cassegrain

Advantages (+)

- Typically long focal length 'folded' to a smaller size
- Lower coma and astigmatism
- Easy to maintain, with well-collimated design sealed from the atmosphere
- Compact and portable

Disadvantages (-)

- Central obstruction due to secondary mirror decreases light gathering power
- Relatively more expensive than reflectors

Types of mounts

Mounts are classified initially according to the type of coordinate system that they use. Among the equatorial mounts, there are 2 basic types, namely the German Equatorial and Fork Mounts.

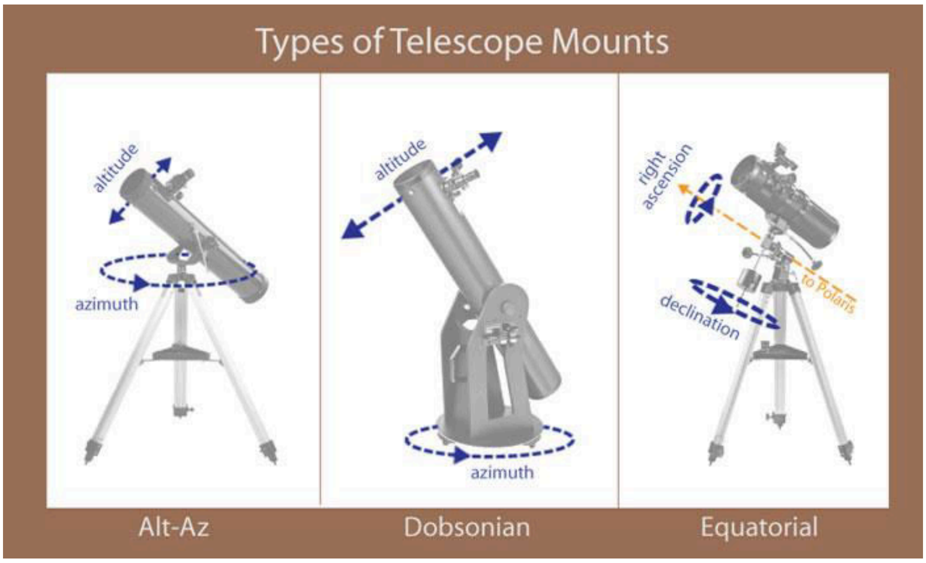
Alt-Azimuth Mounts

The alt-azimuth mount is the simplest type of mount that has only 2 planes of motion, namely altitude (up and down) and azimuth (left and right). Due to the relative difficulty of

[8] https://en.wikipedia.org/wiki/Schmidt–Cassegrain_telescope
[9] https://commons.wikimedia.org/wiki/File:Maksutov-Cassegrain-Telescope.svg

tracking objects with this mount, it is generally more suited for viewing objects rather than activities which would require tracking, such as astrophotography.

Equatorial Mounts



Different Types of Telescope Mounts and their planes of movement

The equatorial mount moves mainly along planes of declination and right ascension adjusted to a particular latitude. As such, if one aligns the mount with Polaris (or North in Singapore's case), moving the telescope along the right ascension axis will allow the telescope to follow the path of the stars through the sky. Due to this, the equatorial mount is useful for long period observations and deep sky astrophotography. [10]

Telescope accessories

Finderscope

A finderscope is an aiming device used in astronomy, typically a small auxiliary telescope mounted on the main telescope along the same line of sight. The finderscope usually has a smaller magnification than the main telescope, providing a much larger field of view, useful for manually aiming (also called slewing) a telescope and locating a desired astronomical object. Some finderscopes have crosshairs to aid in accurately pointing the telescope system at a target.

Barlow Lens & Focal Reducer

The Barlow lens is a diverging lens which, used in series with other optics in an optical system, increases the effective focal ratio of an optical system as perceived by all components after it in the system. The practical result is that inserting a Barlow magnifies the image. Conversely, the focal reducer is a lens that reduces the focal ratio of the telescope, hence increasing the FOV and also allowing for shorter exposure camera shots.

[10] http://www.starryhill.org/?page=5&p=telescope

The focal ratio is the ratio of the focal length to the aperture, and is a qualitative measure for lens speed in photography. For telescopes, the Barlow and the focal reducer is usually placed just before the eyepiece.

Filters

A filter is a device that selectively transmits light. In astronomy, monochromatic filters are often employed to remove certain wavelengths of light from objects to more effectively view objects at other wavelengths. For example, in observations of Mercury from higher latitudes, a filter is needed to observe Mercury as it is obscured by the sun. There are also light pollution filters to block out wavelengths of light commonly created by man-made sources such as sodium lamps. They allow for better viewing in light polluted skies like those of Singapore.

Eyepiece

An eyepiece consists of "lens elements" in a housing, with a "barrel" on one end. The barrel is shaped to fit in a special opening of the instrument to which it is attached. The image can be focused by moving the eyepiece nearer and further from the objective. Most instruments have a focusing mechanism to allow movement of the shaft in which the eyepiece is mounted, without needing to manipulate the eyepiece directly. The eyepiece is placed near the focal point of the objective to magnify the image from the objective lens. The amount of magnification is the ratio of the focal length of the optical tube to the focal length of the eyepiece. This means that the shorter the focal length of the eyepiece, the higher the magnification.

Star Diagonal

A star diagonal is a mirror attached to the telescope before the eyepiece. The mirror is placed at a 45 degree angle, allowing for ease of viewing of objects at or near the zenith, which is the point in the sky directly overhead.

Counterweight

A counterweight balances the weight of the telescope. This ensures that the strain on the mount is kept to a minimal and the telescope would not suddenly swing down in the middle of viewing. Using the 2 axes of movement, the counterweight is balanced with the weight of the optical tube and the accessories.

Binoculars

Magnification

The amount of magnification depends on the application the binoculars are designed for. Hand-held binoculars have lower magnifications so that objects can be kept within the field of view without a supporting set-up such as a tripod. A larger magnification leads to a smaller field of view.

Objective Diameter

The diameter of the objective lens (aperture) determines how much light can be gathered to form an image. This number directly affects performance. For the same magnification and optical quality, the larger the diameter, the brighter and sharper the image. An 8×40, then, will produce a brighter and sharper image than an 8×25, even though both enlarge the image an identical eight times. The larger front lenses in the 8×40 also produce wider beams of light (exit pupil) that leave the eyepieces. This makes it more comfortable to view with an 8×40 than an 8×25. It is usually expressed in millimetres. It is customary to categorize binoculars by the magnification × the objective diameter; e.g. 7×50.

Field of view

The field of view of a pair of binoculars is determined by its optical design. It is usually denoted by a linear value, such as how many metres in width can be seen at 1000m, or by an angular value of how many degrees can be viewed.

Constellations

Constellations are “imaginary” things that astronomers made up to help them keep track of stars in the sky during ancient times. They have remained largely unchanged as the stars themselves are so far away that they barely move; so the skies we see today are largely similar to what our ancestors saw.

Each constellation has its own name, like Orion the Hunter or Gemini the Twins. Altogether, there are 88 constellations in the night sky as defined by the International Astronomical Union.[11]

For this booklet to go through all 88 constellations and the deep sky objects within them, it would need an additional 100 pages, which would not be practical. Instead, we recommend picking up books such as Turn Left at Orion in order to gain a greater understanding and appreciation of the night sky. Most importantly, we recommend going out to stargaze once in a while; not only as practice for this competition, but also to remind yourself of the inherent beauty in astronomy.

[11] http://www.ianridpath.com/iaulist1.htm

And	Andromeda	CVn	Canes Venatici	Ori	Orion
Ant	Antlia	Cyg	Cygnus	Pav	Pavo
Aps	Apus	Del	Delphinus	Peg	Pegasus
Aql	Aquila	Dor	Dorado	Per	Perseus
Aqr	Aquarius	Dra	Draco	Phe	Phœnix
Ara	Ara	Equ	Equuleus	Pic	Pictor
Arg	Argo	Eri	Eridanus	PsA	Piscis Austrinus
Ari	Aries	For	Fornax	Psc	Pisces
Aur	Auriga	Gem	Gemini	Pup	Puppis
Boo	Bootes	Gru	Grus	Pyx	Pyxis
Cae	Cælum	Her	Hercules	Ret	Reticulum
Cam	Camelopardalis	Hor	Horologium	Scl	Sculptor
Cap	Capricornus	Hya	Hydra	Sco	Scorpius
Car	Carina	Hyi	Hydrus	Sct	Scutum
Cas	Cassiopeia	Ind	Indus	Ser	Serpens
Cen	Centaurus	Lac	Lacerta	Sex	Sextans
Cep	Cepheus	Leo	Leo	Sge	Sagitta
Cet	Cetus	Lep	Lepus	Sgr	Sagittarius
Cha	Chamaeleon	Lib	Libra	Tau	Taurus
Cir	Circinus	LMi	Leo Minor	Tel	Telescopium
CMa	Canis Major	Lup	Lupus	TrA	Triangulum Australe
CMi	Canis Minor	Lyn	Lynx	Tri	Triangulum
Cnc	Cancer	Lyr	Lyra	Tuc	Tucana
Col	Columba	Men	Mensa	UMa	Ursa Major
Com	Coma	Mic	Microscopium	UMi	Ursa Minor
CrA	Corona Australis	Mon	Monoceros	Vel	Vela
CrB	Corono Borealis	Mus	Musca	Vir	Virgo
Crt	Crater	Nor	Norma	Vol	Volans
Cru	Crux	Oct	Octans	Vul	Vulpecula
Crv	Corvus	Oph	Ophiuchus		

List of IAU Constellations

Asterisms

Asterisms are popular groupings or patterns of stars that are recognized in the night sky. Although this definition is somewhat similar to the definition of a constellation, they are not recognized by the International Astronomical Union. below is a list of some well-established asterisms:

Autumn

- Segment of Perseus - in Perseus
- Northern Fly - in Aries
- Head of Cetus - in Cetus
- Frederick's Glory - in Andromeda
- Square of Pegasus - in Andromeda and Pegasus
- Circlet - in Pisces
- Y of Aquarius (Water Pitcher) - in Aquarius

Summer

- Northern Cross - in Cygnus
- Job's Coffin - in Delphinus
- Summer Triangle - in Cygnus, Lyra, Aquila
- Keystone - in Hercules
- Bull of Poniatowski - in Ophiuchus

- Milk Dipper - in Sagittarius
- Teapot - in Sagittarius
- Coathanger - in Vulpecula

Winter and Spring

- Sickle - in Leo
- Praesepe (also called the Beehive) - in Cancer
- Hydras Head - in Hydra
- Heavenly G - in Orion, Taurus, Canis Major, Canis Minor, Gemini, Auriga
- Sword of Orion - in Orion
- Belt of Orion - in Orion
- The Kids - in Auriga
- Hyades - cluster in Taurus
- Pleiades (also called the Seven Sisters) - cluster in Taurus

Asterisms near the North Celestial Pole

- Big Dipper - in Ursa Major
- Bier - in Ursa Major
- Guardians - in Ursa Minor
- Little Dipper - in Ursa Minor
- Lozenge - in Draco

Can you find the Winter Hexagon?

Most important asterisms

The Winter Hexagon

1. **Sirius**, in Canis Major
2. **Capella**, in Auriga the Charioteer
3. **Aldebaran**, in Taurus the Bull
4. **Rigel**, in Orion the Mighty Hunter
5. **Procyon**, in Canis Minor
6. **Pollux/Castor**, in Gemini the Twins[12]

[12] http://www.eluniversohoy.net/la-via-lactea-la-constelacion-de-orion-y-sirius-desde-el-tibet/

The Summer Triangle

1. **Vega**, in Lyra
2. **Altair**, in Aquila
3. **Deneb**, in Cygnus[13]

Can you find the Summer Triangle?

[13] http://www.skyandtelescope.com/observing/the-summer-triangle-makes-its-midnight-debut/

Solar System

Outside of our Earth reside other planets which also orbit around the Sun, and other celestial objects as well, such as asteroids and dwarf planets. Collectively, these form the solar system, formed 4.6 billion years ago.

How the solar system is formed

The solar system started off as a spinning **giant molecular cloud** (a cloud of dust and gas). Gravitational forces exerted by these gas and dust particles among themselves started pulling them towards each other. As they got closer to each other, the particles also started revolving faster, resulting in a flattening of the dust cloud to form a protoplanetary disc. The centre of the cloud became dense and hot (see chapter "Stars" to know how stars are born), forming a **protostar** from which our Sun was eventually born. Going away from the centre, the solar nebula is less dense and more diffuse, and is where the planets and asteroids form.

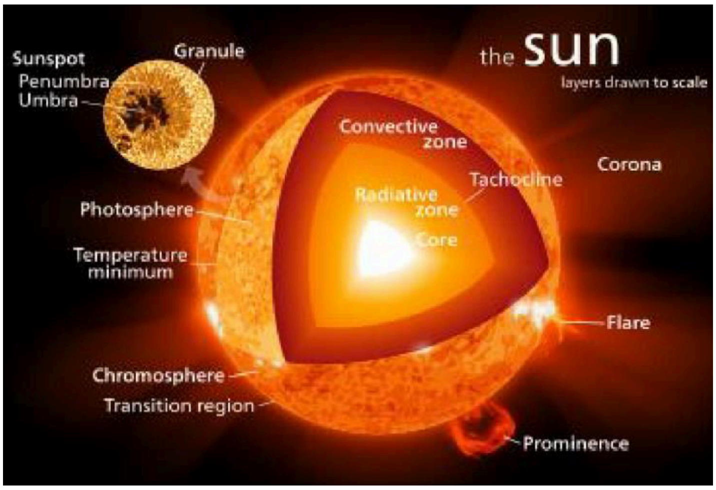
The protoplanetary disc has non-uniform density at different areas. This non-uniformity in the disc is the cause for the formation of planets through **accretion**. At denser areas of the cloud, dust and gas particles will gravitationally attract each other and stick together. As the clumps become larger and heavier, they will continue attracting particles to them and grow in size, eventually forming **planetesimals**, small rocky and icy objects that are the seeds of planets.

These planetesimals continued to gravitationally attract surrounding particles and other planetesimals to themselves, gradually growing in size to form planets. The planets formed closer to the protostar are rocky as any material other than rocks and metals (having high melting points) were vaporised by the intense heat from the protostar. Thus, the inner planets (Mercury, Venus, Earth and Mars) are rocky, terrestrial planets. At the infant stages of planetary formation, they were essentially spheres of molten rock and metal due to the intense heat from the protostar.

Beyond Mars, the temperature is low enough for ice to form. The outer planets (Jupiter, Saturn, Neptune and Uranus) were initially made of rock and ice, until they grew so large that they started to gravitationally draw in large amounts of gases as well, thus they are known as gas giants.

By then, the Sun was fully formed. Remaining debris from the formation of the Solar System scattered. Some formed the asteroid belt, some formed the moons, and others formed the Kuiper Belt and/or Oort Cloud.

The Sun ⊙



Internal Structure of the Sun

Mass / kg	1.989×10^{30}
Mean Radius / km	6.957×10^{27}
Luminosity (Power) / W	3.828×10^{26}
Magnitude	-26.8
Absolute Magnitude	+4.83

Our Sun is a main sequence star with spectral class G2V (see chapter "Stars") and is over 4.6 billion years old. It primarily consists of hot plasma made up of mostly **hydrogen**, some **helium** and little other heavier elements such as oxygen, neon, carbon and iron. Accounting for 99% of the mass of our solar system, it undergoes **nuclear fusion** from hydrogen into helium nuclei (mainly via the proton-proton chain) and generates so much energy that its core temperature reaches 15 million °C. This solar energy is then radiated through the solar system in the form of heat and light.[14]

Sunspots

Sunspots are regions of intense magnetic activity where convection is inhibited by magnetic fields. This allows us to see dark spots on the Sun's bright surface where it is about 4800 °C, comparatively cooler compared to the rest of the Sun's surface (5500 °C). Number of sunspots peaks every 11 years in accordance with the solar cycle.

Planets

A planet is defined by the following criteria by the International Astronomical Union:

- Goes around (orbit) the Sun
- Has enough mass to form a spherical shape by its own gravity,

[14] https://www.pmfias.com/sun-internal-structure-atmosphere/

- Has cleared its neighbouring region of other large objects like planetesimals in its orbit.

Composition

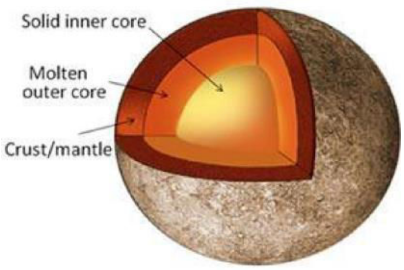
One way of classifying planets is by what they are made of. Mercury, Venus, Earth and Mars are largely made of rock and metals, and are hence categorised as **terrestrial planets**. Out of all the terrestrial planets, only Earth and Mars have moons. Jupiter, Saturn, Uranus and Neptune are largely made of gas and ice, and have significantly larger mass and faster spin than terrestrial planets, hence categorised as **gas giants**. All gas giants have rings, but Saturn's is the most obvious one.

Orbital radius

Another method of classification is by the size of their orbits. There are 2 categories under this form of classification, namely inferior and superior planets. Inferior planets are planets which have smaller orbits than Earth (essentially, Mercury and Venus), while superior planets are planets which have larger orbits than Earth (Mars, Jupiter, Saturn, Uranus, Neptune)

Mercury ☿

Mean diameter / km	4,879.4
Orbital period / days	87.969
Rotational period / days	58.646
Distance from Sun / 10^6 km	57.910
Number of Moons	0
Axial tilt	2.11°
Atmospheric Composition	42% Oxygen (O_2) 29.0% Sodium (Na) 22.0% Hydrogen (H_2)



Internal Structure of Mercury

Mercury is named after the Roman God of Trade and Messaging. It is the smallest planet in the solar system. Mercury has the highest orbital eccentricity of 0.206 among all the planets (its orbit is most elliptical) and its surface temperature has the largest variation in the solar system.[15]

Mercury is heavily cratered due to its barely existent atmosphere, which is due to its small mass, giving it very little gravity to pull in gas particles to form its atmosphere. The Caloris Basin is a large impact crater on Mercury, believed to be caused by the collision of a huge

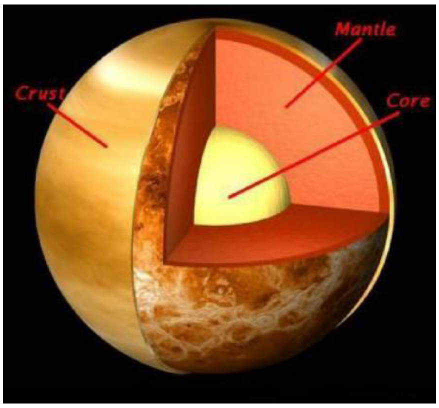
[15] http://messenger.jhuapl.edu/Resources/Science-Highlights/hl_051711.html

meteorite on the surface of the planet so powerful that it is responsible for the unusually hilly terrain around it.

Despite its minuscule mass, it does have an extremely dense iron core which generates a magnetic field much weaker than Earth's (don't confuse this with gravitational field strength).

Venus ♀

Mean diameter / km	12,104
Orbital period / days	224.70
Rotational period / days	-243.02
Distance from Sun / 106 km	108.21
Number of Moons	0
Axial tilt	173.36°, spinning East to West
Atmospheric Composition	~96.5% Carbon Dioxide (CO2) ~3.5% Nitrogen (N_2)



Internal Structure of Venus

Closest to Earth in size and composition, Venus (named after the Roman God of love and beauty) is completely different from Earth in terms of its surface environment.[16] It is the **hottest planet** (with a surface temperature of up to 460°C), despite its greater distance from the Sun than that of Mercury, due to the high concentration of **carbon dioxide** in the atmosphere of Venus. The thick blanket of CO_2, a greenhouse gas, traps the heat received from the Sun and prevents it from escaping into space. This effect is such that even the poles of Venus are hardly cold, varying only a few degrees from its equator. Scientists propose that since Venus has similar size and composition to Earth, Venus used to have an ocean and an atmosphere similar to Earth's, but the runaway greenhouse effect caused a critical increase in CO2 levels, causing the ocean to vaporise and form clouds, which trap more heat. These clouds form a vigorous wind that rages across the whole planet in just 4 Earth days. With high pressure and temperature at Venus surface, the environment on Venus is harsh.

The clouds on Venus are made of sulfur dioxide and sulfuric acid, which gives acidic rain. The clouds do not allow light to pass through and are highly reflective. They also result in the **high albedo**, which is why the planet is often seen as a bright white spot in the

[16] http://solarviews.com/eng/venus.htm

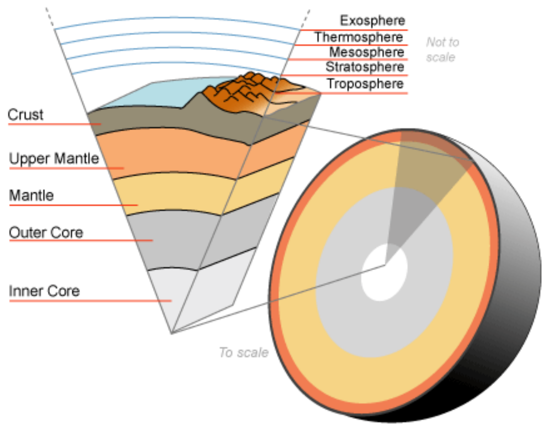
evening/morning sky, the third brightest celestial object after the Sun and Moon. Thus, it is known as the **Morning or Evening Star**.

Since the dense atmosphere of the planet prevents direct observation of the surface, astronomers have made use of radio waves to gradually map out the surface instead. It is observed that Venus has many volcanoes believed to be active. Craters are also seen on the planet surface, though more should have been present initially due to some being wiped out or covered up by solidified lava. Venus is home to Maat Mons, a shield mountain 8 km high, the second highest mountain in the Solar System.

Venus is also one of the planets which rotates in opposite direction to most other planets (retrograde rotation). A day is also longer than a year on Venus.

Earth ⊕

Mean diameter / km	12,742
Orbital period / days	365.26
Rotational period / hours	23.934
Distance from Sun / 106 km	149.60
Number of Moons	1
Axial tilt	23.44°
Atmospheric Composition	78.1% Nitrogen (N_2) 21.0% Oxygen (O_2)



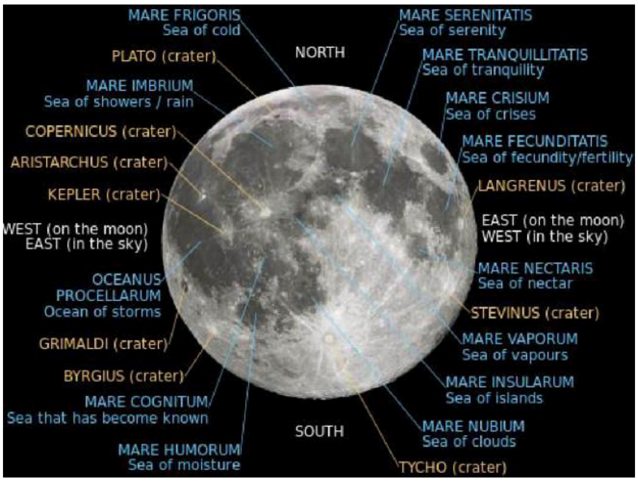
Internal Structure of Earth

The surface of Earth is mostly water which rests upon its rocky solid crust. Crust lies above a hot mantle of silicate rock which in turn lies above a molten iron-nickel outer core. Right in the middle is a solid iron-nickel inner core.[17]

[17] https://simple.wikipedia.org/wiki/Structure_of_the_Earth

Moon ☽

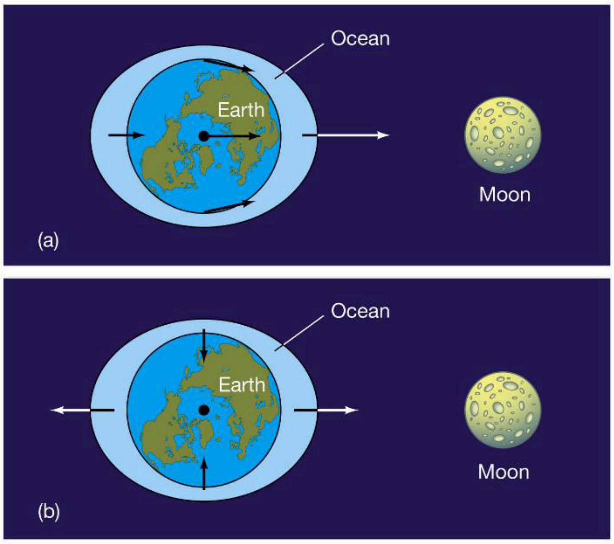
Mean diameter / km	3,474.2
Mean distance from Earth / km	384,400



Seas and Craters on Earth's Moon

The Moon is the only natural satellite of Earth. It has a **synchronous rotation**, as it spins once on its axis for each orbit around the Earth.[18]

This is because the Earth and Moon are so close that they are **tidally locked**, meaning the Earth's gravity tugs on one side of the Moon more strongly than the other, causing the near side of the Moon to always face the Earth.



Tidal Locking between Earth and Moon

The Moon is responsible for high tides and low tides on Earth. The water on the side of Earth nearer to moon experience greater gravitational pull than the centre of Earth, while the water on far side of Earth away from moon experience less gravitational pull than centre of Earth (see part (a)). Thus, there is a net force on the water (see part (b)), resulting in tidal bulges on both sides of the Earth.[19]

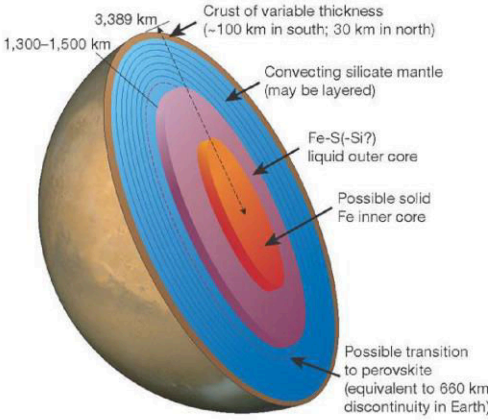
Large dark plains on the Moon surface are known as **maria** (singular: mare; meaning sea) made of basalt, dark, solidified lava from asteroid impacts, with the largest being Oceanus Procellarium. The lighter areas are called **highlands**. Craters and extinct volcanoes are scattered across the surface.

The Moon is hypothesised to have been formed by the **giant-impact theory**. It is said that a large asteroid (about the size of Mars) collided with Earth many years ago, ejecting large amounts of material into space. This cloud of dust/rock cooled and orbited around the Earth to form a ring which eventually accumulated and stuck together to form the Moon that we know today.

[18] https://en.wikipedia.org/wiki/List_of_maria_on_the_Moon
[19] https://writescience.wordpress.com/tag/tides/

Mars ♂

Mean diameter / km	6,778.4
Orbital period / days	686.97
Rotational period / hours	24.623
Distance from Sun / 106 km	227.94
Number of Moons	2
Axial tilt	25.19°
Atmospheric Composition	95.3% Carbon Dioxide (CO_2) 2.7% Nitrogen (N_2) 1.6% Argon (Ar)



Internal Structure of Mars

Named after the Roman God of War (mainly due to its blood red colour of **iron (III) oxide**, aka **rust**), Mars is about half the size of Earth. The internal structure of Mars is likely to be cooler than that of Earth, and hence probably has a solid core, thus it has a weaker magnetic field.[20]

The surface is a little cratered and contains canyons, valleys and volcanoes. It is home to the highest mountain in the solar system, the shield volcano Olympus Mons, and the longest canyon system in the solar system, Valles Marineris.

The atmosphere is too thin and the air pressure too low for liquid water to exist on the planet without vaporising instantly to form vapour. However, scientists believe that water once flowed on Mars due to the presence of valleys and what looks like dried up rivers. The planet is generally cold and dry, with an average surface temperature of -63°C, and experiences seasonal change with an axial tilt similar to Earth. There are frozen carbon dioxide and water ice at the polar ice caps of Mars.

Although its atmosphere consists of more than 90% CO_2, unlike Venus, its thin atmosphere (1% the thickness of Earth's) does not cause a drastic greenhouse effect, thus its average surface temperature is much lower than Earth's. Martian weather is highly dynamic, with season-dependent powerful winds that reach 1 km in height and cause huge dust storms.

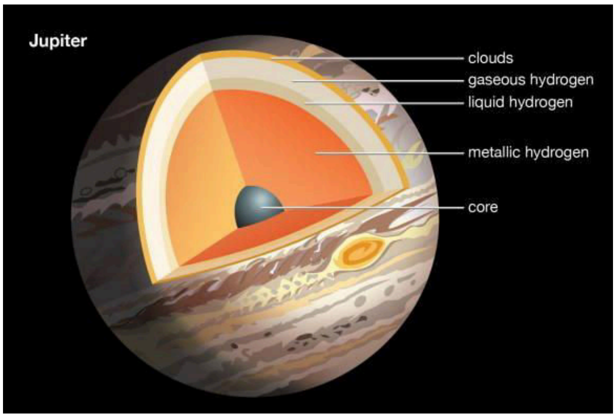
Mars has 2 small potato-like moons known as **Phobos** and **Deimos** (Latin: fear, panic). They are small and irregularly-shaped (due to their small mass and hence their self-gravity is insufficient for them to attain spherical shape). Phobos is observed to be getting closer to Mars,

[20] https://cseligman.com/text/planets/marsstructure.htm

and it is expected that it will eventually (in about 10 million years) pass inside Mars' **Roche Limit** (the distance from the planet when Mars' gravity is stronger than the moon's self-gravity and breaks it apart) and break apart due to tidal forces or collide with Mars, whereas the orbit of Deimos is bringing it further away from Mars.

Jupiter ♃

Mean diameter / km	139,820
Orbital period / days	4,332.6
Rotational period / hours	9.9250
Distance from Sun / 10^6 km	778.57
Number of Moons	67
Axial tilt	3.13°
Atmospheric Composition	89.8% Hydrogen (H_2) 10.2% Helium (He)

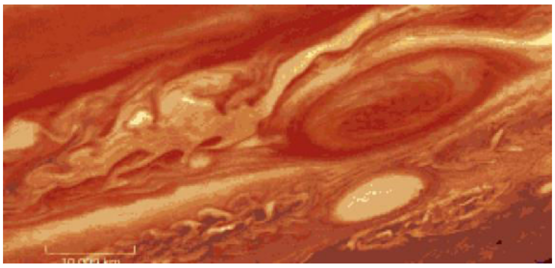


Internal Structure of Jupiter

Named after the King of Roman Gods, Jupiter is the **largest** of all planets in our solar system, with a mass almost 2.5 times that of all other planets combined. Being a gas giant, it has no solid surface, but a dense rocky icy core surrounded by a thick atmosphere of hydrogen and helium. The high pressures exerted by the thick atmospheres actually compress hydrogen at the lower atmosphere into a liquid metallic form (a very good conductor of electricity) at the interior of the planet. There is no clear boundary between the liquid and the highly compressed hydrogen gas in atmosphere. All the heavier elements, including nickel, iron and oxygen, are compressed into its dense rocky core.[21]

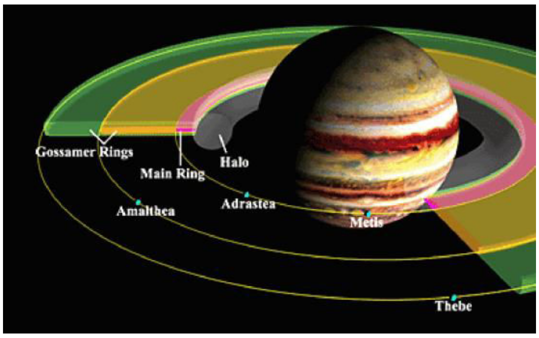
[21] https://www.britannica.com/place/Jupiter-planet/The-interior

The darker bands on Jupiter are warmer regions, known as **belts**, while the pale white bands are known as **zones**. Jupiter's dynamic atmosphere is continuously on the move. This circulatory system, coupled with the rapid rotation of the giant, is the reason for Jupiter's bands as the circulatory system is broken into smaller cells of rising and falling air. It is also the mechanism behind the famous **Great Red Spot**, a long-standing hurricane on the planet, sighted at least 300 years ago from Earth. It is the size of 4 Earths. The planet itself has a uniform temperature due to the circulation of heat.[22]



Great Red Spot of Jupiter

While less commonly known, Jupiter also has a ring system known as the Jovian Ring System that consists of mainly dust. It has three main components: a thick inner torus of particles known as the "halo ring"; a relatively bright, exceptionally thin "main ring"; and two wide, thick and faint outer "gossamer rings", named for the moons of whose material they are composed: Amalthea and Thebe.[23]



Jovian Ring System of Jupiter

[22] https://earthchangesmedia.wordpress.com/2015/10/15/hubbles-planetary-portrait-captures-changes-in-jupiters-great-red-spot/
[23] https://en.wikipedia.org/wiki/Rings_of_Jupiter

Galilean Moons

The four Galilean moons, in ascending order in distance from Jupiter: (from left) Io, Europa, Ganymede, Callisto

Jupiter has 67 known moons, the most prominent and largest of which are the Galilean moons, named after Galileo Galilei who discovered them through his telescope.[24] The Galilean moons are massive enough to attain a spherical shape, whereas the rest of Jupiter's moons are irregular.

Io is similar to Earth's Moon in terms of size and density, containing silicate rock with an iron core. However, the conditions on Io are highly different due to its active volcanism. Its atmosphere mainly consists of sulfur dioxide that is released from volcanoes

Europa is slightly smaller than the Moon. It has a smooth surface of thick layer of ice, under which is possibly, an ocean of water. This deems Europa a possible world for life within our solar system.

Ganymede is the largest moon in the Solar System, even larger than Mercury. It is the only moon that generates its own magnetic field, caused by convection in its molten core. It is made of mostly water ice and silicate rock. A salty ocean is believed to be hidden under its surface.

The above three inner moons are actually tied up in an **orbital resonance**, when Ganymede orbits once, Europa orbits exactly twice and Io orbits exactly 4 times.

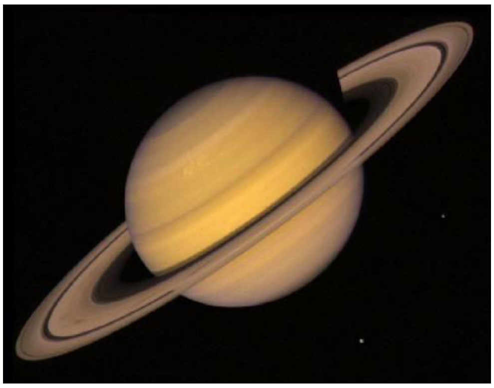
Callisto is also made of equal amounts of ice and rock, but has many craters on its surface.

There is a trend among Galilean moons in which the closer the moon is to Jupiter, the hotter its interior and less water it contains. Io has no ice, Europa has a subsurface ocean, while ice exists on Ganymede and Callisto. This is because the moons experience tidal heating. The closer the moon is to Jupiter, it experiences a greater gravitational force of Jupiter and its core gets stretched further and heats up further.

[24] https://www.jpl.nasa.gov/spaceimages/details.php?id=PIA01299

Saturn ♄

Mean diameter / km	116,460
Orbital period / days	10,759
Rotational period / hours	10.656
Distance from Sun / 106 km	1,433.5
Number of Moons	62
Axial tilt	26.73°
Atmospheric Composition	96.3% Hydrogen (H_2) 3.25% Helium (He)



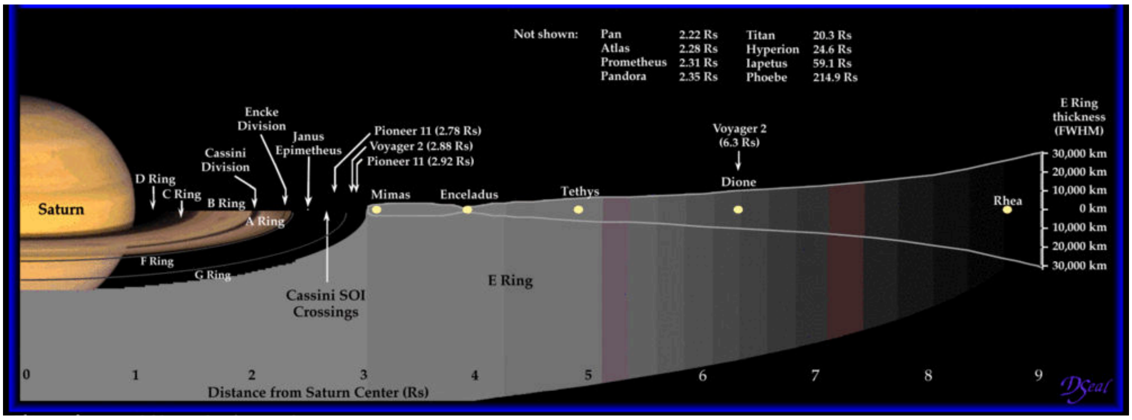
Saturn

Named after the Roman God of Agriculture, Saturn is the 2nd largest planet and the furthest planet known in the ancient times, because it can be seen by the naked eye and telescopes have not been invented yet.[25]

It is also the most oblate (least spherical) due to its fluid internal structure largely composed of gaseous and liquid hydrogen, with a small core of rock and ice, like Jupiter. Do you know that if you placed Saturn in a large bathtub, it would float? It is the only planet less dense than water!

Saturn is well known for its rings, which are grouped into D, C, B, A, F, G, E (in order of distance from Saturn), the most prominent of which are the C, B and A rings. Like that of Jupiter, Saturn's rings also lie within the Roche Limit of the parent planet and hence the **dusty icy particles** in the rings, which range from small grains to large boulders, are unable to aggregate (stick together) to form new moons. These rings may or may not be seen when observed through a telescope, depending on the angle at which Saturn faces Earth. Rings A and B are separated by a region of low ring density known as a **Cassini Division**. Several smaller divisions exist amongst the ring system.

[25] https://nssdc.gsfc.nasa.gov/planetary/factsheet/saturnfact.html



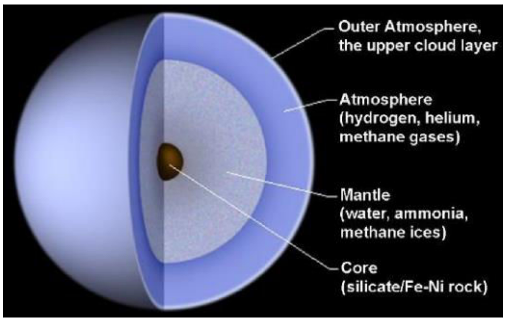
Thickness of Ring with distance from Saturn

Moons

The planet has 62 confirmed moons, the major ones being Mimas, Enceladus, Tethys, Dione, Rhea, Titan, and Iapetus (in order of increasing distance from Saturn). **Titan** is the second largest moon in the Solar System (after Jupiter's Ganymede), and is also the only moon in the solar system known to have an atmosphere. Titan's thick atmosphere of hydrocarbons and methane makes it a potential candidate for life. Temperature on Titan is low enough for methane to exist in liquid state, thus forming lakes on its surface. **Enceladus**, which has a surface of ice and a sub-surface ocean. **Cryovolcanism** is observed on where there are geysers of liquid water are seen sprouting from surface.[26]

Uranus ⛢

Mean diameter / km	50,724
Orbital period / days	30,685
Rotational period / hours	-17.24 (Retrograde)
Distance from Sun / 106 km	2,872.5
Number of Moons	27
Axial tilt	97.77°; near vertical, appears to rotate backwards
Atmospheric Composition	82.5% Hydrogen (H_2) 15.2% Helium (He) 2.3% Methane (CH_4)



Internal Structure of Uranus

[26] http://www.tau.ac.il/~morris/03411203/chapter4/Chapter4_index.html

Named after the Roman God of Sky, Uranus is yet another **gas giant** barely visible to the naked eye, and the first discovered using a telescope.[27] Relatively high levels of methane give Uranus its sky-blue colour as the methane-ice clouds in the planet tend to absorb red visible light, reflecting blue light. The planet appears to be a smooth blue ball, as UV light from the Sun reacts with methane in the atmosphere to produce a hazy effect.

Weather on Uranus is considerably dynamic, with continuous movement of clouds and wind, but most of the solar energy it receives is reflected and doesn't generate enough internal heat to drive complex weather systems.

An axial tilt of about 98° means that Venus and Uranus are the only planets with **retrograde rotation** (rotation in an opposite direction from the planet's revolution). Uranus is a unique planet that rotates on its side, most likely due to an immense collision with another planetesimal late in the formation of Uranus, tilting it on its side and causing it to adopt such an awkward rotation.

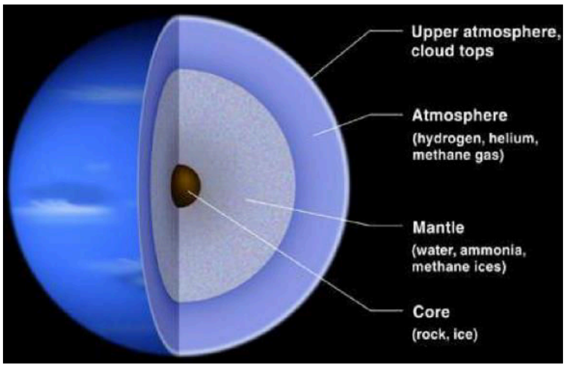
Moons

Uranus has 27 moons, the more significant ones being Miranda, Ariel, Umbriel, Titania, and Oberon (in order of increasing geographical proximity from Uranus). **Miranda** displays evidence of past geological activity, unusual for a celestial body this far from the Sun.

[27] https://en.wikipedia.org/wiki/Uranus

Neptune ♆

Mean diameter / km	49,244
Orbital period / days	60,189
Rotational period / hours	16.11
Distance from Sun / 106 km	4,495.1
Number of Moons	14
Axial tilt	28.32°
Atmospheric Composition	80.0% Hydrogen (H_2) 19.0% Helium (He) 1.5% Methane (CH_4)



Internal Structure of Neptune

Named after the Roman King of the Seas, Neptune is the coldest planet (-220°C cloud-top temperature!) first discovered mathematically from the irregularities of Uranus' orbit.[28] It is similar to Uranus in almost every aspect, but has an axial tilt of only 28.3° and a far more bluish tint, with 1.2 times more methane compared to Uranus. It also has a surprisingly dynamic atmosphere of colossal storms and extremely fast winds despite its cold atmosphere. This is likely to be due to Neptune's internally generated heat.

Constant monitoring by spacecrafts reveals significant storm-like features on the planet. One famous Earth-sized storm was the **Great Dark Spot** which was discovered in 1989, but it had disappeared when the Hubble Space Telescope tried to find it again in 1996.

Neptune's rings (the Galle, LeVerrier, Laselle, Arago and Adams rings) are similarly very faint and difficult to observe.

Moons

Neptune has only 2 major moons – Triton and Nereid. **Triton** has a very round and in synchronous orbit around Neptune (the same face of Triton faces Neptune all the time) and is known for cryovolcanic activity; its ice volcanoes spew frozen nitrogen and can form a thin atmosphere on Triton while it is exposed to the Sun. Its backwards orbit around Neptune relative to all the other moons suggests that the system of moons could have been severely perturbed by a massive planetesimal coming into close proximity with the planet in the past or that it is a captured Kuiper Belt object (see below). **Nereid** follows an extremely large orbit around Neptune and the eccentricity of its elliptical orbit is very high.

[28] https://www.universetoday.com/21596/what-is-neptune-made-of-1/

Special relativity

*Please do not worry (too much) if you do not grasp the mathematics in this section. This part of the booklet primarily serves to introduce the idea of special relativity to all of you, not to make you experts in this area. Thus, what is important is that you understand the difference between classical physics and special relativity.

The Special Theory of Relativity was first developed in early 20^{th} century. In laymen terms, it is due to the relative perception of things (hence the name *relativity*), which results in many unintuitive effects. It turns out that the classical physics we are learning in school now is incorrect, or rather approximate, and Special Relativity makes this more accurate. Special Relativity is a special case of General Relativity, and only applies in the absence of acceleration.

Frames of reference

Observations made in different frames of reference will be different. Imagine a person on the ground and a person in a car driving by. The person in the car (we denote this as the frame of the person in the car) observes the background moving backwards and the person on the ground moving backwards. On the other hand, the person on the ground observes his background to be stationary and the car is the one that is moving. This illustration shows that observations are all relative.

Postulates

The first postulate states that the laws of physics is the same in all inertial frames. What this implies is that an action that happens in one frame must also happen in another frame. For example, should an egg crack in one frame, the egg must also crack in another frame. Causality must also be obeyed. For instance, if the egg cracks because someone dropped it, the egg cannot crack before the act of the person dropping it in any other frame.

The second postulate states that the speed of light in vacuum is constant. This means that it is independent of the frame which is measured in and all observers agree on the same value.

Fundamental effects

In special relativity, unintuitive effects arise as a result of the 2 postulates. These are: the loss of simultaneity, time dilation as well as length contraction.

Loss of simultaneity

Two events that are simultaneous in one frame may not necessarily be simultaneous in another frame. We define simultaneous two events happening at the same time, or with $\Delta t = 0$. An example of how this would look like would be the opening of MRT train doors. The opening of MRT train doors might be simultaneous for an observer standing still on the platform but for an observer running along the platform, he would not observe these events to be simultaneous. In special relativity, time is in fact dependent on space (which you will see later on in the Lorentz Transformations), which is why simultaneity is not invariant unlike in classical physics.

Time dilation

This is the most interesting effect and concerns how time actually varies with reference frame, as opposed to the classical idea that time is absolute. For an event that happens at the same place, $\Delta x = 0$, the time measured in the moving frame S', is related to the time measured in the rest frame S (we consider frame S' to be moving at velocity v with respect to frame S):

$$t_{observed} = \gamma t_{proper}$$

t_{proper} is proper time, the time measured in the frame of clock, or in frame S' in this case, while $t_{observed}$ is the time observed in frame S, and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

Length contraction

Length (or the difference between two spatial coordinates) is relative as well, so the length of a ruler will be different in different frames. For an event that happens at the same time, $\Delta t = 0$, the length measured in the moving frame S', is related to the length measured in the rest frame S (we consider frame S' to be moving at velocity v with respect to frame S):

$$\frac{l_{proper}}{\gamma} = l_{observed},$$

where l_{proper} is proper length, the length measured in the frame of the object you are moving, or in frame S'. in this case, while $l_{observed}$ is the length measured in frame S.

Lorentz Transformations:

If we were to plot the combined effects of all these on Cartesian Coordinate systems with frame S' moving at constant velocity v with respect to frame S, then we have the Lorentz Transformation Equations, which relate the coordinates in one frame to another:

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$

$$\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right)$$

A very important point to note about the Lorentz Transformations is that time is affected by space and vice versa, and it is this intertwinement of space and time that leads to many of the counter intuitive phenomena we observe above

Velocity Addition:

Consider the following scenario: you have a person on top of a car running at $0.4c$ with respect to the car, and the car moving at $0.7c$ with respect to the ground. Using classical physics, we find that the person is moving at $1.1c$ with respect to the ground, which is impossible. Turns out, we need to modify our formula for the addition of velocities to the following:

$$V = \frac{u + v}{1 + \frac{uv}{c^2}}$$

where the object is moving at speed u in frame S' moving at speed v with respect to frame S. V is then the speed of the object in frame S.

Relativistic dynamics

One of the most well known equation $E = mc^2$ is a result of special relativity. However, it is usually misinterpreted as an equation describing the relationship between rest mass and rest energy, which is a special case of this equation. In fact, m is not our conventional mass, but refers to the moving mass of the particle. The moving mass is given by $m = \gamma m_0$ where m_0 is the rest mass. Since the total energy is given by the sum of its kinetic energy and rest energy, we can write the kinetic energy as follows:

$$E_K = (\gamma - 1)m_0 c^2$$

Stars

A star is a massive, luminous ball of plasma that is held together by gravity. The sun, of course, is the nearest star to Earth and is the main source of energy for Earth. Stars come in various sizes, colours, brightness and temperatures.

Stars are powered by a process known as nuclear fusion. Nuclear fusion requires a minimum temperature to occur, and only stars with a mass of at least 0.08 solar masses are able to sustain nuclear fusion. Stars below the 0.08 solar masses mark are known as "failed stars" and exist as **brown dwarfs**.

Stars neither implode nor explode because they are in **hydrostatic equilibrium**. Hydrostatic equilibrium occurs when the inward pull of gravity is balanced by an outward pressure. In the case of stars, the outward pressure comes in the form of radiation pressure generated from nuclear fusion.

Energy Generation in Stars

Nuclear fusion was discovered as the source of energy for the Sun as well as other stars by Hans Bethe in the 1930s, who worked out the steps for the different kinds of nuclear fusion present in stars. In nuclear fusion, nuclei are fused together to form heavier elements, generating energy in the process. The amount of energy released in nuclear fusion is much greater than that of chemical burning or gravitational collapse, giving the Sun's lifetime of around 10 billion years.

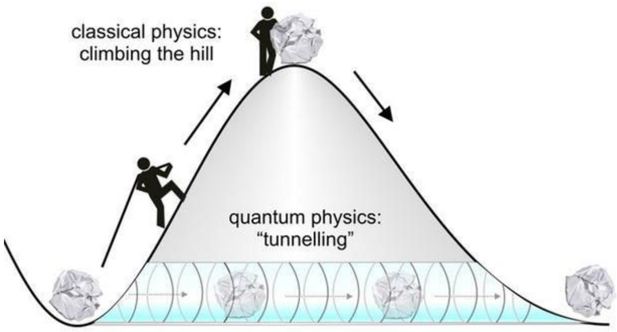
There are three main types of nuclear fusion that occur in stars. Depending on the mass of the star, different types of fusion will be dominant in the core of the star.

Proton-Proton (PP) Chain

The first is the **proton-proton (PP) chain**, which fuses hydrogen into helium. This reaction begins at around **6 million K** and usually occurs in stars of one solar mass or below. It is the primary energy source of our Sun. There are other rarer reaction pathways that involve the heavier elements, but for clarity's sake, below is the one that converts hydrogen directly into helium.

Keeping in mind that hydrogen and other elements in stars exist as plasma rather than atoms, the fusion of hydrogen will involve fusing 2 protons (nuclei of hydrogen) together. Considering the immense repulsion (also known as the **Coulomb barrier**) between like charges at such small distances, it seems impossible that fusion can even occur. In fact, it was initially calculated that temperatures in the sun were a thousand times too low for protons to have enough energy to overcome this barrier.

We then turn to quantum mechanics to explain this phenomenon. It turns out that particles can "tunnel" through potential barriers, allowing them to exist in places classically impermissible. Thus, a very minute number of protons will actually "tunnel" through the Coulomb barrier and react with each other.[29]

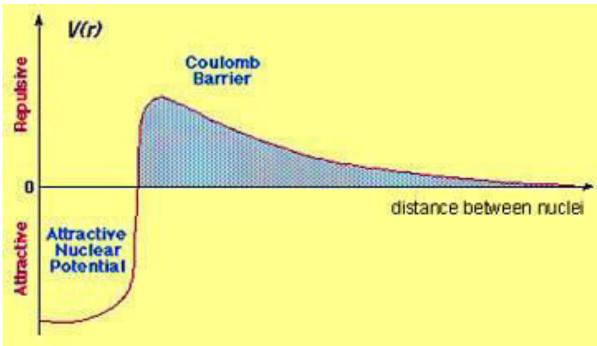


Quantum Tunnelling

We can illustrate this with someone trying to push a heavy mass over a hill. Normally, the guy (i.e. the proton) would not have enough energy to push the heavy mass over the hill to meet its colleague on the other side. However, quantum tunneling says that it's possible for some protons to reach the other side without needing to climb over the Coulomb hill/barrier. This ensures that protons do react (hooray!), but at a very, very, very slow rate, since the probability predicted for quantum tunneling is very small.

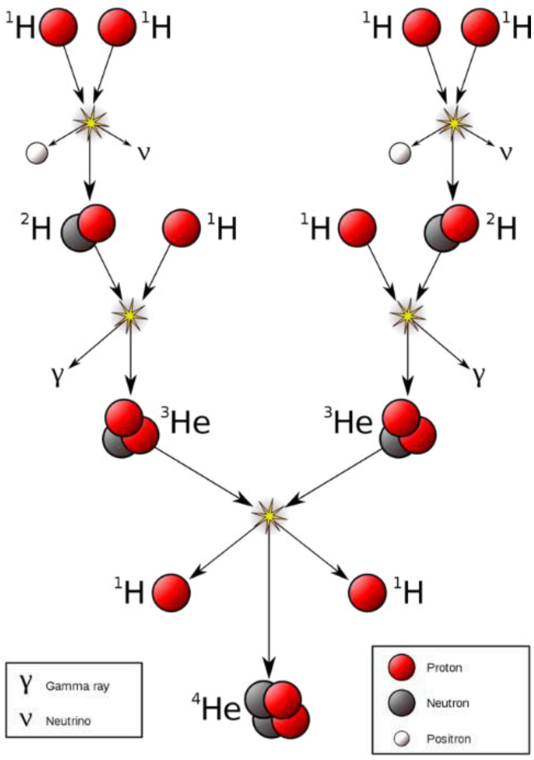
The graph above shows the electric potential becoming very high as the distance between the two protons decreases. Tunneling allows the protons to cross a "small" distance to reach the region of attractive nuclear potential, thereby reacting.[30]

With that in mind, we can introduce the PP chain, with the steps outlined below. At each step energy is released in the form of photons.[31]



Increase of Electric Potential with decrease in distance, allowing for reaction through tunnelling

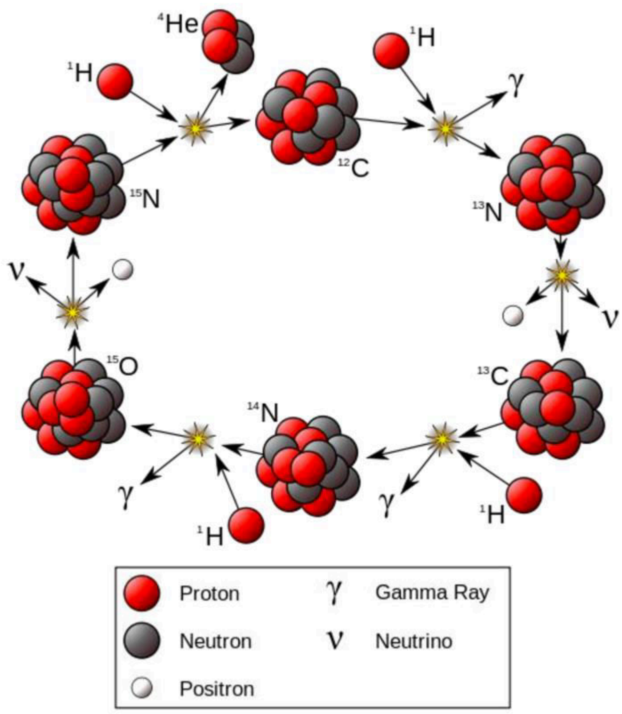
[29] https://newatlas.com/time-electron-quantum-tunneling/50784/
[30] http://www.daviddarling.info/encyclopedia/C/Coulomb_barrier.html
[31] https://en.wikipedia.org/wiki/Proton–proton_chain_reaction



Proton-Proton chain reaction

Carbon-Nitrogen-Oxygen (CNO) Cycle

This cycle involves elements heavier than hydrogen and helium, and is dominant in stars slightly heavier than the sun at around 1.3 solar masses. Of course, this PP chain still operates alongside this cycle in the star, but the CNO cycle is relatively faster. As you can see, the carbon-12 nucleus is regenerated, and four hydrogen atoms are indirectly converted into helium at the end. This cycle can thus continue "fusing" hydrogen to helium.[32]

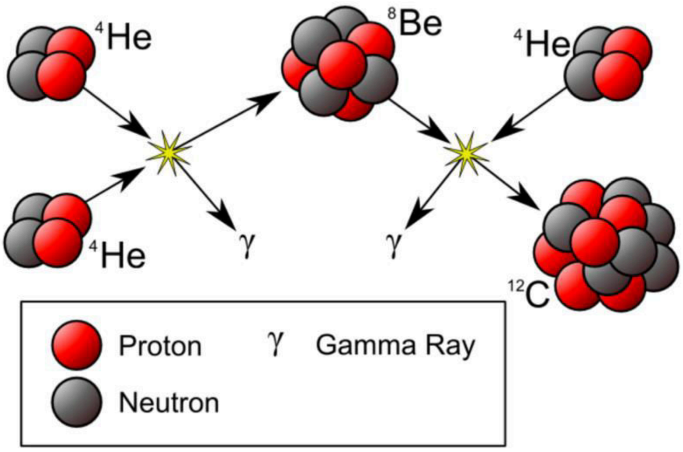


CNO Cycle Reaction

[32] https://en.wikipedia.org/wiki/CNO_cycle

The Triple-Alpha Process

This process is actually pretty rare as it involves a lot of energy, so it happens mainly in high-mass stars and red giants:



Triple-Alpha process

You can see why. Here we combine 3 helium-4 nuclei to form a carbon-12 nucleus, releasing energy in the process. The repulsion, and thus the Coulomb barrier, is stronger because we are effectively fusing helium which has a higher charge. The temperature needs to be much higher to supply enough energy for this reaction to take place fast enough.[33]

Energy Production

In nuclear physics, the mass of atoms are given in **atomic mass units**, or **amu** or **u** for short. The carbon-12 atom is defined to be exactly 12 u in mass, and so a hydrogen atom, or one proton, is around 1 u in mass. A proton by itself (i.e. a hydrogen nucleus) actually has a mass of 1.007276466812 u, for reasons that you will see.

In the PP chain, where four hydrogen nuclei (protons) are converted into one helium nucleus (2 proton, 2 neutrons), the sum of masses of the four protons is 0.7% larger than the mass of one helium nucleus. This **mass defect** is actually converted to energy according to **$E=mc^2$** in the PP chain. This energy is also known as the binding energy, which comes from the **strong nuclear force** holding the nuclei together.

> **Q**: Calculate how much energy is released in a direct conversion from 2 protons and 2 neutrons to a helium-4 nucleus, given that the mass of a proton is 1.67 x 10-27 kg and the mass of a neutron is 1.672 x 10-27 kg.

> **Q**: Given that two protons weigh 1.00728 u each, and that two neutrons weigh 1.00866 u each, find the total energy of two protons and two neutrons. Now, given that a helium-4

[33] https://commons.wikimedia.org/wiki/File:Triple-Alpha_Process.svg

nucleus weighs 4.00153 u, calculate the energy of a helium-4 nucleus. Compare the two energies calculated.

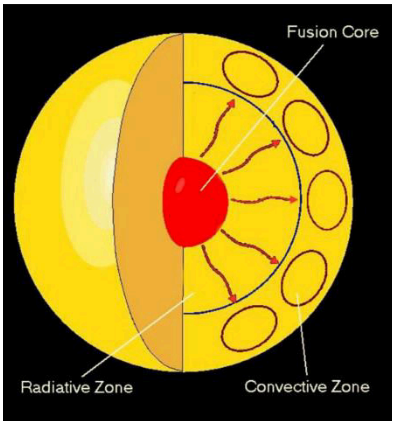
(1 u = 1.66054 x 10-27 kg)

We will explore the concept of binding energy again later on in stellar evolution, but for now we have solved the mystery of how stars produce energy. However, this energy is only created within the core of the star, where temperatures reach millions of Kelvins. How does it get out of the star and into our eyes?

Energy Transfer in Stars

For simplicity, let us use a simplified model of our Sun's interior to explain how energy transfer occurs in stars.[34]

This is the structure of the sun beneath the photosphere, that is, its surface. We have already covered how the energy is generated in the core.



Internal Structure of the Sun

The Radiative Zone

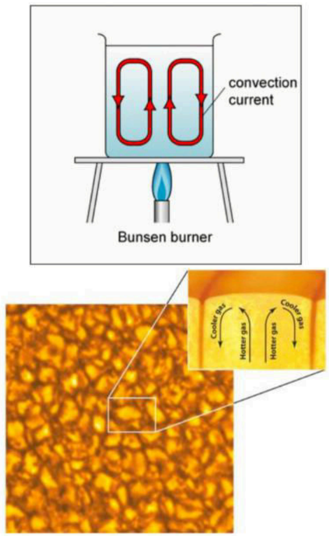
On its journey out, energy in the form of photons first enters the radiative zone. As the name suggests, radiation is the main form of energy transfer here – though not quite the form that you would expect! Keeping in mind that the sun is very dense, photons don't travel straight through as depicted in the diagram above. Rather, they travel in a zigzag manner. This zigzag motion is caused by the **photons "bouncing"** around within this dense layer. Note that this layer of plasma is so hot that it is fully ionized (all electrons are stripped from atoms to form nuclei), so the constituent particles interact strongly with light.

Overall, due to the sheer density of the radiative zone, a photon takes a **few million years** to reach the edge of the radiative zone from the core. This is in spite of the fact that photons travel at the speed of light. Also, the bouncing gradually causes the photon to lose energy over time, and its wavelength will decrease, causing it to fall from the gamma range to the visible-UV range in the electromagnetic spectrum.

[34] https://www.st-andrews.ac.uk/~bds2/ltsn/ljm/JAVA/SUN/Suninter.html

The Convective Zone

At this point, **temperatures are sufficiently low for electrons and atomic nuclei to recombine and form atoms**. These atoms will then absorb the photons and heat up. Convection takes place here, and it's very much akin to heating water and watching the hotter and less dense regions rise up to the surface while cooler and regions sink. This creates convection currents that are easily visible to the eye. The same thing happens in the sun, and this is how the energy is transported to the surface.[35,36]

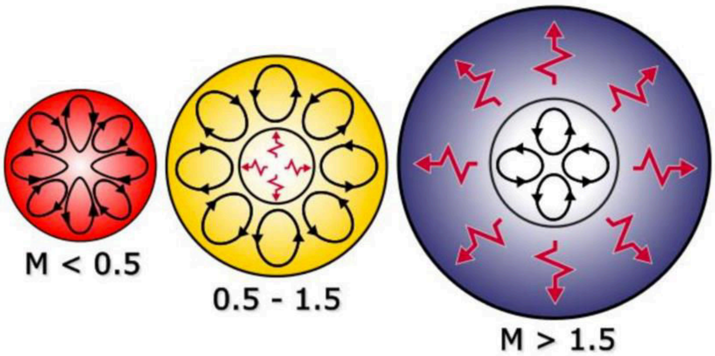


Convection Currents on the Surface of the Sun

Upon reaching the surface of the star, energy is radiated into space which we then see mainly as light.

Convective and radiative zones in other stars

Now that we know more about our sun, how do other stars transfer energy? It turns out that other stars also have radiative and convective zones, just possibly in a different order:[37]



Convective and Radiative Zones in stars of different masses

The middle star depicts a sun-like star, with the zones as mentioned. For a **lower mass star**, the star is less dense and cooler. Hence, outside of the core, **most of the star is convective** since the electrons and atomic nuclei have already combined.

For **higher mass and hotter stars**, it gets complicated. **While energy transfer outside the core is mostly radiative**, it turns out there is a convective zone **within the core**. For convection to occur we need a steep temperature gradient, that is, the temperature must fall fast enough across a certain distance. Why does convection only exist in cores of high-mass stars? This is because the CNO cycle is dominant in high-mass stars, which generates much more energy compared to the PP chain. Calculations have shown that the rate of reaction for the CNO cycle scales with temperature to the 17^{th} order of magnitude, whereas the rate of reaction for the PP chain scales with temperature to the 4^{th} order of magnitude.

[35] https://socratic.org/questions/what-are-convection-currents-and-what-causes-them
[36] http://year11science.weebly.com/conduction-convection-and-radiation.html
[37] https://commons.wikimedia.org/wiki/File:Star_types.sv

This helps to explain why in higher mass stars such a steep temperature gradient exists, allowing for convection to occur within the core.

Stellar Spectra

All stars can, to a first approximation, be thought of as blackbodies. **Blackbodies are objects which absorb and emit all wavelengths of light across the whole electromagnetic spectrum**. However, depending on the temperature of the surface of the star, some wavelengths are emitted more strongly than others. For example, the Sun emits a lot of visible light and UV radiation and much less of gamma rays and radio waves.

Peak wavelength emitted

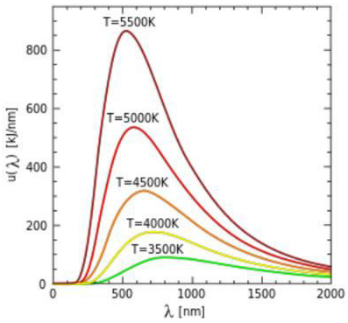
Wien's displacement formula gives the peak wavelength that a blackbody radiates:

$$\lambda_{max} = \frac{0.002898}{T},$$

where T is the temperature of the surface of the star (from which radiation is emitted). The constant 0.002898 m·K is known as Wien's Displacement Constant.

Q: Given that the star Vega has a surface temperature of 9600 K, calculate the peak wavelength of EM radiation that it emits.

Now say you experimentally sample the intensity of a blackbody at many different wavelengths. If you do this for all wavelengths, you will get Planck's distribution, the maximum of which can be found using Wien's displacement formula.



Planck's Distribution of energy density per unit wavelength

Different stars have different surface temperatures, which give different blackbody distributions with different peak wavelengths. This explains why hotter stars appear blue – their peak wavelengths are the shortest. Of course, it's not just blue light that is reaching us: the star only appears blue because that is the dominant form of visible light that is reaching us. The same applies for reddish, cooler stars.[38]

Spectral Lines

If we lay out the blackbody spectrum, including the range of colours of visible light, we get what's called a **continuous spectrum** (all wavelengths are emitted and received):[39]

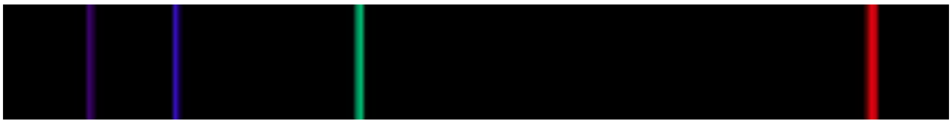
[38] https://www.e-education.psu.edu/astro801/content/l3_p5.html
[39] https://continuousspectrum.com

Continuous Spectrum

If we pass a continuous spectrum of radiation/light through a cloud of gas before receiving it, we would find an **absorption spectrum**:[40]

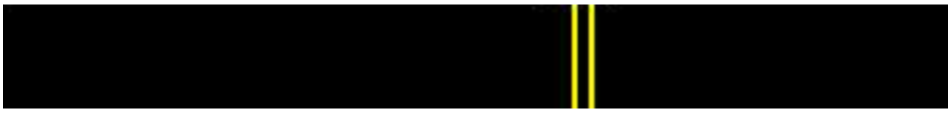
Absorption Spectrum, with dark lines due to absorption of specific wavelengths of light

A curious thing happens: dark lines appear in the spectrum! These dark lines are wavelengths of radiation that are absorbed by the cloud of gas, and thus are prevented from reaching us. Conversely if the cloud of gas is heated up strongly enough, it can emit its own radiation at specific wavelengths, and we get an **emission spectrum**:[41]



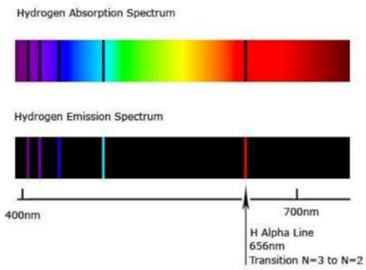
Emission Spectrum, capturing wavelengths of light emitted

From the absorption/emission spectrum, we can tell what kind of elements are in the cloud of gas. For example, if the cloud of gas is made up of **sodium**, the spectrum should look like (sodium-vapour lamps are yellow too):[42]



Spectral Lines of Sodium

Most elements are not that simple though. For example, here is hydrogen:[43]



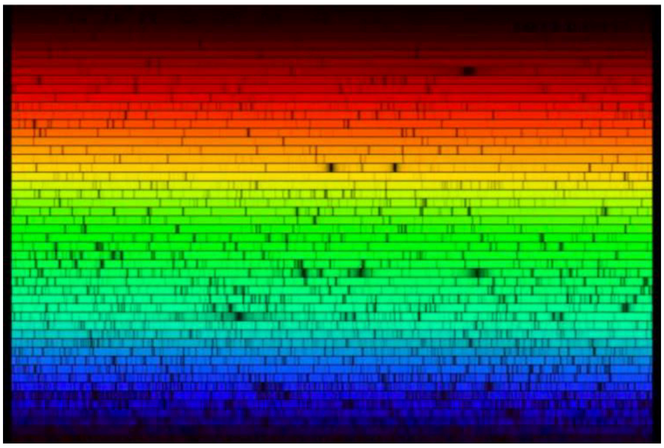
Spectral lines of hydrogen

Think of each element as having a signature which corresponds to its spectra. Each absorption/emission line is reflective of the amount of energy required for electrons in the atom to jump from one state to another. The lines thus allow us to deduce electronic properties of the atom.[44]

---

[40] https://www.khanacademy.org/partner-content/nasa/measuringuniverse/spectroscopy/a/absorptionemission-lines
[41] http://slideplayer.com/slide/4461211/
[42] http://staff.on.br/jlkm/astron2e/AT_MEDIA/CH04/CHAP04AT.HTM
[43] https://www.khanacademy.org/partner-content/nasa/measuringuniverse/spectroscopy/a/absorptionemission-lines
[44] https://www.eso.org/public/images/sunspectrum-noao/

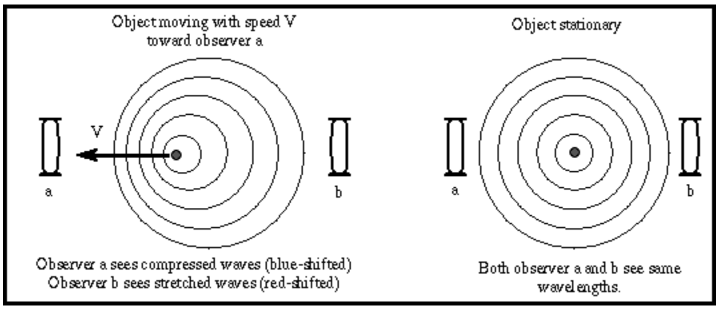
In reality, spectral lines are way more complicated. These are absorption lines from the sun. The whole thing is actually one entire spectrum from red to violet, cut up into strips and then stacked on top of each other. Many elements' absorption lines exist in there. By comparing the spectrum of an unknown mixture of elements with known spectra of certain gases, we can determine the nature of the unknown elements. We can also determine the abundance of elements in the sample based on the strength of its lines. This was how **helium** was discovered, through **spectroscopic analysis** of the sun.



Spectrum emitted by Sun

The Doppler Effect

The Doppler Effect is named after Christian Doppler. A classic way to demonstrate this effect is by observing the siren of an ambulance as it passes by you. When the ambulance is approaching you, you hear a higher pitch of sound; when the ambulance is moving away from you, you hear a lower pitch of sound. Since sound is a wave, we can pictorially represent the higher and lower pitches by smaller and larger wavelengths respectively.[45]



Shift in Wavelength due to Doppler Effect

The diagram above aids in understanding how to derive the changing pitch of sound. Note that the sound is emitted at a fixed frequency. After emitting the first wave, the ambulance would have "caught up/raced ahead" a little (depending on which side you are), and so the next wave would be emitted at a nearer/further distance from the first wave. Thus the wavelength of the sound effectively changes due to the motion of the source. Since the speed of sound is **constant with respect to a stationary observer, if the wavelength changes, then so must the frequency and pitch of the sound heard.** You can refer to the derivation of the wavelengths and the changed frequency above.

However, the above applies only for a stationary observer. What about a moving observer? In that case, the moving observer might conversely also receive the sound waves at lower or higher frequency, since he can move a little after receiving the first sound wave

[45] http://www.astronomynotes.com/light/s10.htm

and then receive the second sound wave with delay too, causing him to hear a lower pitch. By taking into account this delay in the same manner as above, the final expression for Doppler shifted sound frequency is:

$$f' = \frac{u+v_o}{u+v_s} f_s,$$

where u is the speed of sound, v_o is the speed of the observer, v_s is the speed of the source, f' is frequency observe and f_s is the original frequency of sound emitted by the source.

Note: Be careful with the signs of the velocity values. When in doubt, take the direction from the observer to the source as positive, and vice versa.

When we're dealing with Doppler shifted **radiation/light**, we can apply the same arguments and principles above for a moving source. Also, for light, only the velocity of the source **relative to the observer** is important, as given by special relativity. The equation here is: (we are more concerned with the wavelength rather than the frequency when dealing with electromagnetic radiation, since we don't actually hear radiation…)

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$$

Here, v is the relative speed between source and observer, c is the speed of light, λ0 is the original wavelength of the radiation and Δλ is the change in wavelength of the radiation that we want to find.

However, this only applies for **low velocities**. At higher velocities, we have to use special relativity to derive the Doppler Effect. The relativistic Doppler Effect equation is:

$$1 + \frac{\Delta\lambda}{\lambda_0} = \frac{f_s}{f'} = \sqrt{\frac{c+v}{c-v}}$$

The Doppler shift can also be observed in spectroscopy. Due to a source (usually a star or galaxy) moving away from us, the lines have a longer wavelength, and are **red-shifted**. Astronomers use **redshift** to calculate the receding velocity of galaxies. By knowing the velocity, we can then determine the proper distance from us to the galaxy using Hubble's law.

Luminosity and Flux

The idea of luminosity came about to allow astronomers to quantify the total output of radiation. In simple terms, luminosity is the rate of energy released by a star or an object. It turns out that for blackbodies there is a relation called the Stefan-Boltzmann law which allows us to calculate the luminosity:

$$L = 4\pi R^2 \sigma T^4$$

where R is the radius of the star, T is the surface temperature, and $\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$ is the Stefan-Boltzmann's constant.

$$b = \frac{L}{4\pi D^2}$$

This expression gives the brightness/intensity of a star at some distance D away from it. This follows **the inverse square law of light intensity** as it goes further away from its source. Notice how the brightness is expressed as a flux, that is, the rate of energy output per unit area (in this case, the area is the area of the sphere with radius equal to the distance between the observer and the source since light from an object spreads out in all directions i.e. in a spherically symmetrical manner).

The Magnitude System

Apparent Magnitude

In today's world, we use a logarithmic scale to measure brightness. We define every increase of magnitude by 5 grades to be a 100 times decrease in brightness. That is, a sixth magnitude star is 100 times dimmer than a first magnitude star. Since the scale is logarithmic, this means that every increase in magnitude by one implies a decrease in brightness by 2.512 (or to be more precise, the fifth root of 100, as you can guess).

If the above system sounds confusing, here's a mathematical way of expressing all of the above features of the magnitude system. Use this for converting differences in magnitudes of two stars to a ratio of their intensities.

$$\frac{b_1}{b_2} = 100^{\frac{1}{5}(m_2 - m_1)}$$

Now for a reality check. Let's investigate the **apparent magnitude** of some objects, that is, their magnitude as viewed by us. Our sun has an apparent magnitude of -26.74, and the moon -12.74. Both of these objects are much brighter compared to other objects in the sky. Sirius, the brightest star in the night sky, has a magnitude of -1.47. So now you may ask, if the above formula only gives the ratio of brightnesses, how do we determine the value of the apparent magnitude?

Well, astronomers have calibrated 0.00 to be the magnitude of Vega. Everything else takes reference to Vega's magnitude.

Under perfect sky conditions, the human eye can see objects with apparent magnitude 6.0. Under Singapore sky conditions however, severe light pollution only allows us to see magnitude 2.0 and below. This explains why we can't see things like the Andromeda Galaxy, at magnitude 3.44, at all, because its apparent magnitude is above the **limiting magnitude of the sky**.

Absolute Magnitude

Astronomers also wanted a system to quantify brightness without it being dependent on the distance to the observer. Thus, absolute magnitude is defined as the **apparent magnitude when viewed from 10 parsecs, or 32.6 light years away.** Since every star's absolute magnitude is a measure of its brightness from the same distance away, it can be used to compare the relative luminosity of the stars.

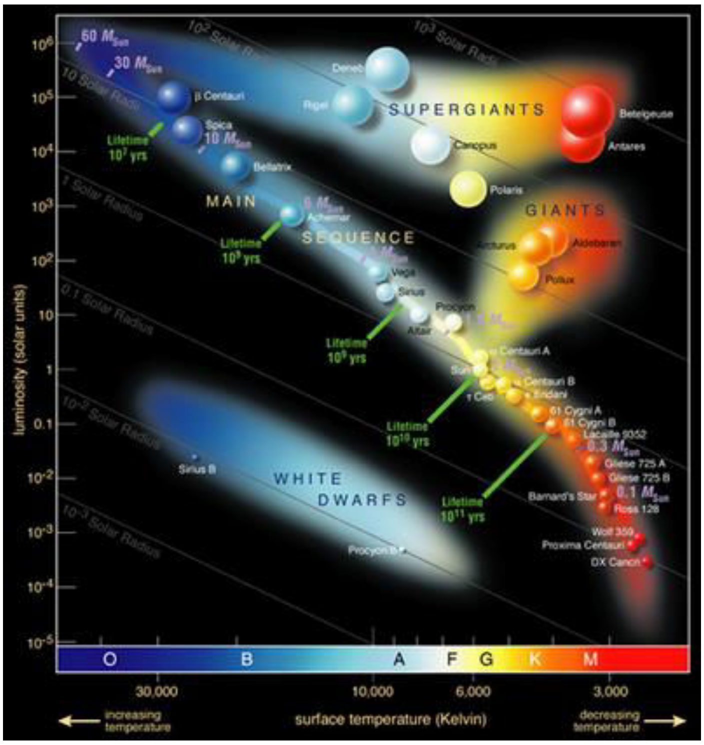
You know that the brightness decreases with the inverse-square law, and you also know how the decrease in brightness is reflected by an increase in magnitude. Combining the two equations, we get an expression for the absolute magnitude of a star:

$$(m - M) = 5log_{10}(d) - 5$$

Here m is the apparent magnitude, M is the absolute magnitude, and d is the distance to the star in parsecs. You can see what happens if you substitute d = 10 parsecs in, the apparent and absolute magnitudes become equal.

Hertzsprung-Russel diagram

You may have noticed a pattern in the stars covered earlier: blue stars are more luminous than yellow stars, which are more luminous than red stars. If we plot all the stars on a single graph, with the y-axis as luminosity, and the x- axis as colour, we get the Hertzsprung-Russell diagram, named after Ejnar Hertzsprung and Henry Norris Russell:



Hertzsprung-Russell Diagram (HR Diagram)

Most of the stars lie on what we call the **Main Sequence**, which runs from top left to bottom right. Main sequence stars are stars that have achieved equilibrium after coming out of the protostar stage (elaborated later). There are of course giants, supergiants, and white dwarfs which we will talk about later, since these are stars in the later stages of their lifetime.[46]

The H-R diagram can have different versions. The y-axis can either be luminosity or absolute magnitude as they are equivalent indicators of measuring how bright a star is. The x-axis can either be the temperature of a star or its **spectral class**.

Spectral Class

A **spectral class** is a letter denoting the star's ionisation state. In simple terms, it just means how hot the star's surface temperature is. In order of decreasing surface temperature,

[46] https://en.wikipedia.org/wiki/Hertzsprung–Russell_diagram

there are the O, B, A, F, G, K and M classes for the Morgan-Keenan System of classification. Most stars are classified under this system. **O and B represent the bluest, hottest stars, A and F represent the white stars, G yellow, K orange and M red**. A commonly used mnemonic to remember this is: **Oh, Be A Fine Girl, Kiss Me**. But since Singapore's education system is an extremely demanding one compared to other countries', I guess for most readers this leaves a much better impression: **Oh Boy, An F Grade Kills Me**

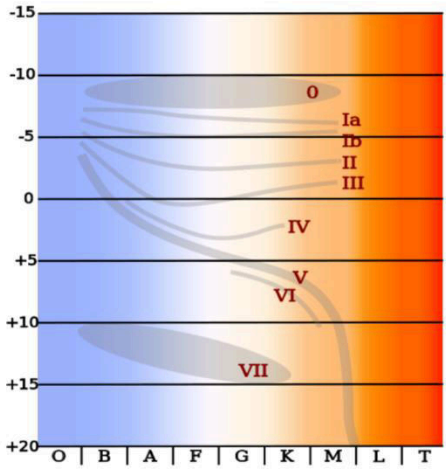
The system can be extended to include other stars. W is a spectral class that comes before O and describes **Wolf-Rayet stars**, which are extremely hot and bluish, sometimes characterised by luminosities so intense that outer layers are occasionally blown away by its own stellar wind.

After M we can insert even more spectral classes for cooler stars. These stars are generally **failed stars** that we call **brown dwarves**, stars that don't have enough energy to start nuclear fusion in their cores. In order of decreasing temperature, there are the L, T and Y brown dwarf stars. Y class brown dwarves in particular are ultra-cool, with surface temperatures going down to < 600 K. We'll come back to them again in stellar evolution.

So... the extended spectral class runs **W, O, B, A, F, G, K, M, L, T, Y**, with another mnemonic! **WOah, Be A Fine Guy/Girl, Kiss Me Later Today, Yolo**

Classifying stars

Stars can be classified based on their spectral class, but within each spectral class there are subdivisions, with 0 being the hottest and 9 being the coolest. So a G0 star is hotter than a G9 star, and an A5 star is cooler than an A3 star. Our sun is considered a G2 star. In the Morgan-Keenan Classification, there is also a luminosity class that is shown in the second H-R diagram on the right.[47]



Spectral Class Classification

I – Supergiants

II – Bright Giants

III – Giants

IV – Sub Giants

V – Main Sequence stars/dwarves

Most stars fall under V, and the convention is to mention the luminosity class after the spectral class. For our sun that would be G2V. For a star like Betelgeuse, it would be M2I, since it's a red supergiant.

[47] https://sigmaorionis.wordpress.com/tag/brown-dwarfs/

Stellar evolution

Astronomers can only observe the star in brief snapshots, as the human civilisation has only been in existence for at most 8000 years, much less than any star's lifetime. However, through many observations we have been able to piece together these snapshots like a jigsaw puzzle, together with some computer simulations, to get a pretty complete picture of how a star evolves.

The beginning

It all starts with a **nebula** (latin for "cloud"), a large cloud of dust and gas. These nebulae dwarf the solar system, with diameters reaching up to tens of light years across. As time passes, the nebula may potentially get disturbed by the shockwaves from a supernova explosion, which triggers a gravitational collapse. Small clumps of the nebula slowly collapse together to form what we call **protostars**.

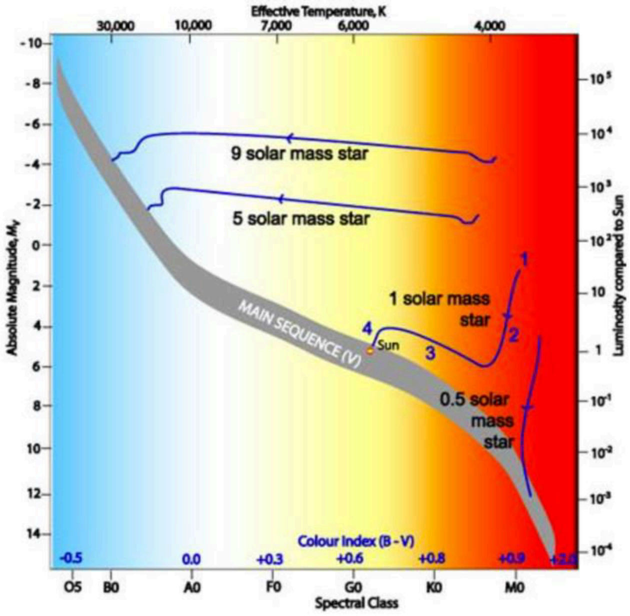
An accretion disk of gas and dust forms around the condensed protostar, which becomes hotter and hotter as it contracts under its own gravity. Eventually, the protostar becomes dense enough to become what's called a **T-Tauri** star, characterised by strong stellar winds that blow outward from it. It is believed that at this stage, the winds are powered by lithium burning within, in a manner similar to the PP-chain. The amount of lithium is small compared to the amount of hydrogen however, and after 100 million years the lithium will get exhausted. Gravitational collapse continues.

Soon enough, the centre of the star becomes hot and dense enough to start nuclear fusion, and the star stops collapsing once the radiation reaches the surface and halts the gravitational collapse. A star's life is all about a battle between the **radiation pressure** from within and **gravitational collapse** of the star itself. The star's entrance into the main sequence marks the start of nuclear fusion halting the gravitational collapse of the star, producing a stable **hydrostatic equilibrium**. If the star collapses more, its density increases, and the radiation pressure becomes stronger, causing it to expand. If there is too much radiation pressure, the star expands outwards and becomes less dense, reducing the rate of nuclear fusion and thus radiation pressure.

Maturing to a main sequence star

On the H-R diagram, the entire process from protostar to main sequence is marked by the **Hayashi** and **Henyey tracks**.

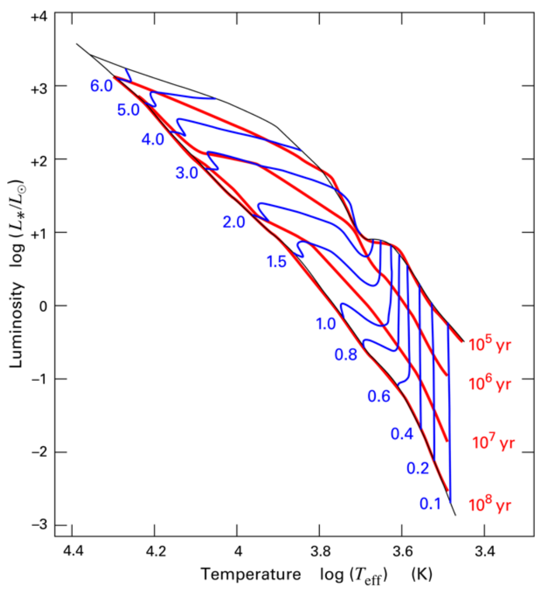
The Hayashi track is a nearly vertical curve that a protostar takes on the H-R diagram. After a protostar ends its contraction and becomes a T Tauri star, it is extremely luminous. The star then follows the Hayashi track downwards, becoming several times less luminous but staying at roughly the same surface temperature, until it enters the Henyey track.[48]



Hayashi Track on HR Diagram

The **Henyey track** by contrast is nearly horizontal in nature. It happens just after nuclear fusion starts, where the star slowly collapses to reach hydrostatic equilibrium. During this time, the star becomes hotter but stays at the same luminosity.[49]

Different stars have different **pre-main sequence tracks** they take. Note that the more massive stars enter the main sequence on the left side, becoming bluish stars, and vice versa for less massive stars. The Hayashi track is dominant for less massive stars, which start out as blazing T-Tauri stars but then settle down into yellow or red dwarves before starting nuclear fusion. The Henyey track starts almost immediately for more massive stars, as they start nuclear fusion earlier owing to their huge size, and then slowly settle into hydrostatic equilibrium.



Henyey Track on HR Diagram

We've already talked about the main sequence stars. Around 90 percent of stars lie on the main sequence in the H-R diagram. Once they exhaust the hydrogen in their cores through nuclear fusion, they will start to leave the main sequence stage and become giants. The more massive a star is, the faster it exhausts the hydrogen in its core. O type stars spend around 60 million years only on the main sequence, while G type stars like our sun spend 10 billion. M type stars

[48] http://stormofscience.blogspot.sg/2013/08/hayashi-tracks-of-protostars-and.html
[49] https://en.wikipedia.org/wiki/Hayashi_track

have lifetimes older than the universe is, and are likely to have lifetimes at 100 billion years. As they say, those who “live fast, die young”.

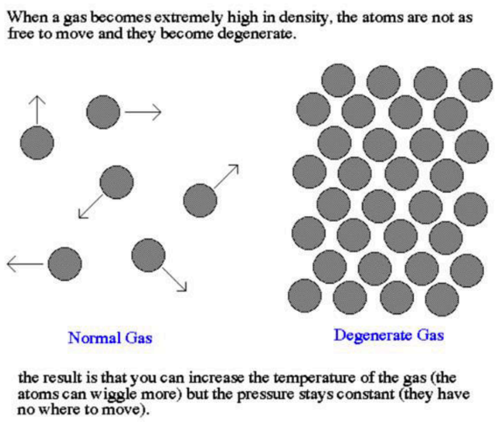
From here on, the evolution of stars of different masses diverge.

Death

The biggest (**Wolf-Rayet** type) stars produce so much radiation pressure that their outer layers are blown out more and more strongly, until finally the whole star explodes into a supernova, leaving nothing behind.

There are also the **brown dwarfs**: stars that make it to the protostar stage but are unable to start nuclear fusion. Strictly speaking, they are not even on the main sequence, and they emit primarily in infrared. They spend the rest of their lives cooling down, relying on gravitational collapse to give out heat, until their temperature becomes so low they’re just floating balls of gas.

For more **normal stars**, after depleting the hydrogen from their cores, these stars begin to contract under gravity again. This continues until the material surrounding the core gets hot enough so as to burn hydrogen once again. This new hydrogen burning shell is larger and has a greater power output than the core during the star’s main sequence. This causes the outer layers of the star to expand from the radiation pressure of this reignited nuclear fusion, as it absorbs more and more photons. In doing so, these outer layers cool down eventually, making these stars **red giants** / **supergiants**. Note that due to the opacity of the outer layers to photons, heat transfer is **convective**.



Degenerate Gas

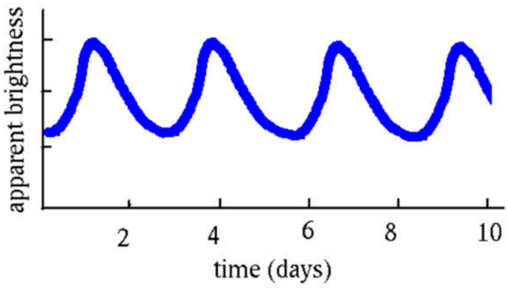
While this is going on, the core becomes hot enough to start helium burning. The triple-alpha process fuses the helium into carbon. At this point the core is made up of a **degenerate gas**, characterised by its extremely high density. A degenerate gas hardly behaves like gas in a main sequence star – its pressure does not depend on temperature. More on its properties later.[50]

For this reason, when helium burning starts, the degenerate gas is **unable to regulate** it by expanding like a normal gas. The temperature just gets higher, accelerating the burning even more. A brief period called the **helium flash** occurs, where so much energy is generated within the core that the energy output of the star is 100 billion times greater than its normal energy output. This happens till the temperature is high enough for the core to behave like a normal gas again, and the cycle of helium burning continues.

[50] http://abyss.uoregon.edu/~js/ast122/lectures/lec12.html

Middleweight stars (0.1 to 8 solar masses)

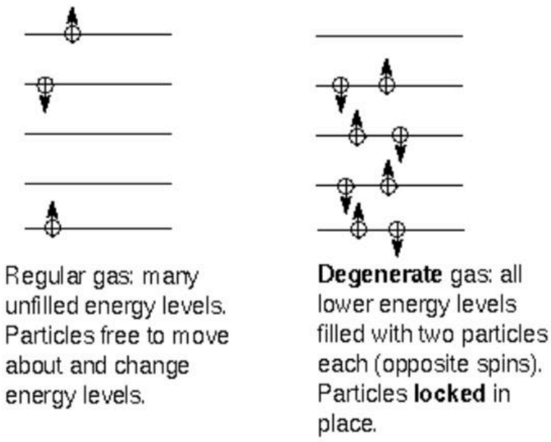
After all the helium has burnt out, we are left with carbon and possibly some oxygen, but the temperature is not high enough for further fusion. The core then collapses under its own gravity, while the surrounding shells/layers continue to fuse helium. The fusion of helium here can cause the outer layers to pulsate as **Cepheid variables** – as the outer layers heat up, they expand till a point where they become opaque to photons, and gravity pulls them back in to heat up again.[51]



Periodic Brightness Fluctuations of Cepheid Variables

There comes a point however, when the helium burning is too much to bear, and the outer layers are expelled into space one by one through pressure waves. The result is a **planetary nebula**, a misnomer due to William Herschel initially thinking these stellar remnants were planets.

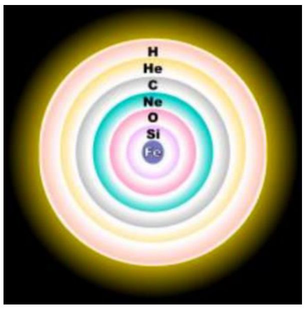
The core of the star is left behind, unable to support nuclear fusion of carbon and oxygen, so it collapses until **electron degeneracy** stops it. At this point the gas is so dense that electrons are literally squeezed to occupy all available energy states. We call the matter inside a sea of electrons. Electron degeneracy then kicks into action through **Pauli's Exclusion Principle**, which prevents electrons from being squeezed so close together that they occupy the same state.[52] This generates a pressure which halts the collapse, giving rise to a **white dwarf** that can be seen at the centre of the planetary nebula.



Difference between regular and degenerate gas

Heavyweight stars (> 8 solar masses)

Stars heavier than 8 solar masses go into the red giant stage in the same way as lighter stars, but are able to fuse carbon and oxygen later on into heavier elements. Layer upon layer of the star gets heated up to fuse heavier elements, until we end up with an onion-like structure.[53]



---

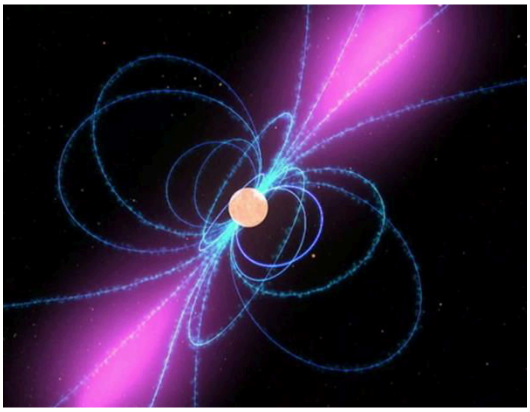
[51] http://www.faculty.virginia.edu/ASTR5110/lectures/photometry/photometry_mags.html
[52] http://www.astronomynotes.com/evolutn/s10.htm
[53] http://www.billtheskyguy.com/faq/

At the centre is a core made up of iron and nickel, two elements with the highest binding energies. This means when iron and nickel are fused together, the binding energy of the product is lower. The fusion process now absorbs energy to "unbind" the nucleus instead of releasing energy as in earlier processes. In other words, iron and nickel have the most stable nuclei.

Elements in different layers of heavy stars

Nuclear fusion is unable to take place, and the core collapses in a dramatic fashion, bringing down all the outer layers with it. This overcomes the electron degeneracy pressure, causing electrons to be combined with protons to form neutrons. Instead of becoming a sea of electrons as in white dwarfs, the star ends up as a **neutron star**. The Pauli Exclusion Principle also applies to neutrons, so a kind of **neutron degeneracy pressure** supports the neutron star against gravity.

The collapse of the core, due to the conservation of angular momentum, also means the star now rotates very, very fast, much like a ballerina pulling her arms in to increase her rotational speed. The period of a neutron star can be as fast as 1 millisecond. This generates strong magnetic fields, like a dynamo. This magnetic field strongly accelerates particles around it, producing narrow beams of X-rays that shoot out from the poles. When these beams are pointed at us, we call these neutron stars **pulsars**.[54]



Pulsars

The outer layers rebound off the core, resulting in a **supernova explosion**. We can illustrate this with the experiment where a tennis ball is placed on a basketball, both of which are then dropped together. The rebound of the much heavier basketball causes the tennis ball to fly upwards at a very high velocity. The same thing happens with the outer layers.

The resulting supernova explosion is titanic and can outshine a galaxy for a few days. We're left with a **supernova remnant** with a neutron star at the centre of it.

Of course, if the core is massive enough (> 3 solar masses), nothing can stop gravity, not

even neutron degeneracy pressure, nor any other force. The core collapses on itself and becomes an infinitely dense and compact object known as a **black hole**.

[54] http://space.mindofamadman.com/2016/04/21/what-are-pulsars/

Solutions to Questions

Q: Calculate how much energy is released in a direct conversion from 2 protons and 2 neutrons to a helium-4 nucleus, given that the mass of a proton is 1.67 x 10^{-27} kg and the mass of a neutron is 1.672 x 10^{-27} kg.

The mass defect is 3.9875 x 10^{-29} kg. Using E =mc^2:

E = (3.9875 x 10^{-29}) (3 x 10^8)^2 = 3.59 x 10^{-12}

While it doesn't seem like much, the amount of energy released can be expressed as 22.4 MeV, where one 1eV (electron volt) is a unit of energy equivalent to 1.6 x 10-19 J. By comparison, the energy needed to ionize a hydrogen atom is 13.6 eV. So a lot of energy is produced out of the PP-chain reaction.

Q: Given that two protons weigh 1.00728 u each, and that two neutrons weigh 1.00866 u each, find the total energy of two protons and two neutrons. Now, given that a helium-4 nucleus weighs 4.00153 u, calculate the energy of a helium-4 nucleus. Compare the two energies calculated.

(1 u = 1.66054 x 10-27 kg)

$$Total\ energy\ of\ two\ protons\ and\ two\ neutrons$$
$$= (6.6846 \times 10^{-27}\ \mathrm{kg})c^2 = 6.106 \times 10^{-10}\ \mathrm{J}$$

$$\begin{aligned} Total\ energy\ of\ a\ helium - 4\, nucleus \\ &= (6.6447 \times 10^{-27}\ \mathrm{kg})c^2 \\ &= 5.980 \times 10^{-10}\ \mathrm{J} \end{aligned}$$

The difference should give you the same energy as in the previous question. The mass defect and the difference in energies are the same thing.

Q: Given that the star Vega has a surface temperature of 9600 K, calculate the peak wavelength of EM radiation that it emits.

Using Wien's displacement formula for peak wavelength:

$$\lambda_{peak} = \frac{0.002898 m \cdot K}{9600 K}$$
$$\lambda_{peak} = 3.02 \times 10^{-7} m$$

Hence the peak wavelength is around 302 nm.

Q: A certain sound source is emitting sound at 1000 Hz and travelling at 100 m/s towards you. Hearing the high-pitched sound, you frantically try to flee from it at 5 m/s. What is the frequency of sound that you end up hearing? Take the speed of sound as 340 m/s.

Substituting into the Doppler effect formula for sound, and taking the direction from observer to source as positive:

$$f' = \frac{340-5}{340-100}(1000\text{Hz}) = \frac{335}{240}(1000\text{Hz})$$
$$\underline{f' = 1400\text{Hz}}$$

Q: The ratio of the change in wavelength to the original wavelength is called redshift. Astronomers have measured the redshift of light emitted by a deep space object to be exactly 1.00. Determine the velocity that the object is receding away from us.

$\frac{\Delta\lambda}{\lambda_0} = 1.00$, as given. Using the relativistic Doppler formula:

$$\sqrt{\frac{c+v}{c-v}} = 2$$
$$\frac{c+v}{c-v} = 4$$

$$1 + \frac{2v}{c-v} = 4$$
$$\frac{v}{c-v} = 1.5$$
$$\frac{c-v}{v} = \frac{c}{v} - 1 = \frac{2}{3}$$

Solving, we get a pretty large velocity:

$$v = \frac{3}{5}(3.00 \times 10^8 m/s) = 1.8 \times 10^8 m/s$$

Q: This one is a pretty tricky question for those new to the magnitude system. I have two stars of magnitude 1.0 and 3.0 respectively. What is the magnitude of their combined brightness if I put them side by side?

Suppose the star of magnitude 3.0 is of brightness 1, in some arbitrary unit. Because the magnitude is a (reverse) logarithmic scale, the star of magnitude 1.0 is of brightness 1 x $(2.512)^{(3.0-1.0)}$ = 6.309. The combined brightness is then 6.309 + 1 = 7.309. This is 7.309 times brighter than the star of magnitude of 3.0. Let the magnitude of the combined brightness be M. Substituting in:

$$2.5lg(7.309) = 3.0 - M$$

$$M = 0.840$$

As you can see, the combined magnitude is less than both of the stars, an unusual aspect of the magnitude system.

Q: Our sun has an apparent magnitude of - 26.74. What is its absolute magnitude?

The sun's distance in parsecs from Earth is very small: 4.848 x 10-6 pc

Using the distance modulus formula:

$$-26.74 - M = 5lg(4.848 \times 10^{-6}) - 5$$

And solving: <u>M = 4.83</u>