

# School of Computer Science and Engineering

## Task 1

Course- B.Tech.

Type- Core

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Course Name- Probability and Statistics

Semester- ODD

### Lab (Week-5): Expected Value, Variance and Joint Probability

1. Two dice are rolled simultaneously. Let the random variable X represent the maximum of the two numbers obtained. Compute the probability distribution of X. (Within Lab)

Answer:  $P(X=k) = (2k-1)/36$ ,  $k=1,2,3,4,5,6$

Code:

```
prob_max = {k: (2*k - 1)/36 for k in range(1, 7)}
```

```
for k, p in prob_max.items():
```

```
    print(f"P(X={k}) = {p:.4f}")
```

2. A school is analyzing students' performance in a science quiz (6 questions total). Let the discrete random variable X = 'Number of Correct Answers.'

Out of 60 students, the teacher recorded the following distribution:

Number of Correct Answers (x)	0	1	2	3	4	5	6
Frequency f(x)	4	7	11	14	13	8	3

a) Find the Mean and variance of the variable X.

Answer:  $x=0,1,2,3,4,5,6$   $f(x)=4,7,11,14,13,8,3$   $f(x) = 4,7,11,14,13,8,3$   $f(x)=4,7,11,14,13,8,3$

Total students = 606060.

So,  $p(x)=f(x)/60$

$= 0.0666667, 0.1166667, 0.1833333, 0.2333333, 0.2166667, 0.1333333, 0.05.$

$E[X] = (0 \cdot 4 + 1 \cdot 7 + 2 \cdot 11 + 3 \cdot 14 + 4 \cdot 13 + 5 \cdot 8 + 6 \cdot 3) / 60 = 181/60 \approx 3.0166667$

$E[X^2] = (0^2 \cdot 4 + 1^2 \cdot 7 + 2^2 \cdot 11 + 3^2 \cdot 14 + 4^2 \cdot 13 + 5^2 \cdot 8 + 6^2 \cdot 3) / 60 = 693/60 = 11.55$

$\text{Var}(X) = E[X^2] - (E[X])^2 = 11.55 - (181/60)^2 = 8819/3600 \approx 2.45$

b) If a student is randomly selected from the class, what is the probability that they scored at least 4 correct answers on the exam?

Answer:  $P(X \geq 4) = (13 + 8 + 3) / 60 = 24/60 = 0.4$

Code:

```
x = [0, 1, 2, 3, 4, 5, 6]
```

```
freq = [4, 7, 11, 14, 13, 8, 3]
```

```
n = sum(freq)
```

```
p = [f/n for f in freq]
```

```
mean = sum(x[i]*p[i] for i in range(len(x)))
```

```
mean_sq = sum((x[i]**2)*p[i] for i in range(len(x)))
```

```
variance = mean_sq - mean**2
```

```
p_at_least_4 = sum(p[i] for i in range(4, len(x)))
```

```
print(f"Mean = {mean:.4f}")
```

```
print(f"Variance = {variance:.4f}")
```

```
print(f"P(X>=4) = {p_at_least_4:.4f}")
```

3. Draw 1500 random number from the following distribution.

X	P(X=x)
0	0.25
1	0.5
2	0.125
3	0.125

You are not allowed to use any built-in function. You have to use the ideas of PMF and CDF.

Answer:

```
import random
```

```
values = [0, 1, 2, 3]
```

```
probs = [0.25, 0.5, 0.125, 0.125]
```

```
cdf = []

cum_sum = 0

for p_val in probs:
    cum_sum += p_val
    cdf.append(cum_sum)

def generate_sample():
    r = random.random()

    for i, threshold in enumerate(cdf):
        if r <= threshold:
            return values[i]

samples = [generate_sample() for _ in range(1500)]
sample_freq = {val: samples.count(val) for val in values}
print("Generated Frequencies:", sample_freq)
```

## Lab 5 Task 2

### Quiz: Expected Value, Variance and Joint Probability

**Level:** 3rd Semester

**Total Questions:** 7

**Duration:** 25 minutes

#### Section A: Multiple Choice Questions

1. **A die is rolled once. Let the random variable  $X$  be the outcome. Then the expected value  $E[X]$  is:**  
a) 3.5  
b)  $35/12$   
c) 2.5  
d)  $25/6$   
Answer: a) 3.5
  2. **For two random variables  $X$  and  $Y$ ,  $\text{Cov}(X,Y) = 0$  always implies independence.**  
a) True  
b) False  
c) True and converse is also true  
d) None of the above  
Answer: b) False
  3. **A discrete random variable  $X$  has PMFs:  $P(X=1)=0.2$ ,  $P(X=2)=0.5$ ,  $P(X=3)=0.3$ . Find  $\text{Var}(X)$ .**  
a) 0.41  
b) 0.49  
c) 0.51  
d) 0.61  
Answer: b) 0.49
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#### Section B: Short Answer Questions

4. **Write the relationship between variance and standard deviation.**  
Answer:  
 $\text{Variance} = \text{standard deviation}^2 \ (\sigma^2)$
  5. **What is marginal probability? How is it related to the joint distribution?**  
 $P(X=x) = \sum_y P(X=x, Y=y)$   
 $P(Y=y) = \sum_x P(X=x, Y=y)$
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#### Section C: Python Coding

## 6. Joint & Marginal Probability Simulation

### Problem Statement:

Simulate rolling two dice 10,000 times. Let  $X$  = outcome of die 1,  $Y$  = outcome of die 2.

- Estimate the joint probability table  $P(X, Y)$ .
- Compute marginal probabilities  $P(X)$  and  $P(Y)$ .

**Answer:** import random

```
trials = 10000
```

```
joint_counts = [[0]*6 for _ in range(6)]
```

```
for _ in range(trials):
```

```
    x = random.randint(1,6)
```

```
    y = random.randint(1,6)
```

```
    joint_counts[x-1][y-1] += 1
```

```
joint_prob = [[joint_counts[i][j]/trials for j in range(6)] for i in range(6)]
```

```
marginal_x = [sum(joint_counts[i][j] for j in range(6))/trials for i in range(6)]
```

```
marginal_y = [sum(joint_counts[i][j] for i in range(6))/trials for j in range(6)]
```

```
print("Joint probability table P(X,Y):")
```

```
for i in range(6):
```

```
    print("{:.4f}".format(p) for p in joint_prob[i])
```

```
print("Marginal P(X):", "{:.4f}".format(p) for p in marginal_x)
```

```
print("Marginal P(Y):", "{:.4f}".format(p) for p in marginal_y)
```

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## 7. Conditional Probability from Simulation

### Problem Statement:

Simulate tossing a coin twice and repeat the experiment 10,000 times.

- Let  $X=1$  if the first toss is a Head, else 0.
- Let  $Y=1$  if the second toss is Head, else 0.

**Estimate  $P(X=1|Y=1)$  using simulation.**

```
import random

trials = 10000

count_Y1 = 0

count_X1_and_Y1 = 0

for _ in range(trials):

    first = random.choice([0,1])

    second = random.choice([0,1])

    if second == 1:

        count_Y1 += 1

        if first == 1:

            count_X1_and_Y1 += 1

if count_Y1 == 0:

    est = None

else:

    est = count_X1_and_Y1 / count_Y1

print("Estimated  $P(X=1 \mid Y=1)$  =", est)
```

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