

School of Computer Science and Engineering

Task 1

Course- B.Tech.

Type- Core

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Course Name- Probability and Statistics

Semester- ODD

Lab (Week-5): Expected Value, Variance and Joint Probability

1. Two dice are rolled simultaneously. Let the random variable X represent the maximum of the two numbers obtained. Compute the probability distribution of X. (Within Lab)

Answer: $P(X=k) = (2k-1)/36$, $k=1,2,3,4,5,6$

Code:

```
prob_max = {k: (2*k - 1)/36 for k in range(1, 7)}
```

```
for k, p in prob_max.items():
```

```
    print(f"P(X={k}) = {p:.4f}")
```

2. A school is analyzing students' performance in a science quiz (6 questions total). Let the discrete random variable X = 'Number of Correct Answers.'

Out of 60 students, the teacher recorded the following distribution:

Number of Correct Answers (x)	0	1	2	3	4	5	6
Frequency f(x)	4	7	11	14	13	8	3

- a) Find the Mean and variance of the variable X.

Answer: $x=0,1,2,3,4,5,6$ $f(x)=4,7,11,14,13,8,3$ $3f(x) = 4,7,11,14,13,8,3$ $3f(x)=4,7,11,14,13,8,3$

Total students = 606060.

So, $p(x)=f(x)/60$

$$= 0.06666667, 0.11666667, 0.18333333, 0.23333333, 0.21666667, 0.13333333, 0.05.$$

$$E[X] = (0 \cdot 4 + 1 \cdot 7 + 2 \cdot 11 + 3 \cdot 14 + 4 \cdot 13 + 5 \cdot 8 + 6 \cdot 3) / 60 = 181/60 \approx 3.0166667$$

$$E[X^2] = (0^2 \cdot 4 + 1^2 \cdot 7 + 2^2 \cdot 11 + 3^2 \cdot 14 + 4^2 \cdot 13 + 5^2 \cdot 8 + 6^2 \cdot 3) / 60 = 693/60 = 11.55$$

$$\text{Var}(X) = E[X^2] - (E[X])^2 = 11.55 - (181/60)^2 = 8819/3600 = 2.45$$

- b) If a student is randomly selected from the class, what is the probability that they scored at least 4 correct answers on the exam?

Answer: $P(X \geq 4) = (13 + 8 + 3) / 60 = 24/60 = 0.4$

Code:

```
x = [0, 1, 2, 3, 4, 5, 6]
freq = [4, 7, 11, 14, 13, 8, 3]
n = sum(freq)
p = [f/n for f in freq]
```

```
mean = sum(x[i]*p[i] for i in range(len(x)))
mean_sq = sum((x[i]**2)*p[i] for i in range(len(x)))
variance = mean_sq - mean**2
p_at_least_4 = sum(p[i] for i in range(4, len(x)))
```

```
print(f"Mean = {mean:.4f}")
print(f"Variance = {variance:.4f}")
print(f"P(X>=4) = {p_at_least_4:.4f}")
```

3. Draw 1500 random number from the following distribution.

X	$P(X=x)$
0	0.25
1	0.5
2	0.125
3	0.125

You are not allowed to use any built-in function. You have to use the ideas of PMF and CDF.

Answer:

```
import random
```

```
values = [0, 1, 2, 3]
```

```
probs = [0.25, 0.5, 0.125, 0.125]
```

```
cdf = []
cum_sum = 0
for p_val in probs:
    cum_sum += p_val
    cdf.append(cum_sum)

def generate_sample():
    r = random.random()
    for i, threshold in enumerate(cdf):
        if r <= threshold:
            return values[i]

samples = [generate_sample() for _ in range(1500)]
sample_freq = {val: samples.count(val) for val in values}
print("Generated Frequencies:", sample_freq)
```

Lab 5 Task 2

Quiz: Expected Value, Variance and Joint Probability

Level: 3rd Semester

Total Questions: 7

Duration: 25 minutes

Section A: Multiple Choice Questions

1. **A die is rolled once. Let the random variable X be the outcome. Then the expected value $E[X]$ is:**
 - a) 3.5
 - b) 35/12
 - c) 2.5
 - d) 25/6

Answer: a) 3.5
2. **For two random variables X and Y, $\text{Cov}(X,Y) = 0$ always implies independence.**
 - a) True
 - b) False
 - c) True and converse is also true
 - d) None of the above

Answer: b) False
3. **A discrete random variable X has PMFs: $P(X=1)=0.2$, $P(X=2)=0.5$, $P(X=3)=0.3$. Find $\text{Var}(X)$.**
 - a) 0.41
 - b) 0.49
 - c) 0.51
 - d) 0.61

Answer: b) 0.49

Section B: Short Answer Questions

4. **Write the relationship between variance and standard deviation.**

Answer:
 $\text{Variance} = \text{standard deviation}^2 (\sigma^2)$
5. **What is marginal probability? How is it related to the joint distribution?**

$P(X=x) = \sum_y P(X=x, Y=y)$
 $P(Y=y) = \sum_x P(X=x, Y=y)$

Section C: Python Coding

6. Joint & Marginal Probability Simulation

Problem Statement:

Simulate rolling two dice 10,000 times. Let X = outcome of die 1, Y = outcome of die 2.

- Estimate the joint probability table P(X, Y).
- Compute marginal probabilities P(X) and P(Y).

Answer: import random

```
trials = 10000
```

```
joint_counts = [[0]*6 for _ in range(6)]
```

```
for _ in range(trials):
```

```
    x = random.randint(1,6)
```

```
    y = random.randint(1,6)
```

```
    joint_counts[x-1][y-1] += 1
```

```
joint_prob = [[joint_counts[i][j]/trials for j in range(6)] for i in range(6)]
```

```
marginal_x = [sum(joint_counts[i][j] for j in range(6))/trials for i in range(6)]
```

```
marginal_y = [sum(joint_counts[i][j] for i in range(6))/trials for j in range(6)]
```

```
print("Joint probability table P(X,Y):")
```

```
for i in range(6):
```

```
    print(["{:4f}".format(p) for p in joint_prob[i]])
```

```
print("Marginal P(X):", ["{:4f}".format(p) for p in marginal_x])
```

```
print("Marginal P(Y):", ["{:4f}".format(p) for p in marginal_y])
```

7. Conditional Probability from Simulation

Problem Statement:

Simulate tossing a coin twice and repeat the experiment 10,000 times.

- Let X=1 if the first toss is a Head, else 0.
- Let Y=1 if the second toss is Head, else 0.

Estimate $P(X=1|Y=1)$ using simulation.

```
import random

trials = 10000

count_Y1 = 0

count_X1_and_Y1 = 0

for _ in range(trials):

    first = random.choice([0,1])

    second = random.choice([0,1])

    if second == 1:

        count_Y1 += 1

        if first == 1:

            count_X1_and_Y1 += 1

    if count_Y1 == 0:

        est = None

    else:

        est = count_X1_and_Y1 / count_Y1

    print("Estimated  $P(X=1 | Y=1) =$ ", est)
```
