Lab 7
Quiz: Sampling data and analyzing distributions: Poisson, Geometric

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Level: 3rd Semester
Total Questions: 14
Duration: 30 minutes

Section A: Multiple Choice Questions

1. The Poisson distribution is generally used to model:

a) The number of successes in a fixed number of trials
b) The number of occurrences in a fixed interval of time or space
c) The waiting time until the first success
d) The variance of a normal distribution
Answer: b) The number of occurrences in a fixed interval of time or space

2. If $X \sim Poisson(\lambda)$, then $E[X] = ?$
a) 0
b) λ
c) λ^2
d) $\sqrt{\lambda}$
Answer: b) λ

3. For a Poisson distribution with mean λ, the variance is: a) 0
b) $\sqrt{\lambda}$
c) λ
d) λ^2
Answer: c) λ

4. A call centre receives on average 4 calls per hour. The probability that exactly 2 calls are received in an hour is:

a) e^(-4) * $4^2/2!$
b) e^(-2) * $2^4/4!$
c) e^(-2) * $2^2/2!$
d) e^(-4) * $2^4/4!$ Answer: a) e^(-4) * $4^2/2!$

5. Which of the following is not a property of a Poisson distribution?

a) Mean = Variance
b) Probability of two or more events in an infinitesimal interval is negligible
c) Trials are independent
d) It is symmetric about the mean Answer: d) It is symmetric about the mean

6. The geometric distribution models:

a) Number of trials until the first success
b) Number of successes in a fixed number of trials
c) Number of arrivals in a fixed time interval
d) The spread of a normal distribution
Answer: a) Number of trials until the first success

7. If X ~ Geometric(p), then P(X=k) = ?

a) p^k (1-p)
b) (1-p)^(k-1) * p
c) (1-p)^k
d) C(n,k) * p^k (1-p)^(n-k)
Answer: b) (1-p)^(k-1) * p

8. The expected value of a geometric random variable with probability of success p is:

a) 1/p
b) (1-p)/p
c) 1/(1-p)
d) p
Answer: a) 1/p

9. The variance of a geometric distribution with success probability p is: a)

1/p

b) $(1-p)/p^2$
c) $p/(1-p)^2$
d) $1/p^2$
Answer: b) $(1-p)/p^2$

10. Which of the following distributions is memoryless?

a) Normal distribution
b) Binomial distribution
c) Poisson distribution
d) Geometric distribution Answer: d) Geometric distribution

Section B: Short Answer Questions

11. Define the Poisson distribution and state its probability mass function (PMF). Also give conditions under which the Poisson distribution is used.

Answer: The Poisson distribution describes the probability of a given number of events occurring in a fixed interval of time or space, provided these events happen at a known constant mean rate and independently of the time since the last event.

The PMF is:

$$P(X=k)=\lambda^k*e^(-\lambda) /k!$$

Conditions:

- Events occur independently.
- Average rate ($\lambda\lambda$) is constant.
- Two events cannot occur at exactly the same instant.
- The probability of more than one event in an infinitesimal interval is negligible.

12. Define the geometric distribution and state its PMF. A basketball player has a probability of 0.4 of making a free throw. Find the expected number of attempts until his first success.

Answer: : The geometric distribution gives the probability that the first success occurs on the k-th trial, where each trial is independent and has probability p of success.
Geometric PMF P(X=k)=
$(1-p)^{k-1}*p$
Basketball Example: Expected Number of Attempts
Given p=0,
E[X]=1/p=1/0.4=2.5
Expected number of attempts = 2.5.

Section C: Python Coding

13. Write a Python program to find the probability of exactly 4 occurrences when λ=2.5 in a Poisson distribution.

Answer:

```
from math import exp, factorial

lmbda = 2.5 k
= 4
prob = (lmbda**k * exp(-lmbda)) / factorial(k)
print("Prrobability: ",prob)
```

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14. Find the probability that the first success occurs on the 3rd trial in Geometric distribution with p=0.3. Answer: p = 0.3 k = 3

prob = (1 - p) ** (k - 1) * p print("Probability: ",prob)

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Task 2

Lab (Week -7)

Sampling data and analyzing distributions: Poisson, Geometric (Lab Exercise)

Question 1: Generating random sample

Generate 1000 random samples from Poisson (λ=3) and compute their sample mean and variance.

Answer: # Question 1: Generate 1000 random Poisson samples (λ=3), compute sample mean and variance

```
import numpy as np

poisson_samples = np.random.poisson(lam=3, size=1000)
poisson_sample_mean = np.mean(poisson_samples) poisson_sample_variance
= np.var(poisson_samples)

print("Question 1 Sample Mean:", poisson_sample_mean)
print("Sample Variance:", poisson_sample_variance)
```

Question 2: Poisson Distribution – Call Centre Simulation

A call center receives on average 5 calls per hour.

(a) Simulate the number of calls received in 12 hours.

(b) Generate 1000 random samples and estimate the probability of exactly 4 calls in an hour.

(c) Compare the sample mean and variance with the theoretical values. Answer:

```
import numpy as np from scipy.stats import poisson calls_12hr = np.random.poisson(lam=5, size=12)
print("Q2 (a): Calls in 12 hours:", calls_12hr) samples_q2 = np.random.poisson(lam=5,
size=1000) prob_estimate_q2 = np.mean(samples_q2 == 4) theoretical_prob_q2 =
poisson.pmf(4, mu=5) print("\nQ2 (b): Probability of 4 calls in 1 hour") print(f"Estimated
(Simulation): {prob_estimate_q2:.4f}") print(f"Theoretical: {theoretical_prob_q2:.4f}")
sample_mean_q2 = np.mean(samples_q2) sample_var_q2 = np.var(samples_q2)
print("\nQ2 (c): Sample Mean and Variance vs Theoretical") print(f"Sample
Mean: {sample_mean_q2:.4f}, Theoretical Mean: 5") print(f"Sample Var:
{sample_var_q2:.4f}, Theoretical Var: 5")
```

Question 3: Geometric Distribution – Basketball Free Throw

A basketball player has a success probability of 0.25 for a free throw.

(a) Simulate 50 games to record the number of attempts until first success.

(b) Compute the average number of attempts and compare with theoretical expected value.

Answer:

```
import numpy as np

p = 0.25 samples_q3 =
np.random.geometric(p, size=50)
avg_attempts_q3 = np.mean(samples_q3)
theoretical_exp_q3 = 1 / p

print("Q3: Geometric(p=0.25) - Basketball Free Throw")
print(f"Average attempts (Simulation): {avg_attempts_q3:.4f}")
print(f"Theoretical Expected Value: {theoretical_exp_q3:.4f}")
```

Expert Level Question

Question 4: Poisson Distribution – Daily Failures in Factory A factory produces light bulbs with an average of 3 failures per day.

(a) Simulate failures for 30 days.

(b) Count days with more than 5 failures.

(c) Compare with theoretical Poisson probability.

Answer:

```
import numpy as np from scipy.stats import
poisson samples_q4 = np.random.poisson(lam=3,
size=30) print("Q4 (a): Daily Failures for 30 Days:\n",
samples_q4) count_more_than_5 =
np.sum(samples_q4 > 5) print("\nQ4 (b): Days with
more than 5 failures:", count_more_than_5)
theoretical_prob_q4 = 1 - poisson.cdf(5, mu=3)
print("\nQ4 (c): Probability of more than 5 failures")
print(f"Theoretical: {theoretical_prob_q4:.4f}")
print(f"Estimated (30 days simulation):
{count_more_than_5/30:.4f}")
```