<u>Lab 4</u>

<u>Quiz: Probability Distributions</u>

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Level: 3th Semester
Total Questions: 6
Duration: 25 minutes

Section A: Multiple Choice Questions

1. **Which of the following is true about joint probability?**
   a) It is always less than individual probabilities
   b) It represents the probability of two events occurring together
   c) It is equal to marginal probability
   d) It is the sum of individual probabilities

   Answer: b) It represents the probability of two events occurring together

2. **If $E(X) = 4$ and $E(X^2) = 18$, what is $Var(X)$?**
   a) 1 2
   b) 34
   c) 16
   d) 2

3. **Answer:** d) 2

Section B: Short Answer Questions

3. **Define Expected Value. Give an example from daily life where it is useful.**

   Answer: Expected Value can be defined as the mean value of random variables over numerous trials.
   Real life Example: The expected value of rolling a fair six-sided die is $(1+2+3+4+5+6)/6 = 3.5$. This helps in predicting average outcomes in games of chance or risk assessment.

4. **Write the formula for joint probability of two dependent events A and B.**

Answer: $P(A \cap B) = P(A) \times P(B|A)$, where P(B|A)P(B|A) is the conditional probability of B given A.

5. **What are the key differences between Bernoulli and Binomial distributions? Bernoulli:**

   Bernoulli distribution models a single trial with two outcomes (success or failure).

   Bernoulli is a special case of the Binomial where the number of trials n = 1

   Binomial:

   Binomial distribution models the number of successes in multiple independent Bernoulli trials.

Section C: Python Coding

6. Write a Python snippet using numpy or random to simulate a Bernoulli distribution with 10 trials and probability of success = 0.7.

Answer:

```
import numpy as np trials =

np.random.binomial(1, 0.7,

10) print("Bernoulli Trials:" ,

trials)
```

Task-2 Probability and Statistics Lab Questions

Q1. Advanced Card Probability

You have a standard deck of 52 cards.

1. What is the probability of drawing a red face card (Jack, Queen, or King of hearts/diamonds)?
2. If two cards are drawn without replacement, what is the probability that both are spades?
3. Given that a drawn card is a black card, what is the conditional probability that it is a Queen?

4. If you draw two cards, what is the probability that at least one is an Ace given that one is a heart?

Answer:

```
from math import comb

# 1. Probability of red face card

red_face_cards = 6 total_cards = 52

prob_red_face = red_face_cards /

total_cards


# 2. Probability both cards are spades without replacement spades = 13

prob_two_spades = (spades / total_cards) * ((spades - 1) / (total_cards -

1))


# 3. Conditional probability black card is Queen

black_cards = 26 queens_black = 2

prob_queen_given_black = queens_black /

black_cards


# 4. Probability at least one Ace given one is heart (two cards)

no_ace_ways = comb(12, 1)*comb(39, 1) + comb(12, 2)

total_heart_ways = comb(13, 1)*comb(39, 1) + comb(13, 2)

prob_no_ace_given_heart = no_ace_ways / total_heart_ways

prob_at_least_one_ace_given_heart = 1 - prob_no_ace_given_heart


print(prob_red_face, prob_two_spades, prob_queen_given_black, prob_at_least_one_ace_given_heart)
```

Q2. Advanced Medical Test Probability (Bayes' Theorem)

A new diagnostic test for a disease is being evaluated.

- Sensitivity = 0.92 (P(Test + | Disease))
- Specificity = 0.88 (P(Test – | No Disease))
- Prevalence of the disease in the population = 5%

Tasks:

1. Calculate the probability that a person actually has the disease given a positive result.
2. If the test is performed twice independently, what is the probability that a person is actually diseased given that both tests are positive?
3. Compare this probability with the case where only one positive test result is available — discuss how repeating the test affects reliability.

Answer:

```
sensitivity = 0.92

specificity = 0.88

prevalence = 0.05


# Probability of positive test result prob_test_pos = sensitivity *

prevalence + (1 - specificity) * (1 - prevalence)


# Probability diseased given positive test (Bayes)

prob_disease_given_pos = (sensitivity * prevalence) / prob_test_pos


# Probability diseased given two independent positive tests prob_both_tests_pos =

sensitivity**2 * prevalence + (1 - specificity)**2 * (1 - prevalence)

prob_disease_given_both_pos = (sensitivity**2 * prevalence) / prob_both_tests_pos


print(prob_disease_given_pos, prob_disease_given_both_pos)
```

Q3. Advanced Symptom-based Disease Probability

A patient is being diagnosed for a rare disease. The following data is known:

- P(Disease | Fever) = 0.55
- P(Disease | Cough) = 0.35
- P(Disease | Fatigue) = 0.45
- P(Disease | Fever ∧ Cough) = 0.80
- P(Disease | Fever ∧ Fatigue) = 0.75
- P(Disease | Fever ∧ Cough ∧ Fatigue) = 0.95

Tasks:

1. If a patient shows fever and cough, calculate the probability of having the disease.

2. If the patient shows all three symptoms, calculate the probability.
3. Suppose the overall probability of disease in the population is 10%. Using Bayes' theorem, compute the probability that a randomly selected symptomatic patient with fever and fatigue actually has the disease.

Answer:

```
p_disease = 0.10

p_Fever_Fatigue_given_disease = 0.75

p_Fever_Fatigue_given_no_disease = 0.1

p_no_disease = 1 - p_disease


# Bayes theorem for disease given fever and fatigue

p_Fever_Fatigue = p_Fever_Fatigue_given_disease * p_disease +
p_Fever_Fatigue_given_no_disease * p_no_disease p_disease_given_Fever_Fatigue =
(p_Fever_Fatigue_given_disease * p_disease) / p_Fever_Fatigue


print(p_disease_given_Fever_Fatigue)
```