MATH 137   Assignment 4, Part 2   Due: 11 am, Friday, October 15

Your assignment consists of two separate parts. Part 1 is available online at

$$\text{http://mapleta.uwaterloo.ca}$$

and is due at 4 pm on Thursday October 14.

Part 2 consists of the problems below. Place part 2 of your assignment in the correct drop box outside MC 4066, corresponding to the class section in which you are registered.

Hand in your solutions to the following 5 problems.

1. Let $f(x) = x^3 + bx^2 + cx + d$ be a general cubic polynomial with the coefficient of x^3 adjusted to be 1.

   (a) Show that $f(x) > 0$ when x is very large. Also show that $f(x) < 0$ when $x < 0$ and $|x|$ is very large.
   (Hint: Write $f(x) = x^3(1 + b/x + c/x^2 + d/x^3)$ and determine the sign of the part in brackets when $|x|$ is very large.)

   (b) Use part (a) to prove that every cubic polynomial has at least one real root.

   (c) Does every odd degree polynomial have at least one real root? Briefly explain your answer.

2. Is there a real number a such that the following function is continuous on $\mathbb{R}$?
   $$f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x \geq a, \\ 1 - x & \text{if } x < a. \end{cases}$$

3. Evaluate the following limits if they exist.

   (a) $\displaystyle\lim_{x\to\infty} \arctan(x - x^4)$

   (b) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

   (Hint: Compare with Exercise #23 in Section 2.6.)

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(c) $\displaystyle\lim_{x \to -\infty} \left(x + \sqrt{x^2 + 2x}\,\right)$

   (Hint: Compare with Exercise #25 in Section 2.6.)

4. Determine whether $f'(0)$ exists when

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

5. Suppose f is a function with the property that $|f(x)| \leq x^2$ for every real number x.

   (a) Show that $f(0) = 0$.
   (b) Show that $f'(0) = 0$.