Finding the Central Spanning Tree in a graph

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***Abstract*— Consider a simple, undirected graph G.In this paper we propose algorithms to first find all the spanning trees in a graph and then find the central spanning tree in the graph.**

I. INTRODUCTION

An interconnection of points is known as Graphs , or more formally , A graph **G** is a set of **E** and **V** , where **E** denotes the set of edges and **V** denotes the set of Vertices . Here a vertex v represents a point or node and an edge e is a connection between two vertices. v_i and v_j are known as endpoints of that edge if an edge e_{ij} connects the vertices v_i and v_j.

A) Adjacency Matrix
The adjacency matrix, sometimes also called the connection matrix, of a simple labeled graph is a matrix with rows and columns labeled by graph vertices.If there exists an edge from v_i to v_j ,the corresponding entry in the adjacency matrix is 1,otherwise the entry is 0. For a simple graph with no self-loops, the adjacency matrix must have 0s on the diagonal. For an undirected graph, the adjacency matrix is symmetric.

B) Cycle in a graph
A cycle in a graph is a closed walk(alternating sequence of vertices and edges with no edge being repeated) in which each vertex except the terminal vertex appears once.A cycle is also known as a circuit.

C) Tree
A tree is a connected graph without any circuits.There is one and only one path between any two vertices in a tree.A tree with n vertices always has n-1 edges.

D) Spanning Tree
A tree T is said to be a spanning tree of a connected graph G if T is a subgraph of G and T contains all the vertices of G.

E) Branches
The edges of a graph which are also the edges of a spanning tree are said to be the branches of a graph with respect to that spanning tree.

F) Chords
The edges of a graph which are not a part of the edges of a spanning tree are said to be the chords of a graph with respect to that particular spanning tree.

G) Distance between two spanning trees in a graph
The distance between two spanning trees T_1 and T_2 is defined as the number of edges of G present in one tree but not in the other.The distance between two spanning trees in a graph is a metric.

H) Central Tree
For a spanning tree T_0, let max $d(T_0,T_i)$ denote the maximal distance between T_0 and any other spanning tree of the graph.Then T_0 is called the central tree of G if max $d(T_0,T_i) \leq$ max $d(T,T_j)$ for every other tree of the graph.

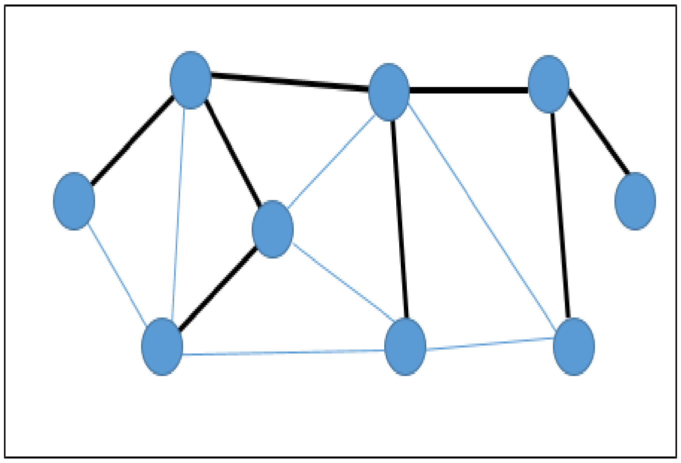


Fig. 1. Example of Spanning tres in a graph

II. MOTIVATION

Efficient polynomial time algorithms are well known for the minimum spanning tree problem. We use these algorithms to find one particular spanning tree and then use different approaches to list all possible spanning trees in a graph. Once we get all the possible spanning trees,we use these trees to find the central spanning tree in the graph.We discuss all the proposed algorithms below.

III. METHODS AND DESCRIPTION

We propose three approaches to find all the spanning trees in a graph.

A) **Cyclic Interchange Method**
We first need to find atleast one spanning tree in the graph and we will then build the other spanning trees on top of that. We know that Prim's algorithm exists to find the minimum spanning tree. Working of Prims Algorithm has been discussed in the proposed algorithms section of the paper.

To get all possible spanning trees in this particular graph, we do the following steps :-

- Make all the edges of the graph of the same weight.
- Use Prims Algorithm to get a spanning tree.
- Add any chord to the spanning tree.This forms a fundamental circuit in the tree.
- Remove any branch from the fundamental circuit.
- Repeat the process for all the different branches in the fundamental circuit.
- After generating all the trees with a particular chord in it, we can repeat the process for all the different chords

B) **Backtracking Method**
This proposed method uses the programming paradigm of backtracking.In order to decide whether or not to add an edge to a partially constructed spanning tree, the algorithm checks that the candidate edge:

- Has a higher numeric index than its predecessor
- Does not have both edges in the same connected component of the tree

The first condition prevents duplicates because the edges are always listed in ascending order. The second one ensures that the tree does not contain too many edges, because an edge is superfluous if both of its endpoints are in the same connected component.

C) **A Cut-Set based algorithm**
In this approach we first make all the edges of the graph of the same weight.We then use Prim's algorithm to generate a spanning tree T.Deleting e from T divides it into two connected components with vertex sets V1 and V2.We order the edges of a spanning tree in a particular manner and add them to a set.We the perform the following steps :-

- Pick a particular edge from the set of spanning tree edges.
- Create two set F and R with F containing all the edges before the particular edge in the set of spanning tree edges and R contains all the edges which have been removed from the original spanning tree so far including the picked edge.
- We then find the substitute of the particular edge from the cutset defined for this edge based on the original spanning tree.
- The substitute edge selected should not belong to the set F or R.
- We repeat this process recursively for all the spanning trees generated till no new trees can be generated.

We can use any of the algorithms suggested above to generate all possible spanning trees,Once the trees have been generated we can use algorithm 4 proposed in the proposed algorithms section to find the central spanning tree for the graph.The algorithm basically finds the tree that has the minimum maximum distance from any other spanning tree in the graph. .

IV. PROPOSED ALGORITHMS

Algorithm 1 Prims Algorithm

```
1: Create set **mstSet** to keep track of all vertices already included in the minimum spanning tree.
2: Assign a key value to all vertices in the input graph. Initialize all key values as INFINITE. Assign key value as 0 for the first vertex so that it is picked first.
3: while mstSet doesn't include all vertices do
     • Pick a vertex u which is not there in mstSet and has minimum key value.
     • Include u to mstSet.
     • Update key value of all adjacent vertices of u. To update the key values, iterate through all adjacent vertices. For every adjacent vertex v, if weight of edge u-v is less than the previous key value of v, update the key value as weight of u-v
4: end while
```

Algorithm 2 Generate all possible spanning tree using Backtracking

```
1: if number of vertices in the tree becomes equal to the vertices in the given graph then
2:     print the tree
3: end if
4: for all the edges e in the graph having the index number greater than the previous added edges do
5:
6:     if (the edge e is the first edge) or( the vertices in the edge e are in different connected component ) then
7:         add edge e to the tree
8:         recur for all possible configurations of adding different edges to the tree
9:     end if
10: end for
```

Algorithm 3 Cut-Set based algorithm

```
Comment: T is an (F, R)-admissible MST, which is written as $T = F\cup \{e^{k+1},...,e^{n-1}\}$, with $F = \{e^1,...,e^k\}$
1: for i=k+1,.....n-1 do
2:     Find Cut-Set Cut$e^i$
3:     Find if a substitute $e^{-i}$ exists for $e^i$ in Cut$e^i$
4: end for
5: for i=k+1,....n-1 if substitute exists do
6:     Set $T_i$ = $T\cup e^{-i}$
7: $e^i$ and output $T_i$
8:     F = $F\cup \{e^{k+1},.....,e^{i-1}$ and R = $R\cup \{e^i\}$
9:     Repeat Recursively
10: end for
```

Algorithm 4 Generate central spanning tree in the graph

```
1: Generate all spanning trees using cut-set based algorithm or backtracking algorithm
2: for each tree in spanning trees do
3:     d[i] = -1;
4: end for
5: d($t_i$, $t_j$) = number of non-common branches between two trees
6: for all trees $t_i$ in spanning trees do
7:     for all trees $t_j$ in spanning trees do
8:         d[i] = max(d[i], d($t_i$, $t_j$))
9:     end for
10: end for
11: Then the tree having Minimum of maximum of d($t_i$,$t_j$) for all trees is the central tree.
```

V. IMPLEMENTATION

We have used C++ in the backtracking approach to accept a graph from the user.We accept the graph in the form of an adjacency list.To implement the cut-set based algorithm as well as the final algorithm determining the central spanning tree, we have used python and have used matplotlib library to plot the graphs shown in the results.

VI. RESULTS

Github Repository Link : `https://github.com/piano-man/graph_theory.git`

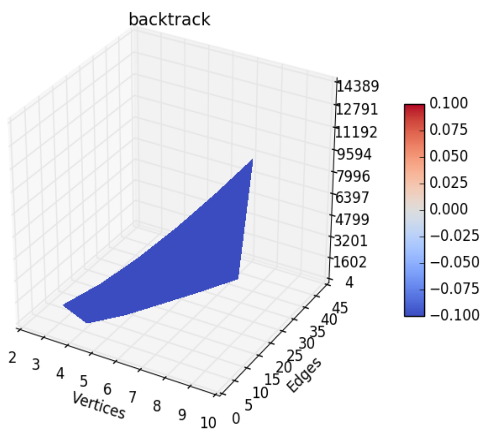


Fig. 2. Time vs vertices vs edges graph for backtracking algorithm)

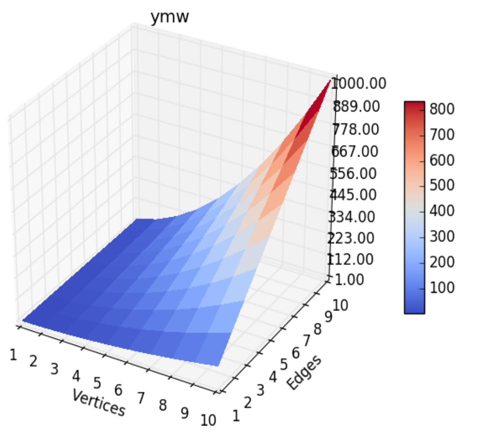


Fig. 3. Time vs vertices vs edges graph for Cut-Set based algorithm

Analysis and comparison of algorithms :-

The time complexity for the cut-set based algorithm code is $O(Nmn)$ where n, m and N stand for the number of nodes, edges and spanning trees, respectively and where Cut(e^i) can be found in $O(m)$ time and at each subproblem it's repeated atmost n times.Time complexity for the Backtracking algorithm is $O(E!)$ where E is the number of edges which is more than the cut-set based algorithm.

The time complexity of the algorithm used to generate the central spanning tree depends on the algorithm used to generate all spanning trees in a graph. If Cut-Set based algorithm is used, the complexity is $O(Nmn + (N^2 * nlog(n)))$.

If the backtracking algorithm is used, the complexity is $O(E! + (N^2 * nlog(n)))$.

In the above expressions,

- $O(nlog(n))$ is the complexity of the distance function which finds the number of edges present in one tree but not in the other.We first convert both the trees to set which takes $O(log(n))$ time and then find the intersection between the two formed sets which takes $O(nlog(n))$ time.
- $O(N^2)$ is the complexity of iterating through all the pairs of the spanning trees.

VII. APPLICATIONS

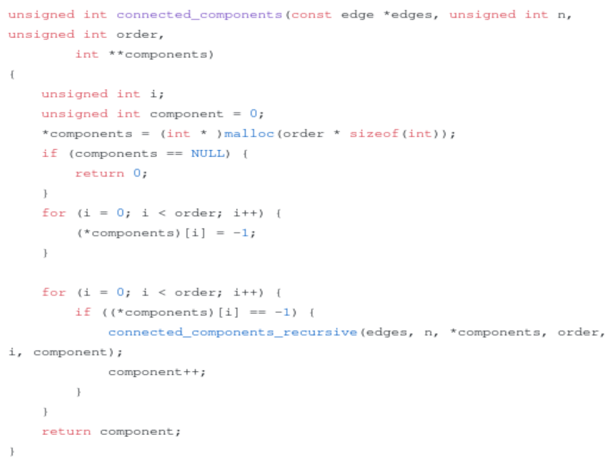
Central trees have widespread applications in the field of social networking where if every account is represented by a tree, central tree could represent an account that is most similar to a group of accounts.

It could also be used to find an account with a maximum number of common friends with a given set of accounts.

VIII. DISCUSSION

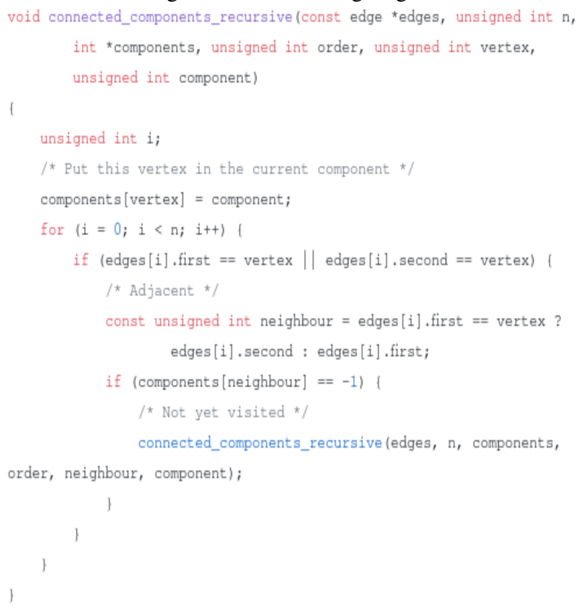
In this paper we used the given algorithms to find all the spanning trees in a graph and then used these algorithms to find the central spanning tree in a graph.The cut-set algorithm is popularly known as the ymw algorithm as it was proposed by Yamada,Katakoa and Watanabe.The algorithm traditionally lists all the minimum spanning trees in a graph but because we make all the edges of the same weight, it effectively generates all the spanning trees.

IX. CODE SNIPPETS



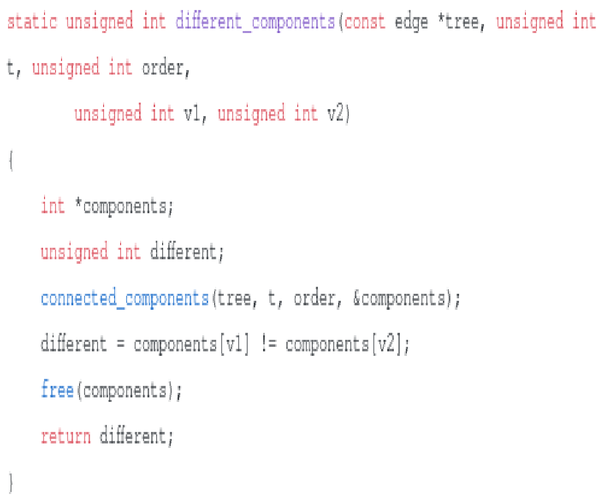
```
unsigned int connected_components(const edge *edges, unsigned int n,
unsigned int order,
        int **components)
{
    unsigned int i;
    unsigned int component = 0;
    *components = (int * )malloc(order * sizeof(int));
    if (components == NULL) {
        return 0;
    }
    for (i = 0; i < order; i++) {
        (*components)[i] = -1;
    }

    for (i = 0; i < order; i++) {
        if ((*components)[i] == -1) {
            connected_components_recursive(edges, n, *components, order,
i, component);
            component++;
        }
    }
    return component;
}
```

Fig. 4. Backtracking Algorithm



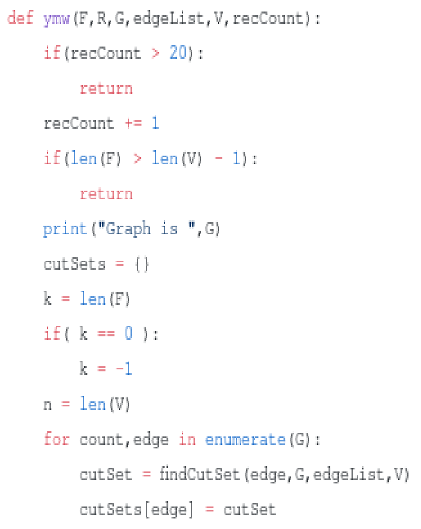
```
void connected_components_recursive(const edge *edges, unsigned int n,
        int *components, unsigned int order, unsigned int vertex,
        unsigned int component)
{
    unsigned int i;
    /* Put this vertex in the current component */
    components[vertex] = component;
    for (i = 0; i < n; i++) {
        if (edges[i].first == vertex || edges[i].second == vertex) {
            /* Adjacent */
            const unsigned int neighbour = edges[i].first == vertex ?
                    edges[i].second : edges[i].first;
            if (components[neighbour] == -1) {
                /* Not yet visited */
                connected_components_recursive(edges, n, components,
order, neighbour, component);
            }
        }
    }
}
```

Fig. 5. Backtracking Algorithm



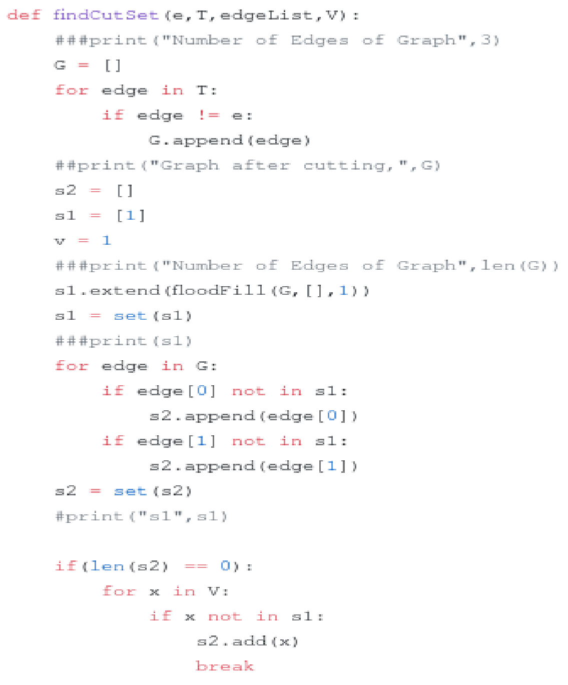
```
static unsigned int different_components(const edge *tree, unsigned int
t, unsigned int order,
        unsigned int v1, unsigned int v2)
{
    int *components;
    unsigned int different;
    connected_components(tree, t, order, &components);
    different = components[v1] != components[v2];
    free(components);
    return different;
}
```

Fig. 6. Backtracking Algorithm



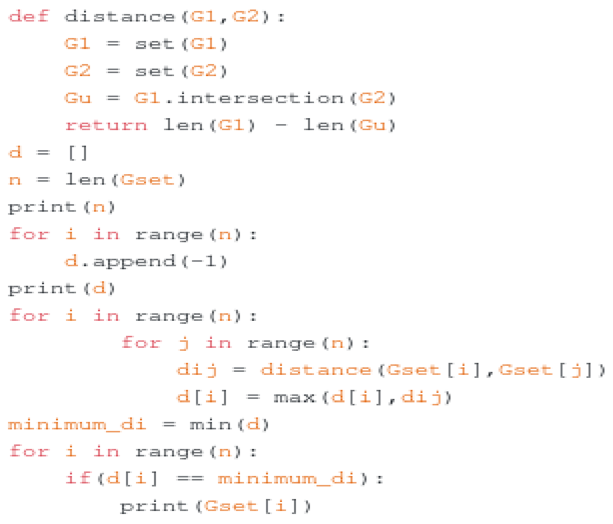
```
def ymw(F,R,G,edgeList,V,recCount):
    if(recCount > 20):
        return
    recCount += 1
    if(len(F) > len(V) - 1):
        return
    print("Graph is ",G)
    cutSets = {}
    k = len(F)
    if( k == 0 ):
        k = -1
    n = len(V)
    for count,edge in enumerate(G):
        cutSet = findCutSet(edge,G,edgeList,V)
        cutSets[edge] = cutSet
```

Fig. 7. Cut-Set based Algorithm



```
def findCutSet(e,T,edgeList,V):
    ###print("Number of Edges of Graph",3)
    G = []
    for edge in T:
        if edge != e:
            G.append(edge)
    ##print("Graph after cutting,",G)
    s2 = []
    s1 = [1]
    v = 1
    ###print("Number of Edges of Graph",len(G))
    s1.extend(floodFill(G,[],1))
    s1 = set(s1)
    ###print(s1)
    for edge in G:
        if edge[0] not in s1:
            s2.append(edge[0])
        if edge[1] not in s1:
            s2.append(edge[1])
    s2 = set(s2)
    #print("s1",s1)

    if(len(s2) == 0):
        for x in V:
            if x not in s1:
                s2.add(x)
                break
```

Fig. 8. Cut-Set based Algorithm



```
def distance(G1,G2):
    G1 = set(G1)
    G2 = set(G2)
    Gu = G1.intersection(G2)
    return len(G1) - len(Gu)
d = []
n = len(Gset)
print(n)
for i in range(n):
    d.append(-1)
print(d)
for i in range(n):
        for j in range(n):
            dij = distance(Gset[i],Gset[j])
            d[i] = max(d[i],dij)
minimum_di = min(d)
for i in range(n):
    if(d[i] == minimum_di):
        print(Gset[i])
```

Fig. 9. Finding the central tree

X. CONCLUSIONS

We can find all the spanning trees in a graph using the methods suggested in the paper and then use these methods to find the central spanning tree in a graph.

REFERENCES

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