Pricing Contingent Coupon Auto-Callable Notes of Common stock of Applied Materials, Inc. using Binomial Tree

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1 Executive Summary

This report presents the valuation of a Contingent Coupon Auto-Callable Note linked to Applied Materials, Inc., using a Binomial Tree model. The objective is to estimate its fair value by modeling stock price evolution and incorporating coupon payments and auto-call conditions.

Our approach constructs a recombining binomial tree, using the Cox-Ross-Rubinstein (CRR) model for stock price movements and backward induction for valuation. Key inputs—risk-free rates (derived from SOFR discount factors), dividend yield, and implied volatility—were sourced from Bloomberg. Forward rates were used to reflect changing interest rate regimes across observation dates.

The **final estimated price of the note** is **$9.7446**, closely aligning with the **term sheet**'s **$9.76**. Sensitivity analysis highlights that the price is particularly influenced by implied volatility, coupon barrier levels, and the number of time steps in the binomial tree. Oscillatory behavior in convergence suggests that the alignment of observation dates with tree nodes impacts accuracy.

The model effectively captures the key features of the structured note. Future improvements could involve refining volatility assumptions across observation periods and optimizing step selection to enhance numerical stability.

2 Introduction

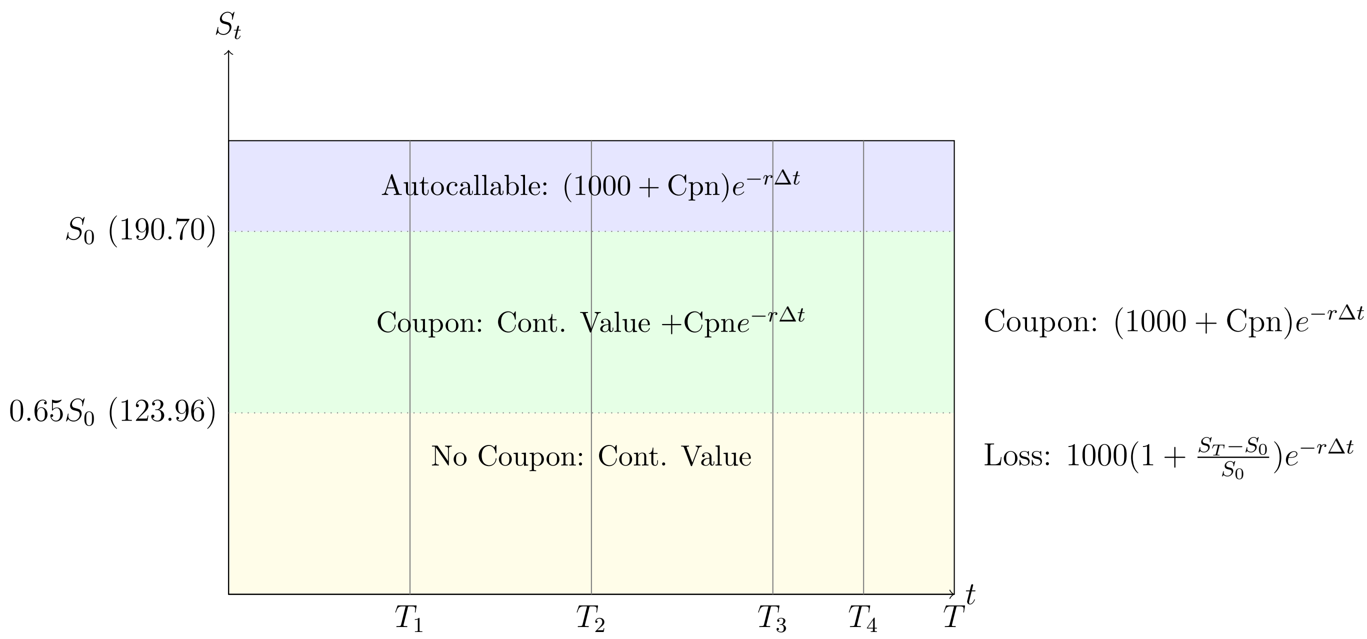
2.1 Product Specifications

Below is a summary of the main details of the instrument from the term sheet.

1. Trade Date: 01/24/2025
2. Original Issue Date (Settlement Date): 01/29/2025
3. Observation Dates: Quarterly
   - T1: 04/23/2025

- T2: 07/23/2025
- T3: 10/23/2025
- T4: 01/23/2026 (Final Valuation Date)

4. Coupon Payment Dates:
   - 04/28/2025
   - 07/28/2025
   - 10/28/2025
   - 01/28/2026
5. Maturity date : 01/28/2026
6. Underlying Financial instrument: Common stock of Applied Materials, Inc.
7. Payment terms:
   (a) If the Notes are not automatically called and the Final Value is equal to or greater than the Downside Threshold: a cash payment at maturity per \$10 principal amount Note equal to \$10 plus the Contingent Coupon otherwise due on the Maturity Date.
   (b) If the Notes are not automatically called and the Final Value is less than the Downside Threshold: a cash payment at maturity that is less than \$10 per \$10 principal amount Note, equal to: \$10 × (1 + Underlying Return)
8. Contingent coupon rate: 11.35%
9. Initial value: \$190.70
10. Downside threshold: \$123.96 or 65.00% of initial value
11. Coupon barrier: \$123.96 or 65.00% of initial value



2.2 Model Brief

The valuation framework employs a Binomial Tree model, consisting of two primary components:

2.2.1 Stock Price Tree Generation

The first component generates a stock price tree based on:

- Initial stock price (S_0)
- Up and down factors (u and d)
- Time to maturity (T)
- Number of steps (N)

For the up/down factors, we implement the Cox-Ross-Rubinstein (CRR) model:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{1}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{2}$$

The CRR model was chosen specifically because it makes the up/down factors independent of the risk-free rate. This is crucial as our note has four distinct observation dates, effectively dividing our analysis period into four regimes. While these regimes theoretically should have different risk-free rates and volatilities, our implementation uses:

- A single volatility measure
- Different forward rates
- Volatility-dependent up/down factors

This approach ensures tree recombination and prevents exponential growth in complexity.

2.2.2 Valuation Tree Construction

The second component builds upon the stock price tree and incorporates:

- Auto-call/coupon payment features at observation dates
- Maturity date conditions against downside threshold

The valuation process employs backward induction, starting from the final valuation date and working backward to time step 0. The resulting value at the root node represents the fair value of the note, incorporating all term sheet specifications and constraints.

2.3 Data Description

This section describes the data sources, conventions, and processing methods used in our valuation model. The key market variables required for the analysis include the risk-free rate, dividend yield, and implied volatility.

2.3.1 Conventions and Notation

To ensure clarity, we define the key symbols used throughout the document:

- $r(0, T)$: Risk-free rate applicable from time 0 to maturity T
- $P(0, T)$: Discount factor corresponding to maturity T
- $F(T_1, T_2)$: Forward rate from T_1 to T_2
- d: Dividend yield
- σ: Implied volatility
- S_0: Initial stock price
- T: Time to maturity in years
- N: Number of binomial steps

1. **Risk-Free Rate**

   - **Interpolation of Discount Factors** We do not have risk-free rates for the exact observation dates. Therefore, we use available discount factors from Bloomberg and interpolate them for our required dates. The discount factor is obtained using linear interpolation between the two closest available discount factors.
   - **Conversion to Spot Rates** Once interpolated discount factors are available, the risk-free rate for each observation date is calculated using:

     $$r(0, T) = -\frac{\ln P(0, T)}{T} \tag{3}$$

     where $P(0, T)$ is the interpolated discount factor for the given maturity T.
   - **Forward Rate Computation** Using the spot rates, forward rates are calculated for each interval using the following formula:

     $$F(T_1, T_2) = \frac{T_2 r(0, T_2) - T_1 r(0, T_1)}{T_2 - T_1} \tag{4}$$

     where $r(0, T_1)$ and $r(0, T_2)$ are the interpolated spot rates for maturities T_1 and T_2.
   - **Expansion of the Risk-Free Rate Array** The computed forward rates correspond to only four observation dates, whereas our binomial tree model requires rates for all 251 time steps. Thus, we expand the risk-free rate array by assigning the computed forward rates to their corresponding time intervals, ensuring proper rate application across all steps in the model.

2. **Dividend Yield (d):** The dividend yield estimates were obtained directly from Bloomberg for AMAT (Bloomberg token for Common stock of Applied Materials, Inc.).

3. **Implied Volatility (σ):** Bloomberg provides implied volatility surfaces for different strikes and maturities, denoted as:

   $$\sigma = f(K, T) \tag{5}$$

   where volatility depends on moneyness (S/K) and time to expiry (T).

   Between different observation dates we could have different varying implied volatilities, but we opted to choose a single volatility so the binomial tree recombines and does not expand disproportionately.

2.4 Data collection and estimation

While fetching the data, we ensured to use relevant dates for fetching the risk-free rate r and dividend yields d with the corresponding the trade and settlement dates. We ensured to set the global date as the trade date while extracting the OIS rates.

1. **No. of steps in Binomial tree (N)**: We selected N = 251 which is the number of trading days between trade date and final valuation date (including both the dates) as we we wanted to model daily cashflow rates to keep the model simple and straightforward at the same time also account for all important dates (observation, coupn payment, and dividend payment dates)

2. **Time step (Δt)**: We computed this value by dividing the time-to-maturity (T) in years by the value of total steps in binomial tree:

$$\Delta t = \frac{T}{N} \tag{6}$$

3. **Dividend Yield (d)**: Continuous dividend yield estimates for AMAT were obtained from Bloomberg. We obtained the value as d = 2.071%.

4. **Volatility Estimation** (σ): We obtained voaltility values for the different observation dates from bloomberg using the OVME command.

Observation dates	**No. of Days**	**Volatility**	
		Moneyness = 100	**Moneyness = 65**
23/04/25	89	37.583	46.701
23/07/25	180	37.464	42.874
23/10/25	272	37.899	41.517
23/01/26	364	37.797	41.494

Based on our assumptions and the data we got from Bloomberg, we have used the volatility $\sigma = 37.797\%$ in our model.

5. **SOFR Rate Estimation ($\mathbf{r(0,}T_1\mathbf{),r(0,}T_2\mathbf{),r(0,}T_3\mathbf{,r(0,}T_4\mathbf{))}$)**: For the different SOFR estimates we used the SWDF command for swap discount factors. We obtained the discount factors for the closest dates available corresponding to the Observation dates. Based on the global date of 01/24/2025 and settle date of 01/24/2025, we obtained the following values for the discount factors:

   Here, T1 corresponds to the first observation date, T2 is the second, T3 is the third and T4 is the final observation date.

   Based on this, we interpolated the discount factors for the observation dates, based on the number of days between the two available dates. After this, we computed the spot rates using:

$$\text{spot\_rates} = -\frac{\ln(\text{discount\_factors})}{\text{obs\_times}} \tag{7}$$

   Since we need Forward rates in each regime, we calculated the forward rates from the above spot rates using:

Trade date	1/24/2025
Date	**Discount factor**
3/28/2025	0.992472
4/28/2025	0.988839
6/30/2025	0.981626
7/28/2025	0.978515
10/28/2025	0.968533
1/28/2026	0.958863

$$F_0(t_1, t_2) = \frac{t_2 r_2 - t_1 r_1}{t_2 - t_1} \tag{8}$$

We arrived at the following forward rates for the different regimes, corresponding to the periods between two observation dates:

$$\begin{aligned} F_0(0, T_1) &= 0.04360046 \\ F_0(T_1, T_2) &= 0.04219659 \\ F_0(T_2, T_3) &= 0.04058528 \\ F_0(T_3, T_4) &= 0.03984763 \end{aligned}$$

The last step was to expand this to an array of size N, so that at each time step, we have a specific forward rate which would be used for the calculation of risk-neutral probability and discount factor.

We used the following commands on Bloomberg to get relevant data:

- OIS: Displays Overnight Index Swap rates and related market data.
- SWDF: Displays swap discount factors for different tenors and currencies. We had to use SWDF when we couldn't directly access the OIS.
- OVME: Provides an overview of market expectations for volatility and skew.

3 Model description

3.1 Stock tree

We perform the stock tree construction using the binomial stock price model. The parameters utilized in this construction are defined as:

- S_0: Initial stock price
- σ: Annual volatility
- Δt: Time step $(= \frac{T}{N})$
- T: Total time to maturity of the tree

- N: Total number of steps

Besides these, we also require the up (u) and down (d) factors. These are calculated using the Cox-Ross-Rubinstein method:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{9}$$

$$d = \frac{1}{u} \tag{10}$$

The stock price tree initiates with S_0 and then employs a double loop on indices i and j, where:

- i represents the time step and thus ranges from 1 to N
- j represents the number of up moves at each step

The stock price at any node (i, j) is given by:

$$S_{i,j} = S_0 \cdot u^j \cdot d^{i-j} \tag{11}$$

This multiplicative process ensures stock prices exist for each node, enabling us to run conditional statements later while building the valuation tree. The resulting lattice structure provides a mapping of all possible stock price paths over the desired time frame.

3.2 Valuation Tree

The valuation tree operates on the same structure as the stock price tree but computes the note's value at each node through backward induction. The continuation value at each node is calculated based on the risk-free discounted value of the expected value of the two future nodes (at the next time step) that evolve from the current node—one corresponding to the up node and the other to the down node—using the risk-neutral probability:

The continuation value at each node is calculated using risk-neutral probabilities:

$$\text{cont. value} = e^{-r\Delta t}[qV_{i+1,j+1} + (1-q)V_{i+1,j}] \tag{12}$$

where:

- $V_{i,j}$ is the option value at node (i, j)
- q is the risk-neutral probability
- r is the risk-free rate
- Δt is the time step

At the final iteration of the time steps, we reach the value at node $(0, 0)$, which represents the price of the note. While this continuation value calculation follows a standard approach, the final option value on a node may change based on the trigger conditions for the note. Broadly, the valuation is performed at each node in two instances: first, at the final valuation date, and second, through backward induction from one time step before the final valuation date to the first node.

3.2.1 Value at Final Valuation Date

The discount factor in this case is the discounting occurring between the maturity date and the Final Valuation date. Thus, at the end of this step, we have estimated the option tree values on the final valuation date step for all the nodes.

Stock condition	Pricing condition	Implementation
Stock price $\geq$ Downside threshold	Principal amount per Note at maturity, and the Contingent Coupon	Option tree value is the discounted value of (issue price and coupon)
Stock price $<$ Downside threshold	Cash payment at maturity of \$10 $\times$ (1 + Underlying Return)	Option tree value is the discounted value of (issue price and return on the stock price); return is a simple %age change from initial level

Table 1: Maturity Payoff Conditions

3.2.2 Backward Induction

The principle behind backward induction follows a straightforward methodology: we estimate the continuation values of the two evolved future nodes and discount them back to the present. In our autocallable note valuation, there are three critical conditions that must be evaluated at each node during the backward induction process on any observation date:

1. **No Coupon Condition**
   If $S_{i,j} < K$ (stock price below coupon barrier):
   - No coupon payment occurs
   - Node value = continuation value only
2. **Coupon Payment Condition**
   If $K \leq S_{i,j} < S_0$ (stock price between coupon barrier and initial value):
   - Coupon payment occurs at next coupon date
   - Node value = continuation value + discounted coupon
3. **Auto-Call Condition**
   If $S_{i,j} \geq S_0$ (stock price at or above initial value):
   - Note is automatically called
   - Node value = discounted value of (principal + contingent coupon)

Additionally, while calculating the continuation values, we require the risk-neutral probability at each time step. The calculation incorporates the risk-free rate, adhering to the no-arbitrage criterion:

$$d < e^{(r-\delta)\Delta t} < u \tag{13}$$

The risk-neutral probability q at each step is given by:

$$q = \frac{e^{(r-\delta)\Delta t} - d}{u - d} \tag{14}$$

where:

- r is the forward rate applicable to that time period

- δ is the continuous dividend yield
- Δt is the time step
- u and d are the up and down factors respectively

Since the barrier conditions have to be checked on each observation date, we utilize the forward rates specific to each zone (between two consecutive observation dates) to estimate the probabilities. This ensures that the discounting and probability calculations accurately reflect the term structure of interest rates across different observation periods.

Now, we begin the backward induction process. Starting from time step 'N-1' (given that we have N time steps), we first calculate the discount factor and the risk neutral probability in that particular zone, using the corresponding risk free forward rate, with 'i' being the current time step. The next step is to check the barrier conditions at each node for every observation date, starting from '0' to i+1 (since there are i+1 nodes at time step i). If the date does not fall in the observation date array, then we simply equate the option tree value with the continuation value. For the dates falling on the observation date, we check for the following conditions:

Stock condition	**Pricing condition**	**Implementation**
Stock price $\geq$ Initial value ('Autocall' condition)	On the Call Settlement Date, a cash payment of the principal amount plus the Contingent Coupon otherwise due for the applicable Observation Date	Option tree value is the discounted value of the issue price and coupon; discounting between the next coupon date and the current observation date
Stock price $\geq$ Coupon barrier price	Contingent Coupon will be paid for that observation date on the relevant Coupon payment date	Discounted value of the coupon added to the continuation value; discounting between the next coupon date and the current observation date
Stock price $<$ Coupon barrier price	No payment of Contingent Coupon	Option tree value is simply the continuation value

Table 2: Observation Date Conditions and Implementation

3.3 Execution of the Stock Tree and Valuation Tree

The implementation requires two main functions, each with specific parameters for constructing the respective trees. Below we detail the required arguments for each function:

Stock Tree Construction

The function `build_stock_price_tree` generates the stock price lattice and returns the stock tree along with movement parameters. It requires:

Parameter	Symbol	Description
Initial stock price	S_0	Market price at valuation date
Time to maturity	T	Total time period in years
Volatility	σ	Annualized volatility of the underlying
Number of steps	N	Total number of discrete time steps

Valuation Tree Construction

The function `build_valuation_tree` implements the backward induction process and returns the option value at the root node. Required parameters:

Parameter	Symbol	Description
Initial stock price	S_0	Market price at valuation date
Stock price tree	–	Output from stock tree construction
Coupon barrier	K	Threshold for coupon payments
Time to maturity	T	Total time period in years
Forward rates	r_t	Vector of rates for each time step
Number of steps	N	Total number of discrete time steps
Observation steps	–	Time steps for observation dates
Up factor	u	Upward movement multiplier
Down factor	d	Downward movement multiplier
Issue price	–	Principal amount of the note
Coupon amount	–	Fixed payment at observation dates
Coupon payment steps	–	Time steps for coupon payments
Dividend yield	δ	Continuous dividend yield

The function returns the fair value of the autocallable note, incorporating all features, including contingent coupons, autocall conditions, and dividend adjustments.

After implementing these two functions, we arrive at a Note valuation of \$9.7446.

4 Discussion

4.1 Results Analysis

Based on the parameter values and the algorithm steps mentioned above, we arrive at a valuation of \$9.7446 per note of an issue price of \$10.0, as compared to the estimates of \$9.76. While our results are quite close, we will study some of the sensitivities and possible sources of errors in our model.

4.2 Sensitivity Analysis

Below we present the sensitivity analysis of the note price with respect to three key parameters: volatility, coupon barrier, and the convergence behavior with increasing time steps.

4.2.1 Volatility Sensitivity

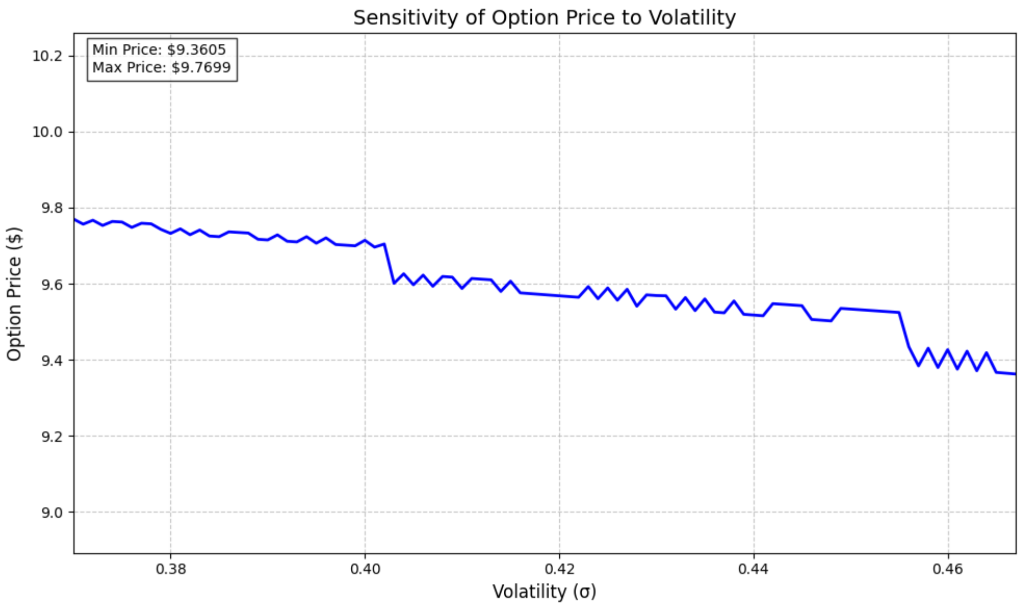


Figure 1: Sensitivity of Option Price to Volatility

Figure 1 demonstrates that the option price exhibits a decreasing trend as volatility increases from 0.37 to 0.467, corresponding to the range of implied volatilities observed at both 65% and 100% moneyness levels. Given our model's approximation of the volatility surface using a single value, two notable trends emerge from this analysis:

Critical Volatility Thresholds The option price experiences significant drops around $\sigma = 0.40$ and $\sigma = 0.45$. These transitions can be attributed to abrupt shifts in the price distribution at these critical volatility levels. The barrier interaction at these thresholds appears to trigger a sudden decrease in the probability of achieving auto-call events and coupon payments, simultaneously increasing the likelihood of missed coupon scenarios.

Volatility Regime Characteristics The analysis reveals distinct pricing behavior across different volatility regimes. In the lower volatility region ($0.37 \leq \sigma \leq 0.40$), the option price demonstrates some stability. This stability can be explained by more concentrated stock price paths, leading to more predictable probabilities of achieving auto-call triggers and coupon payments. In contrast, higher volatility levels exhibit increased price oscillations, reflecting greater path dispersion where even minor volatility changes can cause significant shifts in barrier-hitting probabilities.

4.2.2 Coupon Barrier Sensitivity

The analysis of coupon barrier sensitivity reveals two distinct regimes with markedly different pricing behaviors:

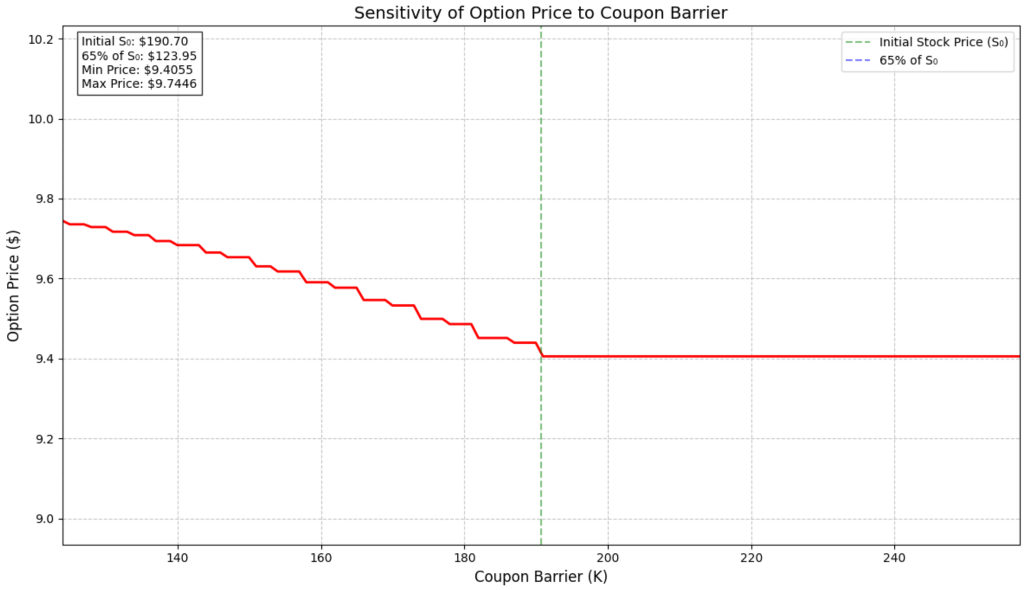


Figure 2: Sensitivity of Option Price to Coupon Barrier

Barrier Below Initial Price ($K < S_0$) The option price exhibits a monotonic decreasing trend as the coupon barrier increases from \$123.95 (65% of S_0) to approximately \$190.70 (S_0). This decline reflects the diminishing probability of achieving coupon payments as the barrier threshold rises, thus reducing the option price.

Barrier Above Initial Price ($K > S_0$) When the coupon barrier exceeds the initial stock price of \$190.70, the option price stabilizes and maintains a constant value, displaying complete insensitivity to further barrier increases. This can be attributed to the dominance of the auto-call feature - when the coupon barrier exceeds the auto-call level (S_0), any price path that would reach the coupon barrier would necessarily trigger an auto-call first. This mechanism effectively creates a valuation ceiling, thus capping the option price.

4.2.3 Sensitivity based on number of steps:

Here are two key trends from the convergence analysis:

Oscillatory Behavior with Increasing Steps The option price exhibits persistent oscillations as the number of steps increases from 251 to 1476, with price values fluctuating between approximately \$9.72 and \$9.78. This sawtoothing pattern can be attributed to the 'odd-even' effect, where certain step sizes align better with observation dates and barrier levels, leading to alternating pricing scenarios.

Non linearity in error: Non-linearity in error emerges from misalignment between tree nodes and critical product features. In our note, the discrete placement of nodes may not perfectly align with coupon barriers (K) and auto-call levels (S0), leading to inconsistent capture of these threshold events. This misalignment becomes particularly pronounced when valuing path-dependent features, where the interaction between observation dates and time steps can significantly impact the final price, and thus cause error.

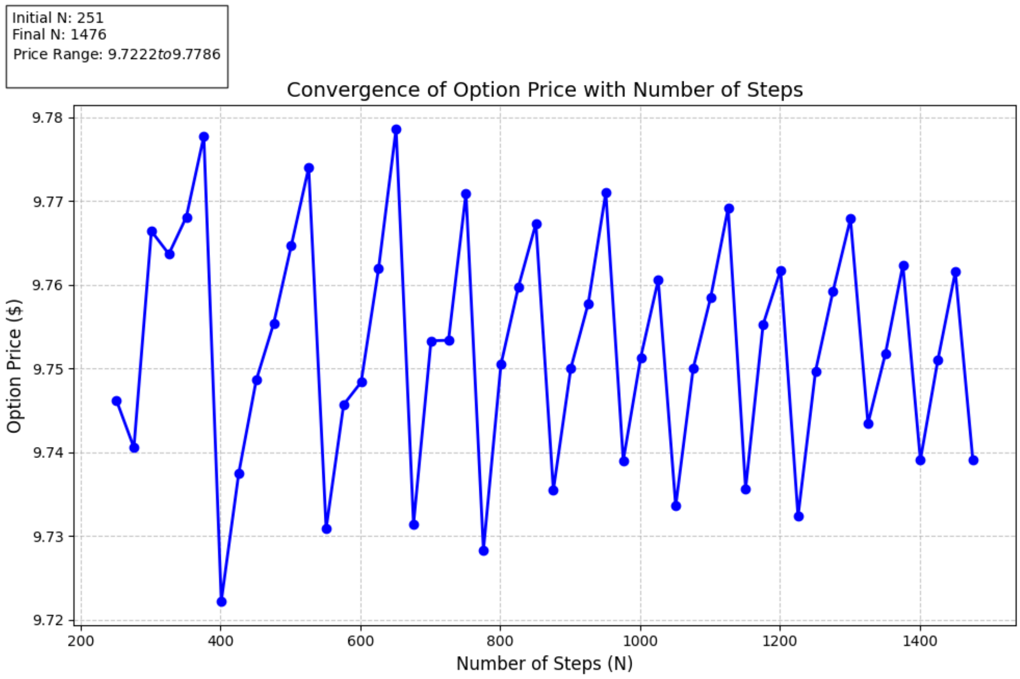


Figure 3: Sensitivity of Option Price to Coupon Barrier

4.3 Possible sources of error

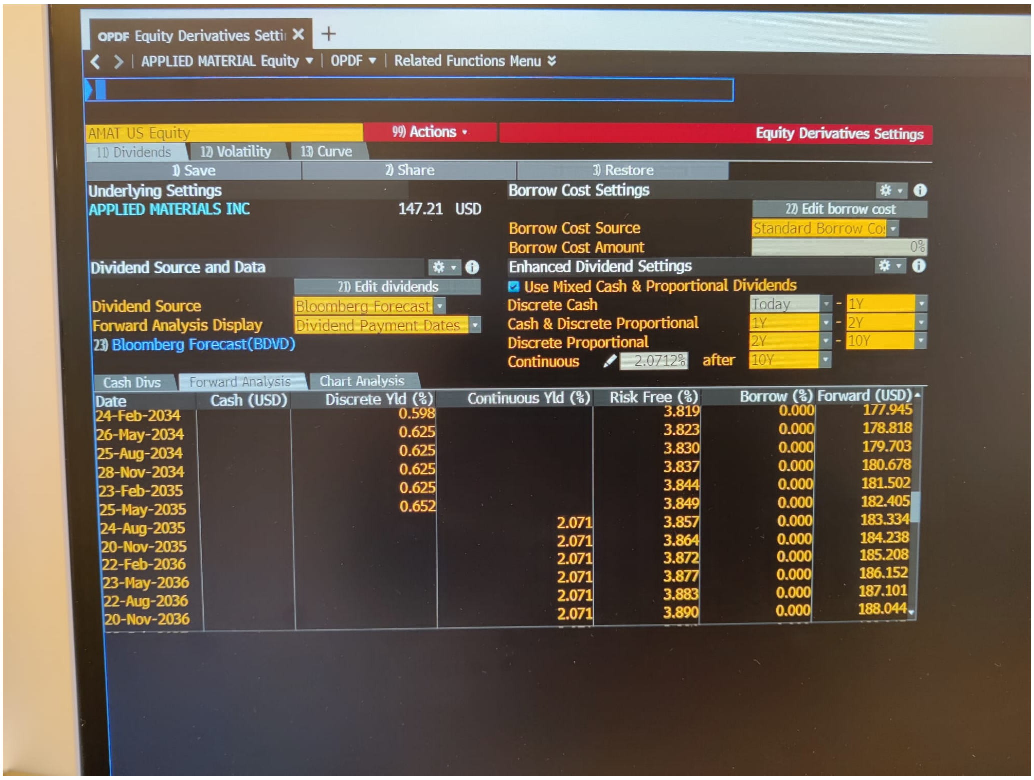
4.3.1 Volatility approximation

In a discrete barrier option, like the one we are dealing with, there are different regimes where the risk free rate and volatility will be different since the barrier criteria depends on certain specific dates. Now, we have used Forward rates to have different rates in different regimes, but we are using a single volatility value (100% moneyness and time to maturity same as the note) for our purposes. This impacts the risk neutral probabilities and thus a lower than expected volatility leads to a lower chance of hitting the barrier thresholds and thus misses out on coupons, thereby lowering the value of the option. On the other hand, a higher volatility will overestimate the price since we will have a higher probability of crossing the thresholds. We are using a single value for ease of computation and arriving at a fair approximation of the value of the note.

4.3.2 Appropriate number of steps

As we have seen, there are non linear errors occurring due to the choice of the steps in our binomial model. Although we are seeing that the values may converge eventually, the convergence is not monotonic. We have hence limited our analysis to match the steps with the number of steps for ease of calculations, rather than trying to achieve a converged value with a very high number of steps, which would be computationally very time consuming (for example, plotting the convergence for steps from 251 to 3001 with increments of 25 steps did not produce any results even after 15mins of runtime).

5 Appendix



References

1. Bloomberg documentation: https://www.bloomberg.com/professional/support/documentation/