

School of Computer Science and Engineering

Task 1

Course- B.Tech.

Type- Core

Priyam Vatsh

S24CSEU2585

Course Code- CSET204

Year- 2025-26

Course Name- Probability and Statistics

Semester- ODD

Lab (Week -5): Expected Value, Variance and Joint Probability

1. Two dice are rolled simultaneously. Let the random variable X represent the maximum of the two numbers obtained. Compute the probability distribution of X. (Within Lab)

Answer: $P(X=k)=(2k-1)/36$, $k=1,2,3,4,5,6$

Code:

```
prob_max = {k: (2*k - 1)/36 for k in range(1, 7)}
```

```
for k, p in prob_max.items():
```

```
    print(f"P(X={k}) = {p:.4f}")
```

2. A school is analyzing students' performance in a science quiz (6 questions total). Let the discrete random variable X = 'Number of Correct Answers.'

Out of 60 students, the teacher recorded the following distribution:

Number of Correct Answers (x)	1	2	3	4	5	6
Frequency f(x)	7	11	14	13	8	3

a) Find the Mean and variance of the variable X.

Answer: $x=0,1,2,3,4,5,6$ $f(x)=4,7,11,14,13,8,3$ $f(x) = 4,7,11,14,13,8,3$ $f(x)=4,7,11,14,13,8,3$

Total students = 606060.

So, $p(x)=f(x)/60$

$= 0.0666667, 0.1166667, 0.1833333, 0.2333333, 0.2166667, 0.1333333, 0.05.$

$E[X] = (0 \cdot 4 + 1 \cdot 7 + 2 \cdot 11 + 3 \cdot 14 + 4 \cdot 13 + 5 \cdot 8 + 6 \cdot 3) / 60 = 181/60 \approx 3.0166667$

$E[X^2] = (0^2 \cdot 4 + 1^2 \cdot 7 + 2^2 \cdot 11 + 3^2 \cdot 14 + 4^2 \cdot 13 + 5^2 \cdot 8 + 6^2 \cdot 3) / 60 = 693/60 = 11.55$

$\text{Var}(X) = E[X^2] - (E[X])^2 = 11.55 - (181/60)^2 = 8819/3600 \approx 2.45$

b) If a student is randomly selected from the class, what is the probability that they scored at least 4 correct answers on the exam?

Answer: $P(X \geq 4) = (13 + 8 + 3) / 60 = 24/60 = 0.4$

Code:

```
x = [0, 1, 2, 3, 4, 5, 6]
```

```
freq = [4, 7, 11, 14, 13, 8, 3]
```

```
n = sum(freq)
```

```
p = [f/n for f in freq]
```

```
mean = sum(x[i]*p[i] for i in range(len(x)))
```

```
mean_sq = sum((x[i]**2)*p[i] for i in range(len(x)))
```

```
variance = mean_sq - mean**2
```

```
p_at_least_4 = sum(p[i] for i in range(4, len(x)))
```

```
print(f"Mean = {mean:.4f}")
```

```
print(f"Variance = {variance:.4f}")
```

```
print(f"P(X>=4) = {p_at_least_4:.4f}")
```

3. Draw 1500 random number from the following distribution.

X	P(X=x)
0	0.25
1	0.5
2	0.125
3	0.125

You are not allowed to use any built-in function. You have to use the ideas of PMF and CDF.

Answer:

```
import random
```

```
values = [0, 1, 2, 3]
```

```
probs = [0.25, 0.5, 0.125, 0.125]
```

```
cdf = []  
  
cum_sum = 0  
  
for p_val in probs:  
    cum_sum += p_val  
    cdf.append(cum_sum)  
  
  
def generate_sample():  
    r = random.random()  
  
    for i, threshold in enumerate(cdf):  
        if r <= threshold:  
            return values[i]  
  
  
samples = [generate_sample() for _ in range(1500)]  
sample_freq = {val: samples.count(val) for val in values}  
print("Generated Frequencies:", sample_freq)
```

Lab 5 Task 2

Quiz: Expected Value, Variance and Joint Probability

Level: 3rd Semester

Total Questions: 7

Duration: 25 minutes

Section A: Multiple Choice Questions

1. A die is rolled once. Let the random variable X be the outcome. Then the expected value $E[X]$ is:
a) 3.5
b) $35/12$
c) 2.5
d) $25/6$
Answer: a)
3.5
 2. For two random variables X and Y , $\text{Cov}(X,Y) = 0$ always implies independence.
a) True
b) False
c) True and converse is also true
d) None of the above
Answer: b)
False
 3. A discrete random variable X has PMFs: $P(X=1)=0.2$, $P(X=2)=0.5$, $P(X=3)=0.3$. Find $\text{Var}(X)$.
a) 0.41
b) 0.49
c) 0.51
d) 0.61
Answer: b) 0.49
-

Section B: Short Answer Questions

4. Write the relationship between variance and standard deviation.
Answer:
 $\text{Variance} = \text{standard deviation}^2 \ (\sigma^2)$
 5. What is marginal probability? How is it related to the joint distribution?
 $P(X=x) = \sum_y P(X=x, Y=y)$
 $P(Y=y) = \sum_x P(X=x, Y=y)$
-

Section C: Python Coding

6. Joint s Marginal Probability Simulation

Problem Statement:

Simulate rolling two dice 10,000 times. Let X = outcome of die 1, Y = outcome of die 2.

- **Estimate the joint probability table $P(X, Y)$.**
- **Compute marginal probabilities $P(X)$ and $P(Y)$.**

Answer: import random

trials = 10000

joint_counts = [[0]*6 for _ in range(6)]

for _ in range(trials):

 x =

 random.randint(1,6) y

 = random.randint(1,6)

 joint_counts[x-1][y-1] += 1

joint_prob = [[joint_counts[i][j]/trials for j in range(6)] for i in range(6)]

marginal_x = [sum(joint_counts[i][j] for j in range(6))/trials for i in range(6)]

marginal_y = [sum(joint_counts[i][j] for i in range(6))/trials for j in range(6)]

print("Joint probability table P(X,Y):")

for i in range(6):

 print(["{: .4f} ".format(p) for p in joint_prob[i]])

print("Marginal P(X):", ["{: .4f} ".format(p) for p in marginal_x])

print("Marginal P(Y):", ["{: .4f} ".format(p) for p in marginal_y])

7. Conditional Probability from Simulation

Problem Statement:

Simulate tossing a coin twice and repeat the experiment 10,000 times.

- **Let $X=1$ if the first toss is a Head, else 0.**
- **Let $Y=1$ if the second toss is Head, else 0.**

Estimate $P(X=1|Y=1)$ using simulation.

```
import random

trials = 10000

count_Y1 = 0

count_X1_and_Y1 = 0

for _ in range(trials):

    first = random.choice([0,1])

    second = random.choice([0,1])

    if second == 1:

        count_Y1 += 1

        if first == 1:

            count_X1_and_Y1 += 1

1 if count_Y1 == 0:

    est = None

else:

    est = count_X1_and_Y1 / count_Y1

print("Estimated  $P(X=1 | Y=1)$  =", est)
```
