Replication Strategy for Buffered PLUS Securities

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1 Executive Summary

This report explores the valuation and replication of Buffered PLUS Securities, a structured note tied to the S&P 500. We replicate its payoff using bonds, European put options, and a digital put, ensuring accuracy with market data.

Our approach builds a portfolio that mirrors the note's structure: zero-coupon bonds adjust cash flows, European puts cover downside risk, and a digital put captures the payoff jump at the buffer level. Key inputs—SOFR-based risk-free rates, dividend yields, and implied volatilities—come from Bloomberg.

Our valuation estimates the note's price at \$957.04, close to the term sheet's \$970.30. Sensitivity analysis highlights the impact of forward rates and implied volatility. Minor discrepancies stem from approximations in digital option pricing and risk-free rate interpolation.

2 Introduction

2.1 Product Specifications

Below is a summary of the main details of the instrument from the term sheet.

1. Pricing date T_0: 01/17/2025
2. Issue date T_1: 01/23/2025
3. Valuation date T_2: 01/29/2027
4. Maturity date T_3: 02/03/2027
5. Underlying index: SP500
6. Payment terms:
    (a) If the final index value is greater than the initial index value:
    \$1,000 + leveraged upside payment
    (b) If the final index value is less than or equal to the initial index value but has decreased from the initial index value by an amount less than or equal to the buffer amount of 10%:
    \$1,000 + (\$1,000 $\times$ absolute index return)

(c) If the final index value is less than the initial index value and has decreased from the initial index value by an amount greater than the buffer amount of 10%: ($1,000 x the index performance factor) + $100

7. Leverage factor: 150%

2.2 Model brief

The instruments we are using are: a) bonds to help construct synthetic positions to ensure a fixed cash flow structure at expiration, b) a digital put to replicate the jump in payoff diagram at 90% of spot index level, and c) European put options, as we are performing valuation from right to left. The slope of the payoff diagram will determine the required number of options required to create the portfolio.

Used Instruments List

Instrument	Strike ($)
Bond	FV: 1,175
Short Put	6,696.29
Long Put	5,996.66
Short Put	5,396.99
Short Digital Put	5,396.99
Bond	FV: 100

Table 1: Portfolio Instruments

2.3 Data Description

We used Bloomberg as the primary source of fetching data. The data points we had to fetch were the risk-free rate (r) for various time periods, dividend yield (d), and implied volatility (σ) for different European options.

2.3.1 Conventions and Interpretations

- **Risk-Free Rate (r):** The risk-free rate is typically derived from SWDF rates. To obtain the continuously compounded risk-free rate, we use discount factors ($P(0,T)$) and computed as follows:

$$r(0,T) = -\frac{1}{T}\ln P(0,T) \tag{1}$$

  For our purposes, multiple risk-free rates are considered:

  - $r(0,T_1)$ - Spot rate at time T_0, maturing at T_1 (Settlement Date)
  - $r(0,T_2)$ - Spot rate at time T_0, maturing at T_2 (Valuation Date)
  - $r(0,T_3)$ - Spot rate at time T_0, maturing at T_3 (Maturity Date)

  From these, we define:

- $r_1 = r(0, T_2)$ - The spot rate for discounting up to valuation.
- $r_2 = F_0(T_1, T_3)$ - The forward rate between settlement and maturity, derived using:

$$F_0(T_1, T_3) = \frac{r(0, T_3)T_3 - r(0, T_1)T_1}{T_3 - T_1} \tag{2}$$

- **Dividend Yield (d):** The dividend yield estimates were obtained directly from Bloomberg for SP500.
- **Implied Volatility (σ):** Bloomberg provides implied volatility surfaces for different strikes and maturities, denoted as:

$$\sigma = f(K, T) \tag{3}$$

where volatility depends on **moneyness (S/K) and time to expiry (T)**.

2.4 Data collection and estimation

While fetching the data, we ensured to use relevant dates for fetching the risk-free rate r and dividend yields d with the corresponding pricing (T_0) and settlement (T_1) dates. We ensured to set the global date as pricing date (T_0) while extracting the OIS rates.

1. Dividend Yield (d): Continuous dividend yield estimates for SP500 were obtained from Bloomberg for T2. We obtained the value as d = 1.1146%.

2. Volatility Estimation (σ): We used the implied volatility data from Bloomberg for the respective European Put options based on the moneyness and time to expiry. The respective moneyness and expiry times were input in the matrix of implied volatility in the OVME screen.

Date	Strike	Moneyness	Type	Long/Short	Time	Implied Volatility (%)
1/17/2025	6696.29	1.11667	Put	Short	T2	14.878
1/17/2025	5996.66	1.00000	Put	Long	T2	17.729
1/17/2025	5396.99	0.90000	Put	Short	T2	19.947
1/17/2025	5396.99	0.90000	Put	Short Digital	T2	19.947

Table 2: Option Portfolio Specifications

The IV for the digital option has been taken to be the same as European Option.

3. SOFR Rate Estimation (r(0,T_1),r(0,T_2),r(0,T_3)): For the different SOFR estimates we used the SWDF command for swap discount factors. Based on the global date of 01/17/2025 and settle date of 01/23/2025, we obtained the following values for the different tenors (screenshot attached in Appendix).

   Since the first entry we got was for 01/29/2025, we used this discount factor for 01/23/2025 (T1) and number of days as 6 (01/23/2025 - 01/17/2025) to arrive at r(0,T1) = 4.3877%.

   For r(0,T2), we interpolated the discount factors for 01/22/2027 and 01/24/2028 to arrive at the discount factor for 01/29/2027 (0.920638). This gave the respective discount rate of r(0,T2) or r1 = 4.0675%.

Date	Discount factor
1/29/2025	0.999279
7/22/2026	0.940243
1/22/2027	0.921339
1/24/2028	0.884606

Table 3: Discount Factors for Different Dates

Rate	Value (%)
$r(0, T_1)$	4.3877
$r(0, T_2)$ or r_1	4.0675
$r(0, T_3)$	4.0669
$F_0(T_1, T_3)$ or r_2	4.0642

Table 4: Risk-free Rates - Input Parameters

For r(0,T3), we interpolated the discount factors for the above dates to arrive at the discount factor for 02/03/2027 (0.920137921). This gave the r(0,T3) as 4.0669%.

Based on r(0,T1) and r(0,T3), we calculated the $F_0(T_1, T_3)$ or r2 as 4.06428%.

We used the following commands on Bloomberg to get relevant data:

- OIS: Displays Overnight Index Swap rates and related market data.
- SWDF: Displays swap discount factors for different tenors and currencies. We had to use SWDF when we couldn't directly access the OIS.
- OVME: Provides an overview of market expectations for volatility and skew.

3 Model

3.1 Bond pricing

While replicating a payoff, we make use of a zero coupon bond to shift the cash flows to a fixed level. For eg, in this case, at a strike price of $6696.29, we need a fixed payoff of $1175. So, we use a ZCB with a face value of $1175 for this part. Similarly, we also need a $100 bond to shift the left part of the payoff vertically, since that is the minimum guaranteed payoff to be received from the structured note at maturity.

3.2 Option pricing

The replicating valuation framework which we will use is the Black-Scholes formula for European options. For put options,

$$P(S_0, 0) = e^{-r_2(T_3-T_1)} \left[KN(-d_2) - S_0 e^{(r_1-d)T_2} N(-d_1) \right] \tag{4}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r_1 - d + \frac{1}{2}\sigma^2)T_2}{\sigma\sqrt{T_2}} \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T_2} \tag{6}$$

Here we use different risk free rates for discounting and the risk-neutral Expected value of the payoffs at maturity. The time period calculations have been shown above.The data for dividend yield has been taken from Bloomberg, while we use the implied volatility of European options with SP500 as the underlying for the σ in the above formula.

For shifting the replicating portfolio's payoff at the strike price of $5396.99, we make use of the digital put option, which can be priced as follows:

$$DP_0 = e^{-rT} AN(-d_2) \tag{7}$$

where,

$$d_2 = \frac{\ln(S_0/K) + (r - d - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{8}$$

3.3 Valuation of each component

Based on the above formula and the data we have estimated, we plug those values in each of the respective components.

- Bond: $1,175 of face value
- Short Put: -$731.453 per option contract (negative because holding a short position in an option is a liability)
- Long Put: $453.629 per option contract
- Short Put: -$288.2 per option contract
- Short Digital Put: -$32.202 per option contract

However, we need the slope of the payoff diagram to estimate the number of options required to replicate the payoff.

The last part is the discount factor. $e^{-r_2(T_3-T_1)}$ is the discount factor using r_2 and $(T_3 - T_1)$.

3.4 Replication Strategy

Using the values and the slope we calculate the value of each portfolio component and add it and discount is using the above formula to arrive at the final value of the replicating portfolio. By law of no arbitrage, we can conclude that the price of the structured note is the same as the value of the replicating portfolio. The value of the structured note is $957.04.

4 Discussion

4.1 Results Analysis

The term sheet provides an estimate of the price of the structured note as $970.30, while based on our replication portfolio approach, we arrive at $957.04. There are some possible reasons of errors/deviations from this estimate, which we will discuss in a subsequent section.

Strike price range	Instrument	Slope	Calculation
>6696.2903	Bond	1	–
5996.66–6696.2903	Short Put	0.2501	1000/Spot price * Leverage = 1000/5996.66*1.5
5396.994–5996.66	Long Put	0.1668	1000/Spot price = 1000/5996.66
<5396.994	Short Put	0.3335	Twice the slope of Long Put to change the direction
<5396.994	Short Digital	1	–
<5396.994	Bond	1	–

Table 5: Slope of the components

Instrument	Strike price	No. of instruments	Value per instrument	Total value
Bond	–	1.0000	1,175.000	1,175.000
Short Put	6,696.2903	0.2501	-731.453	-182.965
Long Put	5,996.6600	0.1668	453.629	75.647
Short Put	5,396.9940	0.3335	-288.199	-96.120
Short Digital Put	5,396.9940	1.0000	-32.202	-32.202
Bond	–	1.0000	100.000	100.000
Total value at maturity				**1,039.360**
Discount factor				0.921
Final discounted value				**957.045**

Table 6: Portfolio Valuation Summary

4.2 Sensitivity analysis

r_2	r_1				
	4.19%	4.29%	4.3877%	4.49%	4.59%
3.8643%	960.46	960.70	960.94	961.18	961.41
3.9643%	958.52	958.75	958.99	959.23	959.46
4.06%	956.57	956.81	957.04	957.28	957.52
4.16%	954.63	954.87	955.10	955.34	955.57
4.26%	952.70	952.93	953.17	953.40	953.64

Table 7: Sensitivity Analysis of Portfolio Value to Interest Rates

The sensitivity analysis of interest rates reveals that the final portfolio value exhibits differential sensitivity to r_1 and r_2. A 400 basis point change in r_1 results in a value range of \$0.94, much lower than the \$7.77 range observed for an equivalent change in r_2. This suggests that while both rates significantly impact the portfolio value, r_2 has a slightly more pronounced effect on the final price.

Option 1 IV	Option 2 IV				
	0.15729	0.16729	0.17729	0.18729	0.19729
0.12878	963.1442	968.328	973.527	978.738	983.958
0.13878	954.9197	960.103	965.302	970.513	975.733
0.14878	946.6624	951.846	957.045	962.256	967.476
0.15878	938.3799	943.564	948.762	953.973	959.193
0.16878	930.0782	935.262	940.461	945.672	950.892

Table 8: Sensitivity Analysis of Portfolio Value to Option Implied Volatilities

The sensitivity analysis reveals that the portfolio value exhibits significant sensitivity to changes in implied volatilities, particularly for Option 1. A 400 basis point change in Option 1's implied volatility results in a value range of \$33.1, which is approximately 60% larger than the \$20.8 range observed for similar changes in Option 2's implied volatility. This asymmetric sensitivity suggests that Option 1's implied volatility should be monitored more closely for risk management purposes.

Option 3 IV	Value
0.17947	973.8635
0.18947	965.5335
0.19947	957.0449
0.20947	948.4195
0.21947	939.6760

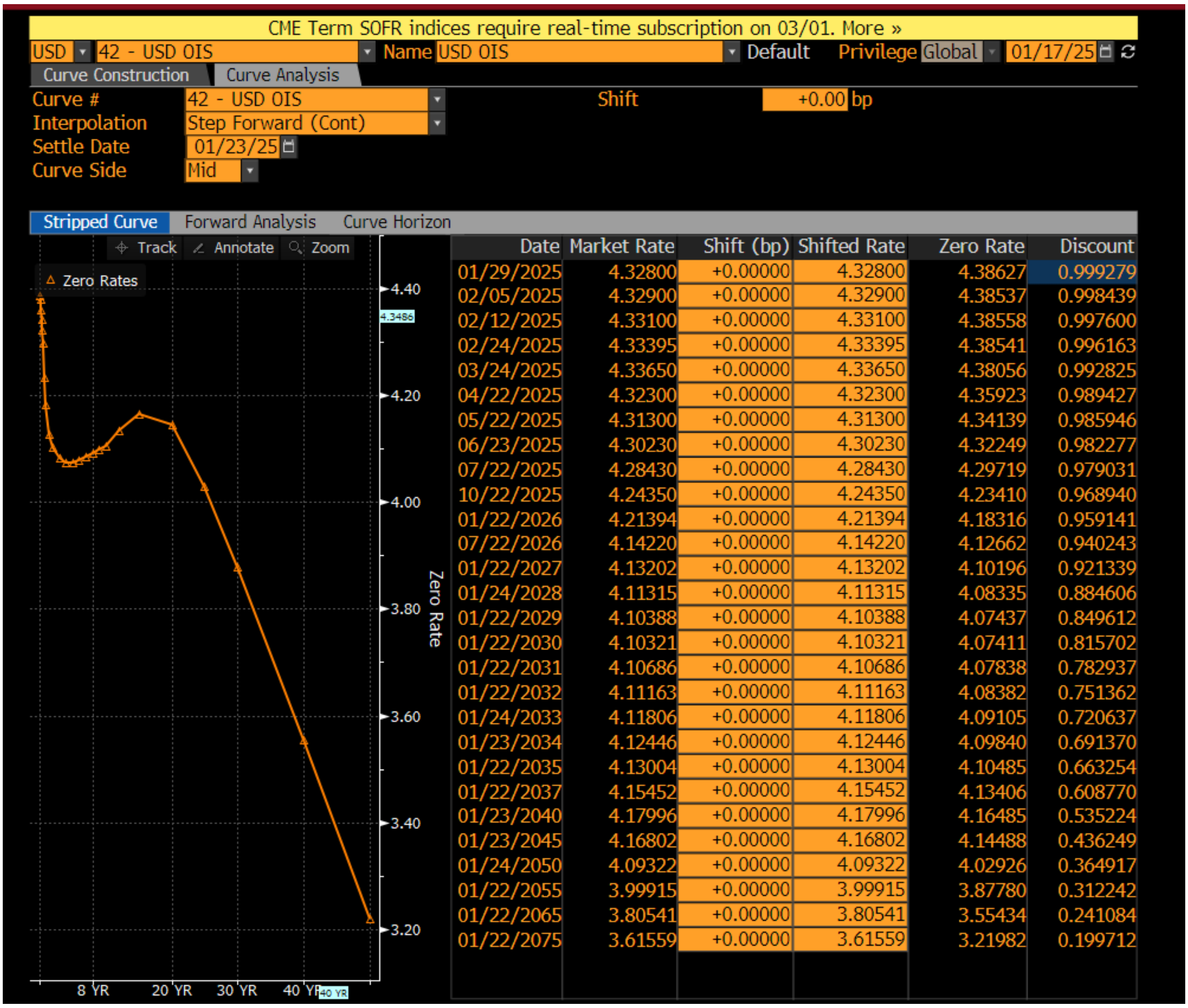
Table 9: Sensitivity Analysis of Portfolio Value to Option 3 Implied Volatility

A 400bps change in Option 3 IV results in a value range of \$34.2, thus showing this IV has the highest impact in terms of senstivity.

4.3 Possible sources of error/variations

- Estimation of r(0,T1): We have estimated r(0,T1), based on the latest date available for the OIS, which was for 01/29/2025 and then used the number of days (01/23/2025 - 01/17/2025 = 7) to estimate the continuous compounding rate. This approximation could lead to some variability in our pricing of options, since this rate is being used to calculate $F_0(T_1, T_2)$, which is r_2.
- Approximation of pricing a digital option: In our current model, we are estimating the volatility for pricing digital option from the implied volatility estimated from European option prices. Given that the payoff from European option is different from a digital option, there may be some error because of this approximation.

5 Appendix



Date	Market Rate	Shift (bp)	Shifted Rate	Zero Rate	Discount
01/29/2025	4.32800	+0.00000	4.32800	4.38627	0.999279
02/05/2025	4.32900	+0.00000	4.32900	4.38537	0.998439
02/12/2025	4.33100	+0.00000	4.33100	4.38558	0.997600
02/24/2025	4.33395	+0.00000	4.33395	4.38541	0.996163
03/24/2025	4.33650	+0.00000	4.33650	4.38056	0.992825
04/22/2025	4.32300	+0.00000	4.32300	4.35923	0.989427
05/22/2025	4.31300	+0.00000	4.31300	4.34139	0.985946
06/23/2025	4.30230	+0.00000	4.30230	4.32249	0.982277
07/22/2025	4.28430	+0.00000	4.28430	4.29719	0.979031
10/22/2025	4.24350	+0.00000	4.24350	4.23410	0.968940
01/22/2026	4.21394	+0.00000	4.21394	4.18316	0.959141
07/22/2026	4.14220	+0.00000	4.14220	4.12662	0.940243
01/22/2027	4.13202	+0.00000	4.13202	4.10196	0.921339
01/24/2028	4.11315	+0.00000	4.11315	4.08335	0.884606
01/22/2029	4.10388	+0.00000	4.10388	4.07437	0.849612
01/22/2030	4.10321	+0.00000	4.10321	4.07411	0.815702
01/22/2031	4.10686	+0.00000	4.10686	4.07838	0.782937
01/22/2032	4.11163	+0.00000	4.11163	4.08382	0.751362
01/24/2033	4.11806	+0.00000	4.11806	4.09105	0.720637
01/23/2034	4.12446	+0.00000	4.12446	4.09840	0.691370
01/22/2035	4.13004	+0.00000	4.13004	4.10485	0.663254
01/22/2037	4.15452	+0.00000	4.15452	4.13406	0.608770
01/23/2040	4.17996	+0.00000	4.17996	4.16485	0.535224
01/23/2045	4.16802	+0.00000	4.16802	4.14488	0.436249
01/24/2050	4.09322	+0.00000	4.09322	4.02926	0.364917
01/22/2055	3.99915	+0.00000	3.99915	3.87780	0.312242
01/22/2065	3.80541	+0.00000	3.80541	3.55434	0.241084
01/22/2075	3.61559	+0.00000	3.61559	3.21982	0.199712

Moneyness	Expiry	12/08/21	08/26/23	05/13/25	01/29/27	10/16/28	07/04/30
111.67	Volatility	N/A	N/A	10.789%	15.024%	17.450%	18.980%
80.00	Volatility	N/A	N/A	27.097%	22.341%	22.412%	22.687%
85.00	Volatility	N/A	N/A	23.540%	21.213%	21.647%	22.119%
90.00	Volatility	N/A	N/A	20.208%	20.085%	20.897%	21.552%
95.00	Volatility	N/A	N/A	17.050%	18.927%	20.140%	20.982%
100.00	Volatility	N/A	N/A	13.933%	17.729%	19.363%	20.400%

References

1. Bloomberg documentation: https://www.bloomberg.com/professional/support/documentation/