

Lab 7

Quiz: Sampling data and analyzing distributions: Poisson, Geometric

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Level: 3rd Semester

Total Questions: 14

Duration: 30 minutes

Section A: Multiple Choice Questions

1. The Poisson distribution is generally used to model:

- a) The number of successes in a fixed number of trials
- b) The number of occurrences in a fixed interval of time or space
- c) The waiting time until the first success
- d) The variance of a normal distribution

Answer: b) The number of occurrences in a fixed interval of time or space

2. If $X \sim \text{Poisson}(\lambda)$, then $E[X] = ?$

- a) 0
- b) λ
- c) λ^2
- d) $\sqrt{\lambda}$

Answer: b)

λ

3. For a Poisson distribution with mean λ , the variance is: a) 0

- b) $\sqrt{\lambda}$
- c) λ
- d) λ^2

Answer: c)

λ

4. A call centre receives on average 4 calls per hour. The probability that exactly 2 calls are received in an hour is:

- a) $e^{-4} * 4^2/2!$
- b) $e^{-2} * 2^4/4!$
- c) $e^{-2} * 2^2/2!$
- d) $e^{-4} * 2^4/4!$

Answer: a) $e^{-4} * 4^2/2!$

5. Which of the following is not a property of a Poisson distribution?

- a) Mean = Variance
- b) Probability of two or more events in an infinitesimal interval is negligible
- c) Trials are independent
- d) It is symmetric about the mean Answer: d) It is symmetric about the mean

6. The geometric distribution models:

- a) Number of trials until the first success
- b) Number of successes in a fixed number of trials
- c) Number of arrivals in a fixed time interval
- d) The spread of a normal distribution

Answer: a) Number of trials until the first success

7. If $X \sim \text{Geometric}(p)$, then $P(X=k) = ?$

- a) $p^k (1-p)$
- b) $(1-p)^{(k-1)} * p$
- c) $(1-p)^k$
- d) $C(n,k) * p^k (1-p)^{(n-k)}$

Answer: b) $(1-p)^{(k-1)} * p$

8. The expected value of a geometric random variable with probability of success p is:

- a) $1/p$
- b) $(1-p)/p$
- c) $1/(1-p)$
- d) p

Answer: a) $1/p$

G. The variance of a geometric distribution with success probability p

is: a) $1/p$

- b) $(1-p)/p^2$
- c) $p/(1-p)^2$
- d) $1/p^2$

Answer: b) $(1-p)/p^2$

10. Which of the following distributions is memoryless?

- a) Normal distribution
- b) Binomial distribution
- c) Poisson distribution
- d) Geometric distribution Answer: d) Geometric distribution

Section B: Short Answer Questions

11. Define the Poisson distribution and state its probability mass function (PMF). Also give conditions under which the Poisson distribution is used.

Answer: The Poisson distribution describes the probability of a given number of events occurring in a fixed interval of time or space, provided these events happen at a known constant mean rate and independently of the time since the last event.

The PMF is:

$$P(X=k)=\lambda^k e^{-\lambda} / k!$$

Conditions:

- Events occur independently.
- Average rate (λ) is constant.
- Two events cannot occur at exactly the same instant.
- The probability of more than one event in an infinitesimal interval is negligible.

12. Define the geometric distribution and state its PMF. A basketball player has a probability of 0.4 of making a free throw. Find the expected number of attempts until his first success.

Answer: : The geometric distribution gives the probability that the first success occurs on the k -th trial, where each trial is independent and has probability p of success.

Geometric PMF

$$P(X=k)=(1-p)^{k-1}p$$

Basketball Example: Expected Number of Attempts

Given $p=0.4$,

$$E[X]=1/p=1/0.4=2.5$$

Expected number of attempts = 2.5.

Section C: Python Coding

13. Write a Python program to find the probability of exactly 4 occurrences when $\lambda=2.5$ in a Poisson distribution.

Answer:

```
from math import exp, factorial
```

```
lmbda = 2.5 k  
= 4  
prob = (lmbda**k * exp(-lmbda)) / factorial(k)  
print("Probability: ",prob)
```

14. Find the probability that the first success occurs on the 3rd trial in Geometric distribution with $p=0.3$. Answer: $p = 0.3$ $k = 3$

```
prob = (1 - p) ** (k - 1) * p print("Probability: ",prob)
```

School of Computer Science and Engineering

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Type- Core

Course Code- CSET204
and Statistics

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Task 2

Lab (Week -7)

Sampling data and analyzing distributions: Poisson, Geometric (Lab Exercise)

Question 1: Generating random sample

Generate 1000 random samples from Poisson ($\lambda=3$) and compute their sample mean and variance.

Answer: # Question 1: Generate 1000 random Poisson samples ($\lambda=3$), compute sample mean and variance

```

import numpy as np

poisson_samples = np.random.poisson(lam=3, size=1000)
poisson_sample_mean = np.mean(poisson_samples) poisson_sample_variance
= np.var(poisson_samples)

print("Question      1      Sample      Mean:",
poisson_sample_mean)     print("Sample      Variance:",
poisson_sample_variance)

```

Question 2: Poisson Distribution - Call Centre Simulation A call center receives on average 5 calls per hour.

- (a) Simulate the number of calls received in 12 hours.**
- (b) Generate 1000 random samples and estimate the probability of exactly 4 calls in an hour.**
- (c) Compare the sample mean and variance with the theoretical values.**

Answer:

```

import numpy as np from scipy.stats import poisson calls_12hr =
np.random.poisson(lam=5, size=12) print("Q2 (a): Calls in 12 hours:", calls_12hr) samples_q2
= np.random.poisson(lam=5, size=1000) prob_estimate_q2 = np.mean(samples_q2 == 4)
theoretical_prob_q2 = poisson.pmf(4, mu=5) print("\nQ2 (b): Probability of 4 calls in 1 hour")
print(f'Estimated      (Simulation):      {prob_estimate_q2:.4f}') print(f'Theoretical:
{theoretical_prob_q2:.4f}') sample_mean_q2 = np.mean(samples_q2) sample_var_q2 =
np.var(samples_q2)

print("\nQ2 (c): Sample Mean and Variance vs Theoretical") print(f'Sample
Mean: {sample_mean_q2:.4f}, Theoretical Mean: 5') print(f'Sample Var:
{sample_var_q2:.4f}, Theoretical Var: 5')

```

Question 3: Geometric Distribution - Basketball Free Throw

A basketball player has a success probability of 0.25 for a free throw.

- (a) Simulate 50 games to record the number of attempts until first success.**
- (b) Compute the average number of attempts and compare with theoretical expected value.**

Answer:

```
import numpy as np
```

```
p = 0.25 samples_q3 =  
np.random.geometric(p, size=50)  
avg_attempts_q3 = np.mean(samples_q3)  
theoretical_exp_q3 = 1 / p  
  
print("Q3: Geometric(p=0.25) - Basketball Free Throw")  
print(f"Average attempts (Simulation): {avg_attempts_q3:.4f}")  
print(f"Theoretical Expected Value: {theoretical_exp_q3:.4f}")
```

Expert Level Question

Question 4: Poisson Distribution - Daily Failures in Factory A

factory produces light bulbs with an average of 3 failures per day.

- (a) Simulate failures for 30 days.**
- (b) Count days with more than 5 failures.**
- (c) Compare with theoretical Poisson probability.**

Answer: import numpy as np from scipy.stats import poisson samples_q4 = np.random.poisson(lam=3, size=30) print("Q4 (a): Daily Failures for 30 Days:\n", samples_q4) count_more_than_5 = np.sum(samples_q4 > 5) print("\nQ4 (b): Days with more than 5 failures:", count_more_than_5) theoretical_prob_q4 = 1 - poisson.cdf(5, mu=3) print("\nQ4 (c): Probability of more than 5 failures") print(f"Theoretical: {theoretical_prob_q4:.4f}") print(f"Estimated (30 days simulation): {count_more_than_5/30:.4f}")