

Lab 6

Quiz: Sampling data and analyzing distributions: Bernoulli, Binomial

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Level: 3rd Semester

Total Questions: 7

Duration: 25 minutes

Section A: Multiple Choice Questions

1. If a coin is tossed 10 times, and the probability of heads is 0.5, what is the expected number of heads: a) 2.5
b) 5
c) 7.5
d) 10

Answer: $n \times p = 10 \times 0.5 = 5$. b) 5

2. Which of the following is true about the relationship between Bernoulli and Binomial distributions?
a) Bernoulli is a special case of Binomial when $n > 1$.
b) Binomial is a special case of Bernoulli when $p = 0.5$.
c) Bernoulli is a special case of Binomial when $n = 1$.
d) they are completely unrelated.

Answer: c) Bernoulli is a special case of Binomial when $n = 1$.

3. In a binomial distribution with $n=8$ and $p=0.3$, what is the variance?
a) 1.68
b) 2.4
c) 0.7
d) 1.2

Answer: $n=8, p=0.3 \Rightarrow np(1-p)=8 \times 0.3 \times 0.7=1.68$ $np(1-p)=8 \times 0.3 \times 0.7=1.68$

a) 1.68

Section B: Short Answer Questions

4. Write the probability mass function (PMF) of a Bernoulli distribution and Binomial distribution.

Answer:

Bernoulli: $P(X=k) = p^k(1-p)^{1-k}$ for $k \in \{0,1\}$

Binomial: $P(X=k) = {}^n C_k p^k (1-p)^{n-k}$ for $k=0,1,2,\dots,n$

5. Explain the significance of the parameter p in Bernoulli and Binomial distributions.

Answer: p represents the probability of "success" in a single trial.

In Bernoulli, p is the probability that the random variable is 1 (success).

In Binomial, p is the probability of success in each of the n independent trials.

Section C: Python Coding

6. Generate 10 random samples from a Bernoulli distribution with $p=0.6$ and print them.

Answer:

```
import numpy as np
```

```
bernoulli_samples = np.random.binomial(1, 0.6, 10)
print("Bernoulli samples:", bernoulli_samples)
```

7. Generate a large sample (10000 points) from a Binomial distribution with $n=20$, $p=0.3$.

Calculate and print the sample mean and variance.

Answer:

```
import numpy as np binomial_samples =
np.random.binomial(20, 0.3, 10000) mean =
np.mean(binomial_samples) variance =
np.var(binomial_samples) print("Binomial sample
mean:", mean) print("Binomial sample variance:",
variance)
```

Task-2

Lab (Week -6)

Sampling data and analyzing distributions: Bernoulli, Binomial (Lab Exercise)

Question 1:

A factory produces light bulbs, and the probability that a bulb is defective is 0.1. If a quality inspector checks 20 bulbs, find using Python:

- The probability that exactly 3 bulbs are defective.**
- The probability that at most 2 bulbs are defective.**

Answer:

```
#Question 1 from scipy.stats import binom,  
nbinom # Question 1: Light bulb defective  
probabilities p_defective = 0.1 n_bulbs = 20  
  
# a) Probability exactly 3 bulbs defective prob_3_defective =  
binom.pmf(3, n_bulbs, p_defective) print(f"\"a) Probability  
exactly 3 defective: {prob_3_defective:.4f}\"\")  
  
# b) Probability at most 2 bulbs defective prob_at_most_2_defective =  
binom.cdf(2, n_bulbs, p_defective) print(f"\"b) Probability at most 2  
defective: {prob_at_most_2_defective:.4f}\"\")
```

Question 2:

A family with 5 children can be modeled as a Binomial distribution where each child is equally likely to be a boy or a girl (assume probability of boy = probability of girl = 0.5). Out of 800 families with 5 children each, answer the following using Python:

- a. How many families would be expected to have exactly 3 boys?
- b. How many families would be expected to have exactly 5 girls?
- c. How many families would be expected to have either 2 or 3 boys?
- d. How many families would be expected to have at least 2 girls?

Write Python code to compute the answers using the Binomial probability formula. Also, verify the results using Answer:

```
#Question 2  
from scipy.stats import binom
```

```
n_children = 5 p_boy  
= 0.5  
n_families = 800  
  
# a) Expected number of families with exactly 3 boys prob_3_boys  
= binom.pmf(3, n_children, p_boy) expected_3_boys = prob_3_boys  
* n_families print(f"\"a) Expected families with exactly 3 boys:  
{expected_3_boys:.0f}\"\") # b) Expected families with exactly 5 girls (0  
boys) prob_5_girls = binom.pmf(0, n_children, p_boy) expected_5_girls
```

```

= prob_5_girls * n_families print(f"b) Expected families with exactly 5
girls: {expected_5_girls:.0f} ")

# c) Expected families with either 2 or 3 boys prob_2_boys = binom.pmf(2,
n_children, p_boy) expected_2_or_3_boys = (prob_2_boys + prob_3_boys)
* n_families print(f"c) Expected families with 2 or 3 boys:
{expected_2_or_3_boys:.0f} ")

# d) Expected families with at least 2 girls (girls >= 2 means boys <= 3)
prob_at_least_2_girls = binom.cdf(3, n_children, p_boy)
expected_at_least_2_girls = prob_at_least_2_girls * n_families print(f"d)
Expected families with at least 2 girls: {expected_at_least_2_girls:.0f} ")

```

Question 3:

You are playing a game where the probability of winning (success) in each trial is $p = 0.3$.

- Write a Python program to calculate the probability that there will be exactly 5 failures before achieving 4 successes. (Hint: Use Negative Binomial distribution formula)
- Write a Python program to calculate the probability of getting exactly 4 wins in 9 trials. (Hint: Use Binomial distribution formula)
- Write a Python program to calculate the probability that the 4th win occurs exactly on the 9th trial. (Hint: Use Negative Binomial pmf for $r=4$ and $k=9-r=5$ failures)

Answer: #Question

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```

from scipy.stats import nbinom, binom

p = 0.3
r = 4 # number of
successes k = 5 # number
of failures
n = 9 # total trials

```

```

# a) Probability exactly 5 failures before 4 successes (Negative Binomial PMF)
prob_neg_binomial_a = nbinom.pmf(k, r, p)
print(f"a) Probability 5 failures before 4 successes: {prob_neg_binomial_a:.4f}")

```

```

# b) Probability exactly 4 wins in 9 trials (Binomial PMF)
prob_binomial_b = binom.pmf(4, n, p)
print(f"b) Probability exactly 4 wins in 9 trials: {prob_binomial_b:.4f}")

```

```

# c) Probability 4th win occurs exactly on 9th trial (Negative Binomial PMF)
prob_neg_binomial_c = nbinom.pmf(k, r, p)
print(f"c) Probability 4th win on 9th trial: {prob_neg_binomial_c:.4f}")

```