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Course: Numerical Methods (CS3010)

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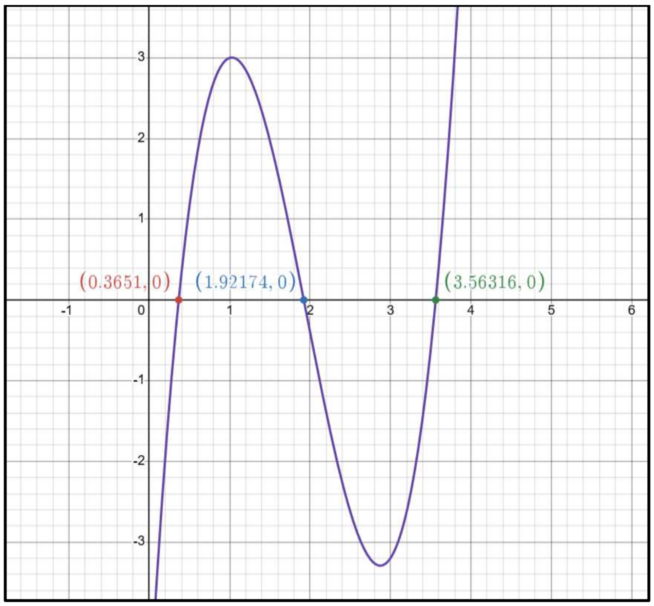
Assignment: Programming Project 3 Report

Using Bisection, Newton-Raphson, Secant, False-Position and Modified Secant methods for locating roots

(a) f(x) = 2x^3 – 11.7x^2 + 17.7x – 5

<u>Graph of the function:</u>



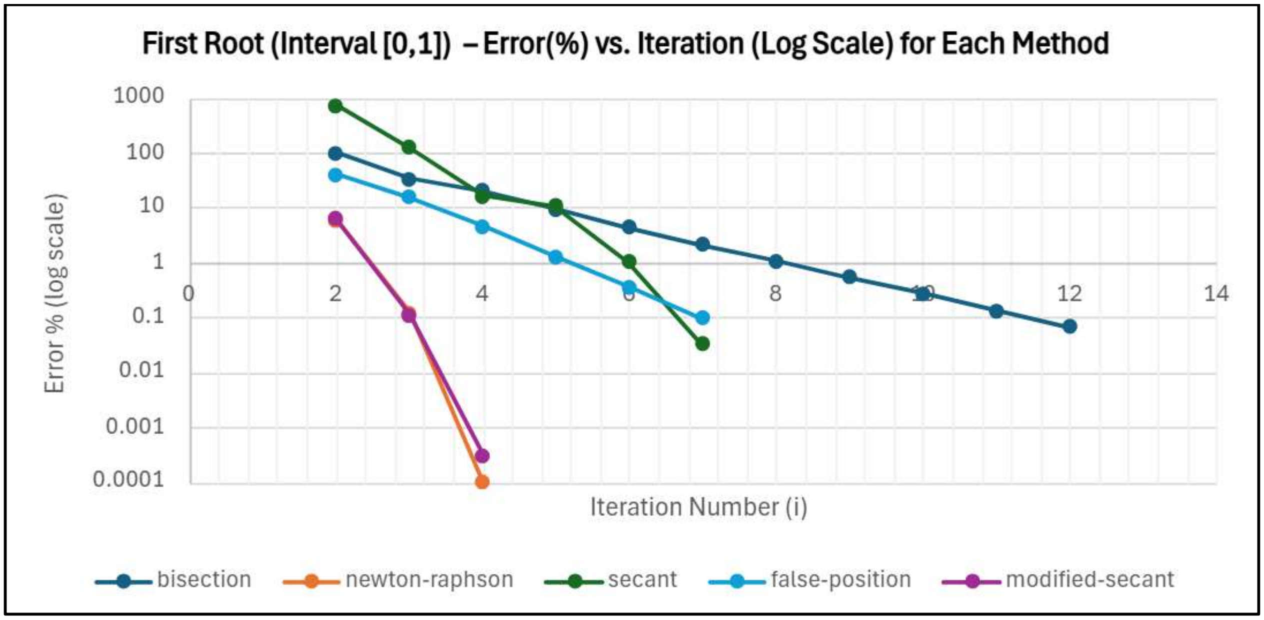
Roots: 0.3651, 1.9217, 3.5631

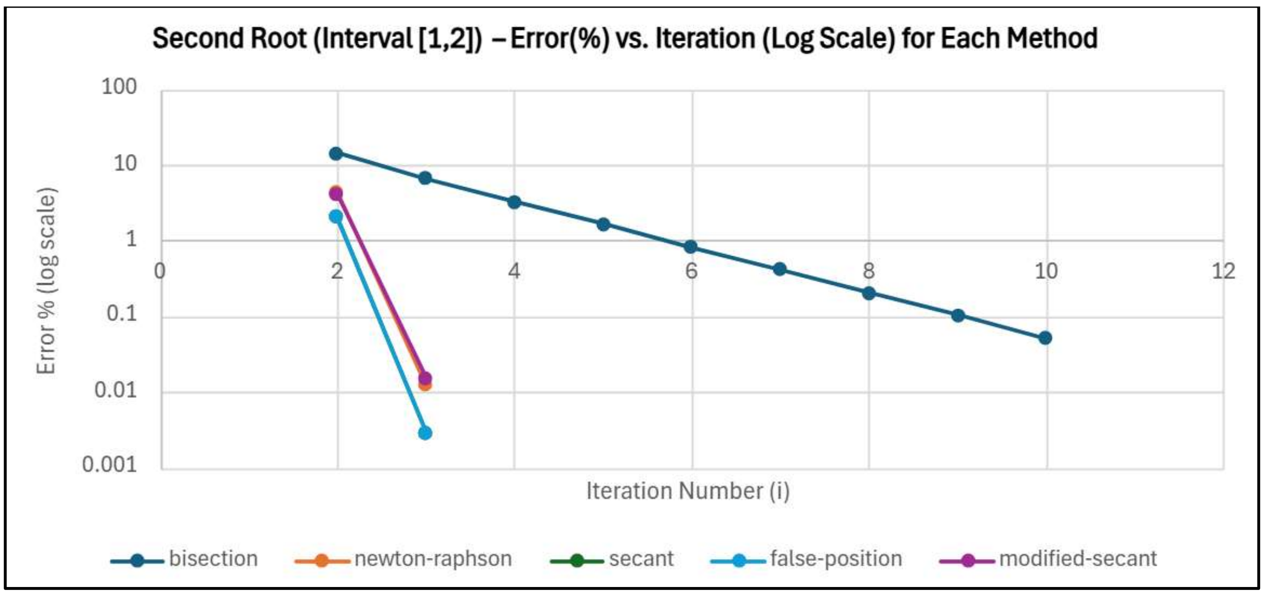
To find the three roots of this polynomial, I use three different intervals:

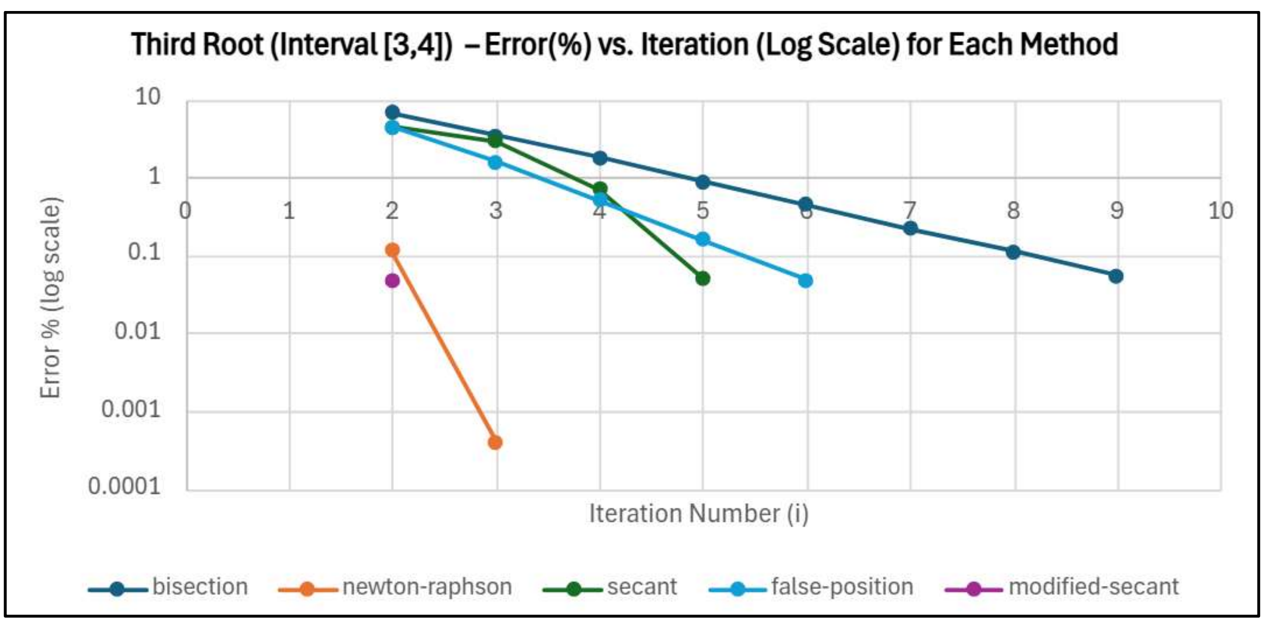
- First Root: the interval [0, 1] with an initial guess of 0.5 for Newton-Raphson and Modified Secant
- Second Root: the interval [1, 2] with an initial guess of 1.5 for Newton-Raphson and Modified Secant
- Third Root: the interval [3, 4] with an initial guess of 3.5 for Newton-Raphson and Modified Secant

<u>Observation based on the graphs:</u>

- The Bisection Method converges to the correct root with more iterations than the other methods.
- The Newton-Raphson Method converges rapidly when the derivative is near the root.
- The Secant and False-Position Methods have nearly identical approximations. I believe it is because they have the same endpoints, and the formulas are similar in the first iteration.
- The Modified Secant Method has convergence behavior like the Newton-Raphson method but with slightly different iteration counts.

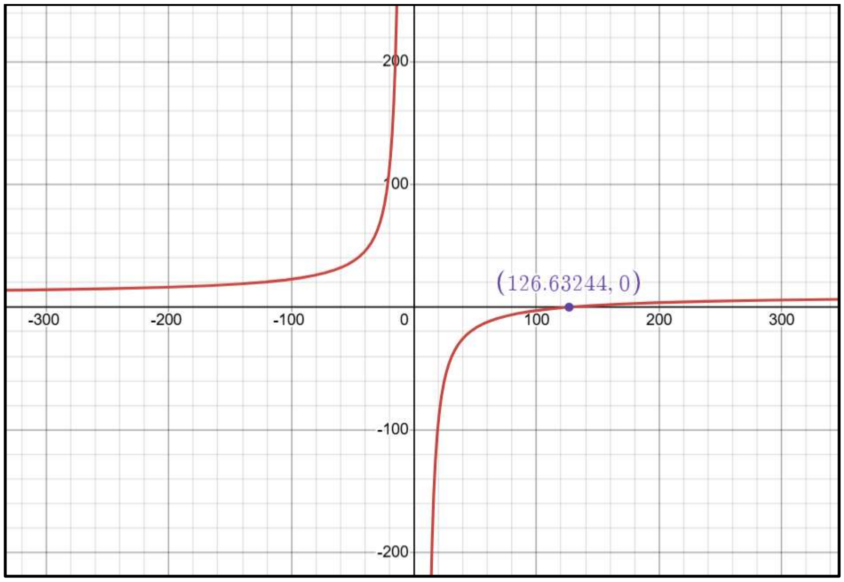

First Root (Interval [0,1]) – Error(%) vs. Iteration (Log Scale) for Each Method
Error % (log scale)
1000
100
10
1
0.1
0.01
0.001
0.0001
0
2
4
6
8
10
12
14
Iteration Number (i)
bisection
newton-raphson
secant
false-position
modified-secant


Second Root (Interval [1,2]) – Error(%) vs. Iteration (Log Scale) for Each Method
Error % (log scale)
100
10
1
0.1
0.01
0.001
0
2
4
6
8
10
12
Iteration Number (i)
bisection
newton-raphson
secant
false-position
modified-secant


Third Root (Interval [3,4]) – Error(%) vs. Iteration (Log Scale) for Each Method
Error % (log scale)
10
1
0.1
0.01
0.001
0.0001
0
1
2
3
4
5
6
7
8
9
10
Iteration Number (i)
bisection
newton-raphson
secant
false-position
modified-secant

(b) f(x) = x + 10 – x cosh(50/x)

Graph of the function:

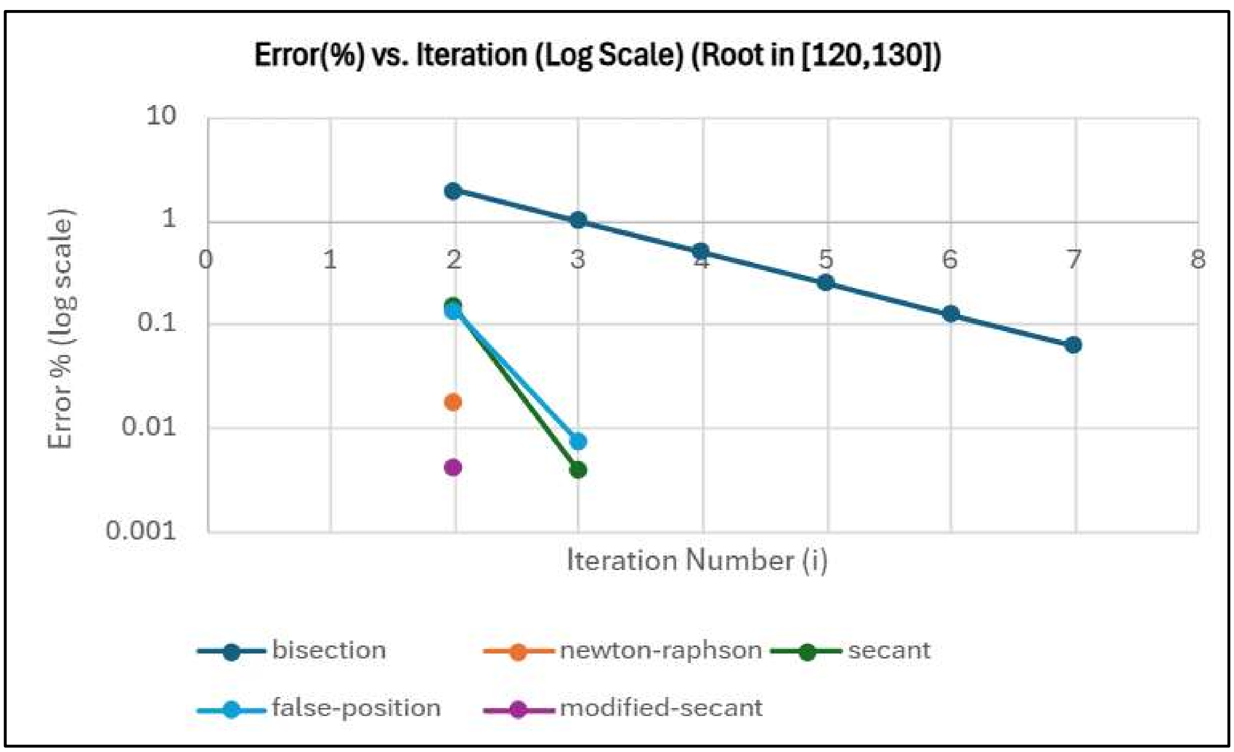


Root: 126.6324

The root lies in the interval [120, 130]. Therefore, I use an initial guess of 125 which is the middle value.

Observation based on the graph:

- The Bisection Method converges to the root with more iterations than the other methods.
- The Newton-Raphson and Modified Secant methods converge rapidly and reach a sufficiently small error in just 2 iterations.
- The Secant and False-Position methods converge in about 3 iterations. Just like the above function, they have nearly identical approximations since they begin with similar initial conditions.



Interesting Behaviors

- The Bisection Method has a slower convergence rate compared to other methods.
- The Secant and False-Position methods yield nearly identical results. This is expected since both methods have algebraic similarity and the same endpoints.
- When a good initial guess is used, the Newton-Raphson and Modified Secant methods perform faster.
- All five methods have error curves that show a steep decline in error on a logarithmic scale. I believe this shows proper convergence to the root.

Data Types Used

The program uses the double data type which is the standard choice for floating point arithmetic in numerical methods. I believe it is suitable to compute small changes in successive approximations and relative errors. Although round-off errors can accumulate, the convergence tolerance (for example, 0.1% in this case) is much larger than the round-off errors.

The following pages show the output of the program (printed from the IDE):

```
C:\Users\646ca\CLionProjects\KellyLwinProgrammingProject3\cmake-build-debug\KellyLw
(a): f(x) = 2x^3 - 11.7x^2 + 17.7x - 5

Root in the interval [0,1]

-----------------------------------------------------------
BISECTION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
-----------------------------------------------------------
 1   0.0000   1.0000   0.5000    -5.0000     1.1750       N/A
 2   0.0000   0.5000   0.2500    -5.0000    -1.2750  100.0000
 3   0.2500   0.5000   0.3750    -1.2750     0.0977   33.3333
 4   0.2500   0.3750   0.3125    -1.2750    -0.5503   20.0000
 5   0.3125   0.3750   0.3438    -0.5503    -0.2169    9.0909
 6   0.3438   0.3750   0.3594    -0.2169    -0.0573    4.3478
 7   0.3594   0.3750   0.3672    -0.0573     0.0208    2.1277
 8   0.3594   0.3672   0.3633    -0.0573    -0.0181    1.0753
 9   0.3633   0.3672   0.3652    -0.0181     0.0014    0.5348
10   0.3633   0.3652   0.3643    -0.0181    -0.0084    0.2681
11   0.3643   0.3652   0.3647    -0.0084    -0.0035    0.1339
12   0.3647   0.3652   0.3650    -0.0035    -0.0011    0.0669
The root 0.3650 has been found in 12 iteration(s)
-----------------------------------------------------------
NEWTON-RAPHSON METHOD
 i       x          f(x)        f'(x)     error(%)
-----------------------------------------------------------
 1   0.50000     1.17500     7.50000       N/A
 2   0.34333    -0.22123    10.37327    5.8484
 3   0.36466    -0.00437     9.96482    0.1200
 4   0.36510    -0.00000     9.95648    0.0001
The root 0.3651 has been found in 4 iteration(s)
-----------------------------------------------------------
SECANT METHOD
 i     x(i-1)      x(i)      x(i+1)     f(x(i+1))    error(%)
-----------------------------------------------------------
 1     0.00000    1.00000    0.62500     1.98047       N/A
 2     1.00000    0.62500   -0.10345    -6.95846  704.1667
 3     0.62500   -0.10345    0.46361     0.89044  122.3137
 4    -0.10345    0.46361    0.39928     0.32927   16.1120
 5     0.46361    0.39928    0.36153    -0.03565   10.4407
 6     0.39928    0.36153    0.36522     0.00119    1.0096
 7     0.36153    0.36522    0.36510     0.00000    0.0325
The root 0.3651 has been found in 7 iteration(s)
-----------------------------------------------------------
FALSE-POSITION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
-----------------------------------------------------------
 1   0.0000   1.0000   0.6250    -5.0000     1.9805       N/A
 2   0.0000   0.6250   0.4477    -5.0000     0.7585   39.6094
 3   0.0000   0.4477   0.3887    -5.0000     0.2298   15.1696
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 4   0.0000   0.3887   0.3716    -5.0000     0.0646    4.5966
 5   0.0000   0.3716   0.3669    -5.0000     0.0178    1.2924
 6   0.0000   0.3669   0.3656    -5.0000     0.0049    0.3557
 7   0.0000   0.3656   0.3652    -5.0000     0.0013    0.0973
The root 0.3652 has been found in 7 iteration(s)
-----------------------------------------------------------
MODIFIED SECANT METHOD
 i       x        f(x+delta*x)      f(x)      error(%)
-----------------------------------------------------------
 1   0.50000     1.21228     1.17500       N/A
 2   0.34242    -0.19524    -0.23071    6.1074
 3   0.36469     0.03218    -0.00403    0.1111
 4   0.36510     0.03624     0.00001    0.0003
The root 0.3651 has been found in 4 iteration(s)

Root in the interval [1,2]

-----------------------------------------------------------
BISECTION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
-----------------------------------------------------------
 1   1.0000   2.0000   1.5000     3.0000     1.9750       N/A
 2   1.5000   2.0000   1.7500     1.9750     0.8625   14.2857
 3   1.7500   2.0000   1.8750     0.8625     0.2383    6.6667
 4   1.8750   2.0000   1.9375     0.2383    -0.0806    3.2258
 5   1.8750   1.9375   1.9062     0.2383     0.0791    1.6393
 6   1.9062   1.9375   1.9219     0.0791    -0.0007    0.8130
 7   1.9062   1.9219   1.9141     0.0791     0.0392    0.4082
 8   1.9141   1.9219   1.9180     0.0392     0.0193    0.2037
 9   1.9180   1.9219   1.9199     0.0193     0.0093    0.1017
10   1.9199   1.9219   1.9209     0.0093     0.0043    0.0508
The root 1.9209 has been found in 10 iteration(s)
-----------------------------------------------------------
NEWTON-RAPHSON METHOD
 i       x          f(x)        f'(x)     error(%)
-----------------------------------------------------------
 1   1.50000     1.97500    -3.90000       N/A
 2   2.00641    -0.43268    -5.09591    4.4188
 3   1.92150     0.00122    -5.11013    0.0124
The root 1.9217 has been found in 3 iteration(s)
-----------------------------------------------------------
SECANT METHOD
 i     x(i-1)      x(i)      x(i+1)     f(x(i+1))    error(%)
-----------------------------------------------------------
 1     1.00000    2.00000    1.88235     0.20090       N/A
 2     2.00000    1.88235    1.92169     0.00028    2.0468
 3     1.88235    1.92169    1.92174    -0.00000    0.0029
The root 1.9217 has been found in 3 iteration(s)
-----------------------------------------------------------
FALSE-POSITION METHOD
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```
 i     a         b         c       f(a)       f(c)    error(%)
-----------------------------------------------------------
 1   1.0000   2.0000   1.8824     3.0000     0.2009       N/A
 2   1.8824   2.0000   1.9217     0.2009     0.0003    2.0468
 3   1.9217   2.0000   1.9217     0.0003     0.0000    0.0029
The root 1.9217 has been found in 3 iteration(s)
-----------------------------------------------------------
MODIFIED SECANT METHOD
 i       x        f(x+delta*x)      f(x)      error(%)
-----------------------------------------------------------
 1   1.50000     1.91590     1.97500       N/A
 2   2.00126    -0.50835    -0.40644    4.1539
 3   1.92145    -0.09674     0.00150    0.0152
The root 1.9217 has been found in 3 iteration(s)

Root in the interval [3,4]

-----------------------------------------------------------
BISECTION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
-----------------------------------------------------------
 1   3.0000   4.0000   3.5000    -3.2000    -0.6250       N/A
 2   3.5000   4.0000   3.7500    -0.6250     2.3125    6.6667
 3   3.5000   3.7500   3.6250    -0.6250     0.6867    3.4483
 4   3.5000   3.6250   3.5625    -0.6250    -0.0069    1.7544
 5   3.5625   3.6250   3.5938    -0.0069     0.3303    0.8696
 6   3.5625   3.5938   3.5781    -0.0069     0.1593    0.4367
 7   3.5625   3.5781   3.5703    -0.0069     0.0756    0.2188
 8   3.5625   3.5703   3.5664    -0.0069     0.0342    0.1095
 9   3.5625   3.5664   3.5645    -0.0069     0.0136    0.0548
The root 3.5645 has been found in 9 iteration(s)
-----------------------------------------------------------
NEWTON-RAPHSON METHOD
 i       x          f(x)        f'(x)     error(%)
-----------------------------------------------------------
 1   3.50000    -0.62500     9.30000       N/A
 2   3.56720     0.04261    10.57710    0.1131
 3   3.56318     0.00016    10.49902    0.0004
The root 3.5632 has been found in 3 iteration(s)
-----------------------------------------------------------
SECANT METHOD
 i     x(i-1)      x(i)      x(i+1)     f(x(i+1))    error(%)
-----------------------------------------------------------
 1     3.00000    4.00000    3.32653    -1.96885       N/A
 2     4.00000    3.32653    3.48127    -0.79592    4.4450
 3     3.32653    3.48127    3.58628     0.24787    2.9279
 4     3.48127    3.58628    3.56134    -0.01908    0.7002
 5     3.58628    3.56134    3.56312    -0.00040    0.0500
The root 3.5631 has been found in 5 iteration(s)
-----------------------------------------------------------
```

```
FALSE-POSITION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
-------------------------------------------------------------
 1   3.0000   4.0000   3.3265    -3.2000    -1.9689       N/A
 2   3.3265   4.0000   3.4813    -1.9689    -0.7959    4.4450
 3   3.4813   4.0000   3.5371    -0.7959    -0.2671    1.5782
 4   3.5371   4.0000   3.5551    -0.2671    -0.0840    0.5065
 5   3.5551   4.0000   3.5607    -0.0840    -0.0259    0.1570
 6   3.5607   4.0000   3.5624    -0.0259    -0.0079    0.0481
The root 3.5624 has been found in 6 iteration(s)
-------------------------------------------------------------
MODIFIED SECANT METHOD
 i       x        f(x+delta*x)      f(x)      error(%)
-------------------------------------------------------------
 1   3.50000    -0.28802    -0.62500       N/A
 2   3.56492     0.40633     0.01845    0.0476
The root 3.5632 has been found in 2 iteration(s)

(b): f(x) = x + 10 - x*cosh(50/x)

Root in the interval [120,130]

-------------------------------------------------------------
BISECTION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
-------------------------------------------------------------
 1 120.0000 130.0000 125.0000    -0.5682    -0.1340       N/A
 2 125.0000 130.0000 127.5000    -0.1340     0.0698    1.9608
 3 125.0000 127.5000 126.2500    -0.1340    -0.0311    0.9901
 4 126.2500 127.5000 126.8750    -0.0311     0.0196    0.4926
 5 126.2500 126.8750 126.5625    -0.0311    -0.0057    0.2469
 6 126.5625 126.8750 126.7188    -0.0057     0.0070    0.1233
 7 126.5625 126.7188 126.6406    -0.0057     0.0007    0.0617
The root 126.6406 has been found in 7 iteration(s)
-------------------------------------------------------------
NEWTON-RAPHSON METHOD
 i       x          f(x)        f'(x)     error(%)
-------------------------------------------------------------
 1 125.00000    -0.13405     0.08323       N/A
 2 126.61058    -0.00177     0.08104    0.0173
The root 126.6324 has been found in 2 iteration(s)
-------------------------------------------------------------
SECANT METHOD
 i     x(i-1)      x(i)      x(i+1)     f(x(i+1))    error(%)
-------------------------------------------------------------
 1   120.00000  130.00000  126.81560     0.01482       N/A
 2   130.00000  126.81560  126.62738    -0.00041    0.1486
 3   126.81560  126.62738  126.63244     0.00000    0.0040
The root 126.6324 has been found in 3 iteration(s)
-------------------------------------------------------------
```

```
FALSE-POSITION METHOD
 i     a         b         c       f(a)       f(c)    error(%)
----------------------------------------------------------
 1 120.0000 130.0000 126.8156    -0.5682     0.0148       N/A
 2 120.0000 126.8156 126.6424    -0.5682     0.0008    0.1368
 3 120.0000 126.6424 126.6330    -0.5682     0.0000    0.0074
The root 126.6330 has been found in 3 iteration(s)
----------------------------------------------------------
MODIFIED SECANT METHOD
 i       x       f(x+delta*x)      f(x)      error(%)
----------------------------------------------------------
 1 125.00000    -0.03108    -0.13405       N/A
 2 126.62732     0.10113    -0.00041    0.0041
The root 126.6325 has been found in 2 iteration(s)

Process finished with exit code 0
```