Optimizing Crane Selection and Location for Multistage Construction Using a Four-Dimensional Set Cover Approach

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Abstract: Crane selection and location planning exists as part of a series of critical decisions faced by construction managers. As today's construction projects grow increasingly complex and complicated construction schemes involving multistaged construction layouts become more common, the problem of crane selection and location planning becomes even more critical to achieving successful project completion. Despite this, current methods do not consider the impact of multiple stages. This paper first shows that neglecting the impact of multiple construction stages leads to suboptimal crane deployment costs. To overcome this, the problem of crane selection and location planning is recast as a four-dimensional set cover problem (4D-SCP). The proposed model is able to provide better solutions when multiple stages are considered, as well as providing a tool to analyze construction methods and work sequences from the perspective of total crane deployment costs. A case study of an academic building with an adjoining workshop is used to demonstrate these advantages. **DOI: 10.1061/(ASCE)CO.1943-7862.0001318.** *© 2017 American Society of Civil Engineers.*

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Introduction

Tower cranes are typically the single biggest equipment investment deployed on a construction site (Al-Hussein et al. 2006), and these represent an average 36% of the total project procurement cost (Yeo and Ning 2006). Their proper selection and location on site is part of a series of critical decisions faced by construction managers, and often determines the success of a project. This decision depends upon several parameters like site conditions, governing regulations, and crane characteristics like type, capacity, height, reach, and cost (Warszawski 1990). If these parameters are not properly accounted for, they can result in collisions and interference between adjacent cranes and built facilities (Yeoh et al. 2016), resulting in various site problems including increased safety hazards, schedule delays, and cost overruns (Shapira and Simcha 2009).

As today's construction projects grow increasingly complex, complicated construction schemes involving phased or multistaged construction site layouts are becoming more common. During planning, managers decompose a complex project into several work zones and allocate specific phases to each zone. This results in a dynamic site model where construction activities change as the project progresses. Such a project decomposition is referred to as construction staging. In view of this, the problem of crane selection and location planning becomes even more critical to achieving successful project completion.

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The literature survey conducted identified that construction staging has not been adequately addressed. Hence, the objective of this paper is to investigate the impact of multiple construction staging on the deployment costs associated with crane selection and layout planning. A hypothesis that disregarding construction stages leads to suboptimal crane selection and layout is put forward. To prove this, an illustrative case is first presented to demonstrate why the current technique of using minimum cost-weighted hook traversal time is not optimal when considering multiple construction stages. To overcome this, the problem of crane selection and layout planning is recast as a four-dimensional set cover problem (4D-SCP). A case study is carried out to demonstrate the real world applicability of the 4D-SCP. Results of the case study are subsequently analyzed to show the cost impact of multiple construction stages. The 4D-SCP allows for different construction methods and work sequences to be analyzed from the total crane-deployment-cost perspective. Finally, the paper concludes with a discussion on limitations and future work.

Literature Survey on Current Tower Crane Selection and Location Planning Models

A literature survey of current tower crane selection and location models is carried out to identify gaps, particularly the existence of papers that adequately addressed the issue of multiple staging on crane selection and location. Mobile cranes have not been included as their operation differs significantly from that of static cranes.

Several different approaches for modeling and solving the problem of crane selection and location planning exist, of which the most popular are simulation and mathematical optimization. Specifically, this literature survey covers the relevant optimization-based approaches, focusing in particular on the objective functions used in defining the problem. Table 1 presents a chronological summary of the surveyed papers and also indicates if the paper specifically deals with single or multiple tower cranes.

Table 1. Survey of Tower Crane Selection and Location Papers

Author (year)	Objective function	Scope
Furusaka and Gray (1984)	Minimize total crane rental cost (including assembly and dismantling)	Multiple cranes
Zhang et al. (1996)	Average hook travel time	Single crane
Zhang et al. (1999)	Multicriteria based on average hook traversal time, number of conflicts, and balanced workload	Multiple crane
Tam et al. (2001)	Minimize cost-weighted hook traversal time	Single crane
Tam and Tong (2003)	Minimize cost-weighted hook traversal time	Single crane
Huang et al. (2011)	Minimize cost-weighted hook traversal time	Single crane
Irizarry and Karan (2012)	Minimize conflict index	Multiple cranes
Lien and Cheng (2014)	Minimize cost-weighted hook traversal time (including labor, assembly, and dismantling)	Multiple cranes
Wang et al. (2015)	Minimize cost-weighted traversal time	Multiple cranes
Marzouk and Abubakr (2016)	Minimize total crane cost and rails rental rate	Multiple cranes

Furusaka and Gray (1984) developed a dynamic programming model based on total crane coverage to solve crane-selection and layout-planning problems, wherein the objective was to minimize the total crane rental cost. One key insight of their approach was the least-cost crane option may change at various floors in a project, which contradicts the normal strategy of allocating the same crane from the first to top floors of a project. However, a major limitation is the model only applies to regular-shaped buildings (Gray and Little 1985).

Mixed-integer linear programming (MILP) models based on minimizing hook travel time for single (Zhang et al. 1996) and multiple (Zhang et al. 1999) tower cranes are another class of mathematical models. Various researchers have adapted this model because of its ability to incorporate crane data as mathematical constraints, making the model more applicable in the real world. This need arose from the observation that slewing and trolley velocities of the crane have an impact on the crane location, which is in turn an important factor to consider during crane operations.

Tam et al. (2001) further incorporated supply locations into the MILP problem. To address the performance issues arising from the added complexity, a genetic algorithm was introduced. Huang et al. (2011) also introduced logistical constraints in the form of homogeneous and nonhomogeneous material flows in their formulation. In both instances, minimization of a cost-weighted hook traversal time was used as the objective function, and the models only addressed single tower crane problems.

Lien and Cheng (2014) adopted a similar model and objective function to Huang et al. (2011), but varied their solution approach by using the particle bee algorithm. They also extended Huang's model from a single tower crane to a predefined number of tower cranes. Wang et al. (2015) incorporated the use of building information models (BIM) and firefly algorithm to solve the crane selection and layout planning problem using the minimum cost-weighted hook traversal time. Marzouk and Abubakr (2016) further considered a mounted tower crane on rails, with the objective of minimizing total crane cost with rail rental. Irizarry and Karan (2012) incorporated the use of geographic information systems (GIS) with BIM, and developed an alternative optimization model where the objective function is a conflict index based on the spatial interferences between crane work zones and facility area.

This survey reveals several gaps and insights in the literature. Firstly, optimization by considering the minimum cost-weighted hook traversal time is the most common model for crane-selection and layout-planning problems, with 7 out of 10 papers surveyed employing this. Secondly, multiple crane selection and location using the minimum cost-weighted hook traversal time model is typically dealt with by predefining the number of cranes deployed, and incrementing this number to determine the mostcost-effective solution. Lastly, all reviewed papers have not considered construction that occurs over multiple stages.

This paper addresses these identified gaps by proposing an alternative model to the minimum cost-weighted hook traversal time model. The authors argue that disregarding schedule information in the form of construction staging may lead to suboptimal crane selection and deployment cost. To demonstrate this, the existing minimum cost-weighted hook traversal time model is adapted so that multiple cranes can be incorporated. Using this model, its shortcomings are demonstrated in the following section.

Illustrative Case: Issues with the Minimum Cost-Weighted Hook Traversal Time Model for Crane Selection and Layout Planning

A simple illustrative example is used. A 20-story 60-m-tall point block with an adjacent 4-story, 12-m car park is to be constructed. The point block is designated as Zone B, while the car park is Zone A. Both blocks have the same footprint of 40 × 40 m (Fig. 1). Zone B contains 25 demand points (D1–D25), while Zone A has 20 demand points (D26–D45), where each demand point is a prefabricated column requiring crane service. Several possible crane locations (L1–L8) and potential supply points (J1–J7) are located around the perimeter of the building. Fig. 1 displays this information at the ground story with individual coordinates shown in meters.

A list of tower cranes are available for use in the project (Table 2). The list contains the various operating characteristics of the tower crane, including its tipload (k_{TL}), maximum capacity (k_{Cap}), maximum radius (k_R), maximum load moment (k_{LM}), height-under-hook (k_{Ht}), slewing (V_ω), trolleying (V_r), and hoisting (V_z) velocities. The operating cost (c) includes the weekly rental and labor rates, while the fixed cost (fc) comprises the assembly, transportation, and dismantling costs. The costs were estimated through interviews with a major crane supplier.

Several schedule based assumptions are made. First, the productivity of each zone is 1 floor per week. Second, this productivity is independent of the crane layout and assumed to be constant. In other words, the duration of each stage is not dependent upon the number of crane resources deployed. Third, the capacity of supply points is not considered, implying the supply points are able to hold any number of construction elements, e.g., prefabricated beams or columns. Last, a construction method is defined over two stages for the blocks. In the first stage, both zones will be carried out simultaneously until the completion of Zone A. Thereafter, work will continue in Zone B until the completion of the project.

In general, there are two possible approaches to evaluate the crane deployment:

Approach 1: Incorporate the staging information and decompose the works into two stages as per the construction method de-

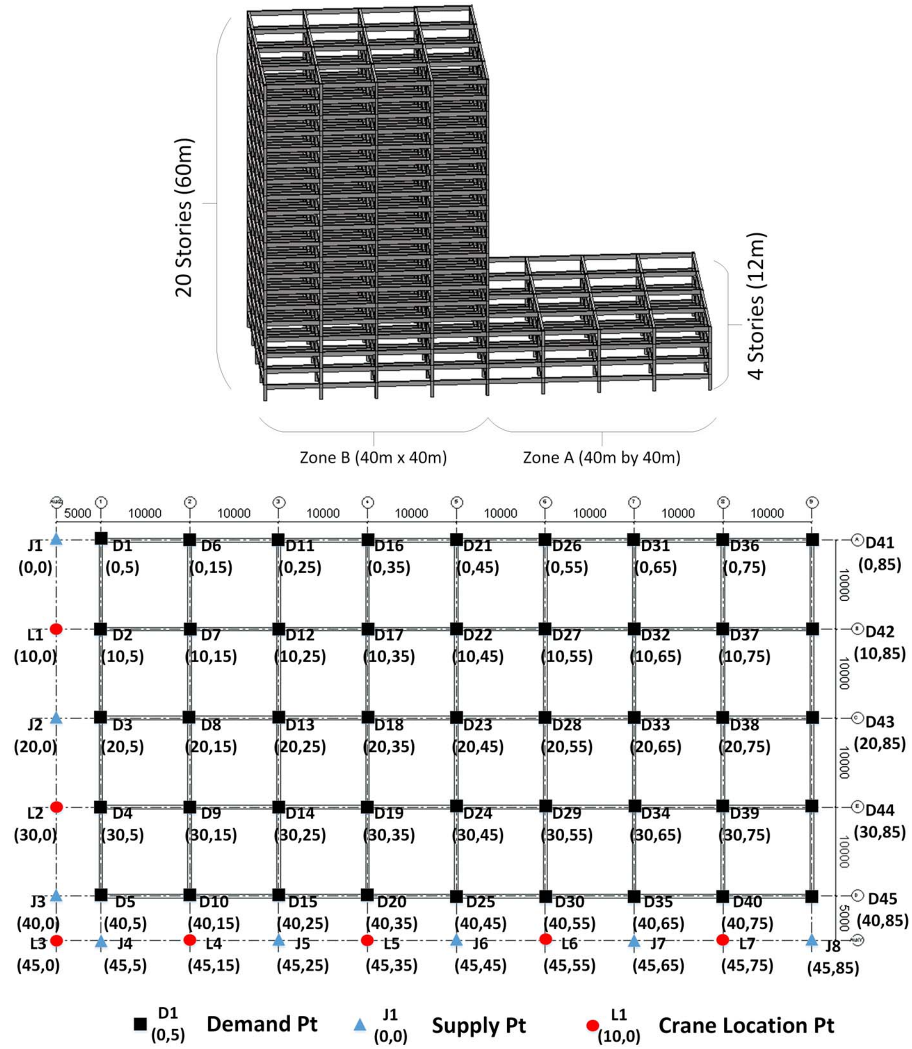

Fig. 1. Illustrative case layout showing zones, heights, demand, supply, and crane locations

fined earlier. Based on the scope of work, Stage I will take 8 weeks to complete, while Stage II will take 16 weeks.

Approach 2: Ignore the staging information, and proceed with both Zones A and B in a single phase. This approach is the traditional approach and is more commonly used in practice (Furusaka and Gray 1984; Chew 2012). In this approach, the combined stage takes 24 weeks.

To determine the impact of multiple stages on the minimum cost-weighted hook traversal time model, the original MILP model from Huang et al. (2011) and Lien and Cheng (2014) was adapted to optimize over multiple cranes. In this model, the decision variables are q_{ijlk} and χ_{lk}, where q_{ijlk} is a binary assignment variable indicating that a demand point i to a supply point j is served by a crane k at location l [Eq. (1)]. χ_{lk} is another binary variable where crane k is located at l [Eq. (2)]

$$q_{ijlk} = \begin{cases} 1 & \text{if demand } i \text{ from supply } j \text{ is served by crane } k \text{ at location } l \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$\chi_{lk} = \begin{cases} 1 & \text{if crane } k \text{ at location } l \text{ is assigned} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The objective of the MILP model [Eq. (3)] is to minimize the cost-weighted operation time, which comprises the fixed cost fc_k and operating cost c_k. The cost-weighted operation time is dependent upon (1) T_{ijlk}, the traversal time of the crane from supply point j to demand point i, and (2) $\overline{c_k}$, the average operating cost of the crane per minute; $\overline{c_k}$ is obtained by dividing the operating cost c_k in Table 2 by an assumed 2,400 min per week

$$\min Z = \sum_i \sum_j \sum_l \sum_k q_{ijlk} \times T_{ijlk} \times \overline{c_k} + \sum_l \sum_k \chi_{lk} \times fc_k \tag{3}$$

The model is further subject to the following constraints. Eq. (4) ensures that every demand point i is assigned to a supply point j by a crane k situated at location l. Eqs. (5)–(7) are used to define the relationship between χ_{lk} and q_{ijlk} such that if any q_{ijlk} takes a value of 1 for any lk, then χ_{lk} must also equal 1. In other words, if an assignment has been made which involves k and l, then the crane k at position l must be selected. M is an arbitrarily big number that is used in conjunction with the auxiliary binary variables $\delta 1_{lk}$ and $\delta 2_{lk}$ to ensure this.

The model is further constrained such that every location l is allowed at most one crane k [Eq. (8)]. An upper limit to the number of cranes N_{cranes} that are available for the site is also defined

Table 2. Types of Cranes

Index	Tipload, k_{TL} (kg)	Maximum capacity, k_{Cap} (kg)	Maximum radius, k_R (m)	Maximum load moment, k_{LM} (m·kg)	Height-under-hook, k_{Ht} (m)	Slewing speed, V_ω (ms^{-1})	Trolleying speed, V_r (ms^{-1})	Hoisting speed, V_z (ms^{-1})	Operating cost, c (\$/week)	Fixed cost, fc (\$)
k1	1,100	2,500	40	50,000	48.6	3.77	40	56	7,000	50,000
k2	1,200	3,000	40	60,000	48.6	3.77	40	48	7,800	50,000
k3	1,100	3,600	45	72,000	48.6	3.77	40	48	8,200	50,000
k4	1,050	5,000	50	76,000	60.1	3.77	45	48	8,500	50,000
k5	1,300	5,000	50	75,000	62	3.77	30	33	8,700	55,000
k6	1,350	3,800	55	95,000	50.1	3.77	60	47	9,200	55,000
k7	1,150	6,000	60	88,200	44	3.77	40	42	9,400	60,000
k8	1,900	5,900	55	146,320	50.1	3.77	60	47	10,300	60,000
k9	1,400	8,000	60	108,800	44.9	4.40	60	58	10,000	65,000
k10	1,800	7,800	60	191,100	65.3	3.77	60	62	10,900	65,000
k11	2,400	10,000	60	166,000	59.7	3.77	50	68	11,200	70,000
k12	1,900	10,000	65	170,000	63.5	3.77	50	73	11,500	75,000
k13	2,050	10,000	75	160,000	64.7	3.77	75	68	12,000	75,000
k14	2,600	12,000	65	180,000	55	4.40	70	79	11,900	75,000
k15	3,200	12,000	70	256,800	57.5	3.46	80	80	15,900	80,000
k16	2,700	16,000	70	257,600	61.9	3.46	80	106	16,500	80,000
k17	2,900	12,000	75	240,000	69.2	3.77	100	110	15,100	80,000
k18	3,200	16,000	75	320,000	61	3.77	80	80	19,700	90,000
k19	2,600	16,000	75	288,000	93.7	3.77	76	80	18,100	90,000
k20	2,700	20,000	75	290,000	64.9	3.46	86	100	18,800	90,000
k21	3,300	16,000	75	272,000	90.7	3.77	80	90	17,300	90,000
k22	3,100	20,000	80	346,000	84.1	4.40	90	92	21,700	100,000
k23	5,400	25,000	80	522,500	66.1	4.40	100	101	31,300	110,000
k24	4,600	40,000	80	528,000	81.5	3.46	67	93	33,600	110,000

[Eq. (9)]. In this illustrative case, $N_{\text{cranes}} = 5$ is arbitrarily defined, as it is expected the optimization solution will have fewer than 5 cranes

$$\sum_j \sum_l \sum_k q_{ijlk} \geq 1 \quad \forall\ i \in I \tag{4}$$

$$\sum_i \sum_j q_{ijlk} - M \times \delta 1_{lk} \leq 0 \quad \forall\ l, k \in L, K \tag{5}$$

$$\chi_{lk} + M \times (1 - \delta 2_{lk}) \geq 1 \quad \forall\ l, k \in L, K \tag{6}$$

$$\delta 1_{lk} - \delta 2_{lk} \leq 0 \quad \forall\ l, k \in L, K \tag{7}$$

$$\sum_k \chi_{lk} \leq 1 \quad \forall\ l \in L \tag{8}$$

$$\sum_l \sum_k \chi_{lk} \leq N_{\text{cranes}} \tag{9}$$

The hook traversal time is dependent upon the crane parameters, as well as the distances of the demand and supply points from the crane location. Fig. 2 shows the physical relationships between these points, as well as the radial and tangential movements of the crane jib from supply to demand points. These relationships may be expressed using the Eqs. (16)–(18) that follow. Eqs. (12)–(14) refer to the time taken for hoisting, trolleying, and slewing, respectively, while Eq. (11) represents the combined horizontal traversal time of the crane hook.

Eq. (10) is the combined hook traversal time, including the consideration of hoisting time. If D_i and S_j are not situated within the crane radius k_{Ht}, or D_i^Z and S_j^Z are not below the height-under-hook of the crane k_{Ht}, then an arbitrarily large number M is assigned. The implication is the demand and supply points are not physically reachable by the crane, and the computed traversal time is consequently penalized. α and β refer to user-defined constants, where α is the degree of coordination of hook movement in radial and tangential directions in the horizontal plane, and β is the degree of coordination of hook movement in vertical and horizontal planes. For comparative purposes in this illustrative case, both α and β are assumed to be 0, indicating complete hook coordination

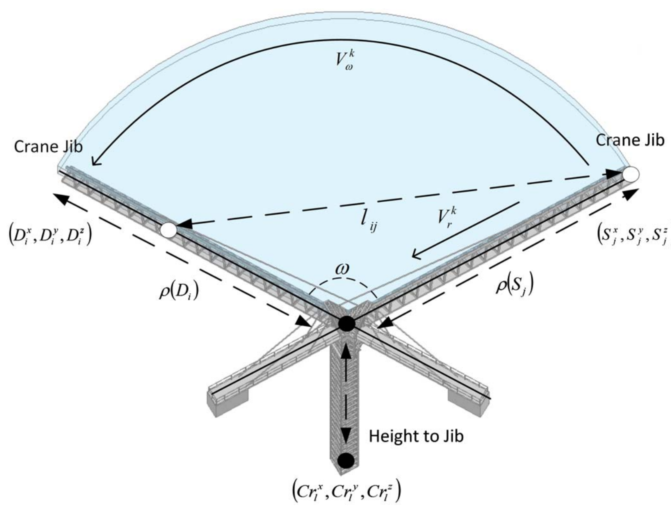

Fig. 2. Three-dimensional crane geometries showing radial and tangential hook movement

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$$T_{ijlk} = \begin{cases} M & \text{if}\, \rho(D_i), \rho(S_j) > k_R \text{ or } D_i^Z, S_j^Z > k_{Ht} \\ \max(T_{lk}^h, T_{lk}^z) + \beta \times \min(T_{lk}^h, T_{lk}^z) & \text{otherwise} \end{cases} \tag{10}$$

$$T_{lk}^h = \max(T_{lk}^r, T_{lk}^\omega) + \alpha \times \min(T_{lk}^r, T_{lk}^\omega) \tag{11}$$

$$T_{lk}^z = \frac{|D_i^Z - S_j^Z|}{V_z^k} \tag{12}$$

$$T_{lk}^r = \frac{|\rho(D_i) - \rho(S_j)|}{V_r^k} \tag{13}$$

$$T_{lk}^\omega = \frac{1}{V_\omega^k} \times \arccos\left[\frac{l_{ij}^2 - \rho(D_i)^2 - \rho(S_j)^2}{2 \cdot \rho(D_i)\rho(S_j)}\right] \tag{14}$$

$$0 \le \arccos\theta \le \pi \tag{15}$$

$$\rho(D_i) = \sqrt{(D_i^x - Cr_l^x)^2 + (D_i^y - Cr_l^y)^2} \tag{16}$$

$$\rho(S_j) = \sqrt{(S_j^x - Cr_l^x)^2 + (S_j^y - Cr_l^y)^2} \tag{17}$$

$$l_{ij} = \sqrt{(D_i^x - S_j^x)^2 + (D_i^y - S_j^y)^2} \tag{18}$$

This MILP model obtains a solution that represents the optimal crane locations and types for a particular construction stage. This solution best supports the crane lifting operation within this stage. However during crane selection and location planning, total crane deployment cost is typically the foremost consideration in the construction manager's decision-making process, after safety. To obtain the total crane deployment cost, the MILP model is repeated for every stage. The operating and fixed costs of all the cranes deployed in each stage, k^{st} are summed according to Eq. (19); $c_{k^{st}}$ and $fc_{k^{st}}$ are the operating costs and fixed costs of Crane k deployed in stage st respectively, while Dur_{st} is the duration of stage st. In the context of this illustrative case, Dur_{st} is in weeks

$$\text{Total crane deployment cost} = \sum_{st}\left[\sum_{k^{st}}(c_{k^{st}} \times Dur_{st} + fc_{k^{st}})\right] \tag{19}$$

Table 3 summarizes the results of applying the MILP model to Approaches 1 and 2. The models were solved using *Gurobi Optimizer*. The MILP model is applied to Stages I and II of Approach 1 separately, and the total crane deployment cost is the sum of crane costs of both stages as per Eq. (19). Similarly, the model is applied to Approach 2, which consists of all demand points from Zones A and B in a single stage.

The optimal solution for Approach 1 consists of deploying Crane k13 at Location L5 in Stage I, followed by Crane k4 at the same location in Stage II. The combined cost is $357,000. On the other hand, the optimal solution to Approach 2 consists of deploying a single k12 crane at Location L6 for a total cost of $351,000. Interestingly, Approach 1 Stage I and Approach 2 do not produce the same crane layout. This is due to the faster trolleying speed of Crane k13, while Crane k12 benefits vertical construction by providing a greater hoisting speed. In this illustrative case, the traditional approach (Approach 2) leads to better results in spite of the addition of construction staging information to Approach 1. This is because the marginal benefits of using the smaller k4 crane as compared to the k12 crane does not outweigh the additional fixed cost from k4.

A more-intuitive solution to this illustrative case is to dedicate individual cranes to each zone. One suggestion is to deploy Crane k4 at Location L5 for Zone B, while deploying another k4 crane at Location L7 for Zone A. The crane at Location L7 may be dismantled immediately after completing the works at Zone A, while the crane at Location L5 will be allowed to span both stages. The total crane deployment cost is computed as 20 weeks × $8,500× 4 weeks × $8,500 + $50,000 + $50,000 = $304,000. The improvement in this solution arises because the crane at Location L5 is a shared crane resource that is allowed to span both stages.

The preceding illustrative case demonstrates the minimum cost-weighted hook traversal time model is not always adequate when staging information is incorporated. The major drawback of this method is that the model solves each stage in isolation, and does not leverage on information of crane types and location in the prior stages on future stages. This consequently results in suboptimal deployment cost solutions as the cranes are unable to be shared across stages. The research challenge is thus to devise a method of adequately incorporating the staging information to optimize the total crane deployment cost and identify opportunities where crane resources can be shared across multiple construction stages in a cost-effective manner.

Proposed Approach to Solve the Crane Selection and Layout Problem with Multiple Construction Stages: 4D-SCP

To overcome the identified research challenge from the illustrative case, it is proposed that the crane selection and layout problem with multiple construction stages be modeled as a four-dimensional cost-weighted set cover problem 4D-SCP.

Table 3. Cost Comparison of Approaches 1 and 2

Comparison	Approach 1, Stage I	Approach 1, Stage II	Approach 2
Solution (crane type and location)	Crane k13 at Location L5	Crane k4 at Location L5	Crane k12 at Location L6
Operating cost	4 weeks × 2 zones × $12,000	16 weeks × $8,500	24 weeks × $11,500
Fixed cost	$75,000	$50,000	$75,000
Subtotal cost	$186,000	$171,000	$351,000
Total deployment cost	Stage I + Stage II: $357,000	Stage I + Stage II: $357,000	$351,000

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Overview of Set Cover Problems

The set cover problem is a classical combinatorial problem, which is well known to be **NP**-complete. In general terms, given a universe U of n elements, and a collection of $S_1, S_2, \ldots, S_m \subseteq U$ where every set S_i has an associated cost $Cost_{S_i}$, a set cover can be defined as a collection of S_i whose union is the entire universe U. The objective is to find the minimum-cost set cover.

Extension of the Set Cover Problem to Handle Multistage Crane Selection and Layout

To understand how the set cover problem may be employed, several analogies from the previous section to the crane-selection and layout-planning problem are drawn on. A two-dimensional (2D) generalization of the illustrative case is used as a starting point for discussion as shown in Fig. 3. In this figure, the universe U refers to all the demand points (marked with black dots). The set S_i is analogous to the physical representation of a crane. More specifically, S_i refers to a single crane of type k situated at a location l. In Fig. 3, two sets, A and B, are shown as circles, which are used to represent Cranes k4 at Locations L5 and L6, respectively. The radius of each circle physically represents the crane radius. A demand point is defined as being covered by a set if the coordinate of the demand point falls within the radius of the set, given its location. Hence, in this figure, the demand points in Zone B are covered by Set B because all the points are within the radius of the crane representing Set B. Similarly, it can be concluded that the demand points in Zone A are covered by Set A. Several points exist that are covered by both Zones A and B; these are points situated in the overlap of both circles. In addition, the cost of each set $Cost_{S_i}$ is analogous to the corresponding crane cost, while the objective of finding the minimum-cost set cover naturally translates into finding the minimum total crane deployment cost.

As an extension of the original set cover problem, time information in the form of construction stages is added to the demand points and crane location points. Adding this information transforms the 2D problem to a three-dimensional (3D) problem, where the third dimension considered is time. Again, the illustrative case is used to demonstrate this transformation (Fig. 3). In the transformed problem, a demand point can be defined by the following coordinate information: (X, Y, Stage). A set is now defined as a crane k at a location l that spans a time period. This time period may consist of a stage or a series of stages. For example, the illustrative case occurs over two stages. The first stage involves Zone A and B over Weeks 0 to 4. Thereafter, the second stage involves Zone B only from Weeks 4 to 20. The time period of Set A can then be defined from Weeks 0 to 4, while the time period of Set B spans both stages from Weeks 0 to 20.

Hence, a demand point is covered by a set if it is situated within the radius of the crane and the stage of the demand point falls within time period spanned by the set. From here, the inclusion of the z-axis (height) is trivial, and the 3D model is easily extended to a 4D-SCP. In this context, a demand point is considered to be covered by a set if it is situated within the radius of the crane, its location is beneath the height-under-hook of the crane, and its stage falls within the time period spanned of the crane. Additional rules exist to further determine if a demand point is covered by a set, and these will be discussed later in the subsequent subsection.

Information Input

The information requirements for using 4D-SCP can be classified into four categories: crane, location points, demand points, and supply points. The required crane information is stored in a table, similar to Table 2. The attributes in this table describe the operating characteristics of the crane, and most may be obtained from crane equipment catalogs.

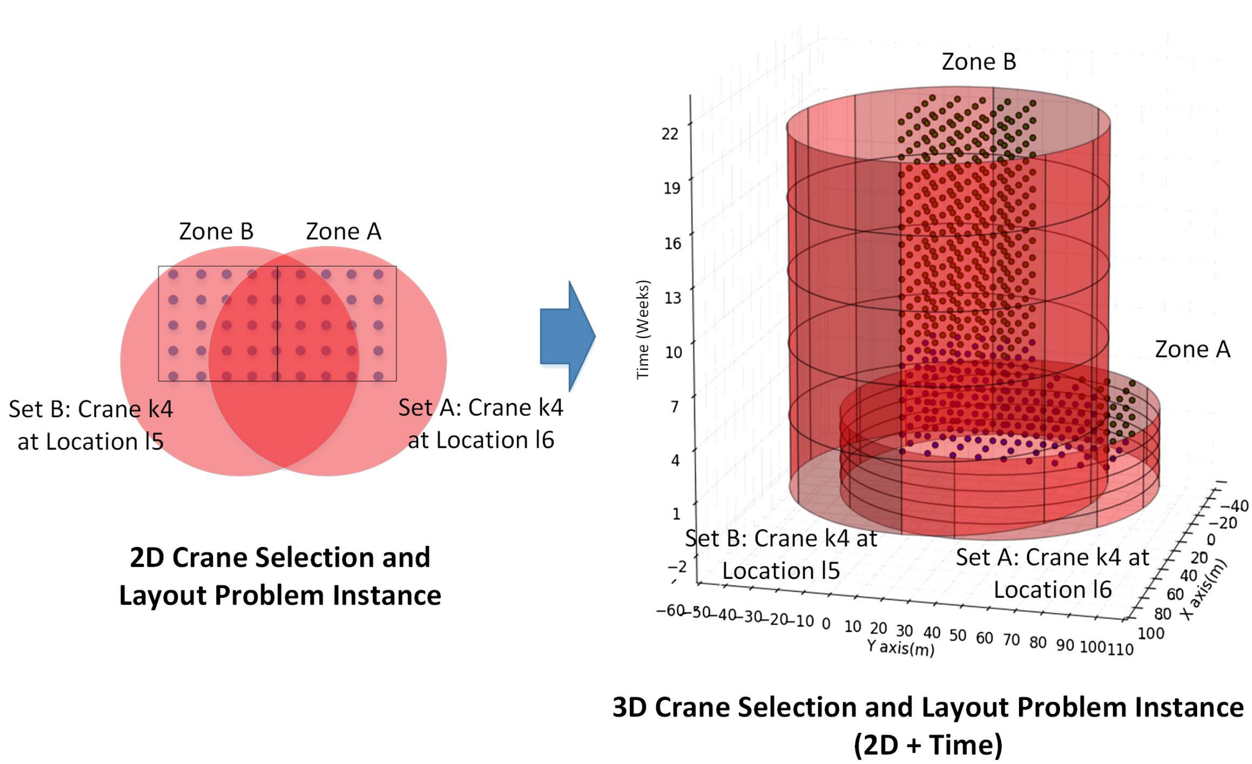

Fig. 3. Applying the set cover problem from 2D perspective to 3D perspective

The location-point and supply-point information consists of possible crane locations and supply locations on site, and are represented as 3D coordinates with time or stage information. In practice, location points are often situated close to the structural elements, particularly if the building height exceeds the allowable freestanding height of the crane. This is to allow for tie-backs to the structure to be constructed. Additionally, the ground conditions at possible locations must also be adequate to sustain the load from the crane. Supply points are typically the coordinates of the pickup points in staging areas and would ideally be located close to the access. The 3D coordinates of these supply and location points may be extracted from BIM if they have been explicitly modeled within, or else inferred from a visual inspection of computer-aided design (CAD) site layout plans.

The demand information consists of 3D coordinates, weight, and stage information. The coordinates may also be extracted from BIM, where they are assumed to be the centroids of the bounding box of the prefabricated construction element. Weights are also estimated by extracting volume from BIM and multiplying by the effective density of the material used, accounting for acceleration due to gravity. In this paper, all models were created in *Autodesk Revit*, and the application programming interface (API) was used to filter, query, and extract the relevant information.

Formulating 4D-SCP as a Mixed-Integer Linear Programming Model

Solving the 4D-SCP problem involves a series of steps as shown in Fig. 4. The first step generates a list of decision variables that enumerates the possible combinations of cranes k, locations l, and stages st. For example, two cranes are available at three possible locations, and construction may take place over three stages, $\{I, II, III\}$. The possible combinations of the three stages is computed as the powerset P of the three stages: *{{I}, {II}, {III}, {I, II}, {I, III}, {II, III}, {I, II, III}}*. The cardinality of P can be computed as $|P| = (2^3 - 1) = 7$. In this paper, a period p refers to one of these possible elements in the powerset P. The implication of the powerset operation is that the crane may occupy any one of the seven available stage configurations. Hence, the total number of possible combinations in the list can be computed as 2 cranes $\times$ 3 locations $\times\, |P| = 42$. Each combination is now encoded as a binary decision variable x_{klp} [Eq. (20)], and stored in a list. When x_{klp} takes the value 1, this indicates that a crane k at location l spanning period p has been assigned. While the powerset operation is exponential, in practice the number of stages in a construction project is usually small

$$x_{klp} = \begin{cases} 1 & \text{if crane } k \text{ at location } l \text{ spans period } p \text{ is assigned} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

The second step (Fig. 4) is a preprocessing step to remove infeasible combinations from the list generated in Step 1. An infeasible combination arises from the lack of reachability between the cranes and the supply points. Each combination is checked that it covers a supply point, by ensuring the supply point falls within the radius of crane k at location l. Combinations that fail this check are deemed infeasible and eliminated from the list of decision variables.

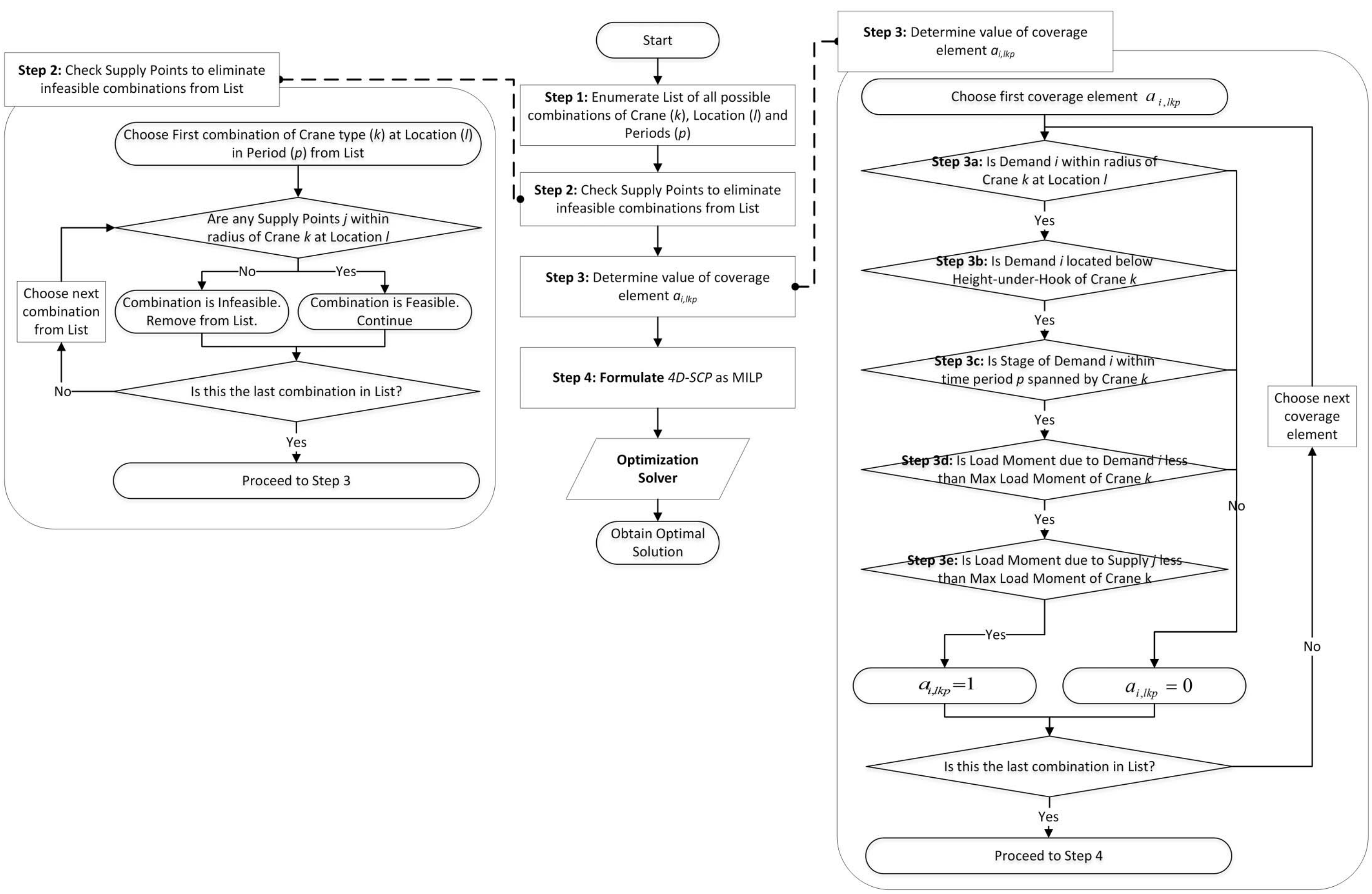

Fig. 4. Flowchart of 4D-SCP

Step 3 comprises rules that determine the value of coverage element $a_{i,lkp}$ [Eq. (21)]; $a_{i,lkp}$ is an element of the set comprising the Cartesian product of the demand point and the combinations of the reduced list from Step 2. The purpose of this coverage element is to indicate if a demand point i can be adequately serviced by a crane k at location l in period p. If so, $a_{i,klp}$ takes a value of 1. To determine the adequacy of service, several crane requirements are accounted for in this paper. Firstly, demand i must be within the crane's service radius. Secondly, demand i must be below the height-under-hook of k, and it must also be within the time period spanned by crane k. The capacity of the crane is also constrained by articulating the load moments of the demand [Eq. (22)] and supply [Eq. (23)] to be less than the maximum allowable load moment. The load moment is approximated as the load (weight) multiplied by radius ρ

$$a_{i,klp} = \begin{cases} 1 & \text{if demand } i \text{ is covered by crane } k \text{ at location } l \text{ in period } p \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

$$\text{Load moment}\,(D_i) = \rho(D_i) \times \text{Weight}_{D_i} \tag{22}$$

$$\text{Load moment}\,(S_j) = \rho(S_j) \times \text{Weight}_{D_i} \tag{23}$$

Other site-specific constraints, like some safety requirements, may also be incorporated in Step 3. For example, pedestrian walking paths or equipment travel paths may sometimes affect the crane-location decision. This can be incorporated as an additional check in Step 3, where if the relevant coordinates defining the travel path falls within the crane radius, the resultant is $a_{i,lkp} = 0$. In other words, the crane cannot be located at that position due to the safety requirement of the travel path. This illustrates a nice feature of the proposed approach, where site-specific constraints may be added as new rules in a modular fashion, without affecting the underlying model.

The final step is to cast 4D-SCP as a MILP. Here, the objective function [Eq. (24)] is the cost-weighted sum of the assigned cranes. The deployment cost c_{klp} of a crane k is calculated in Eq. (25) as a function of the operating cost c_k, fixed cost fc_k, duration of period p, and number of times the crane is assembled and disassembled y

$$\min Z = \sum_{k\in K}\sum_{l\in L}\sum_{p\in P} c_{klp}x_{klp} \tag{24}$$

$$c_{klp} = p_{\text{Dur}} \times c_k + y \times fc_k \tag{25}$$

Eq. (26) constrains the model such that every demand point must be covered by at least one crane (set cover). The model is also constrained such that for any one location, it is only possible to deploy at most one crane k. Moreover, it is only possible to choose one period p per location l [Eq. (27)]. In other words, only one crane that spans a specific combination of stages (defined as p from the possible set of combinations of P) is allowed at one location.

To ensure that the planned duration of each phase p_{Dur} is adequate to support the lifting operations, a productivity constraint is also introduced [Eq. (28)]. The lifting operations is computed by finding the total cycle time taken by all demand points serviced by x_{klp}. The total cycle time is twice the hook traversal time from the demand point to the quickest supply point including loading and unloading times. These are assumed to be constants of 5 min each. μ is the utilization factor of the crane, which is a measure of how often the crane is in use during an operation. Shapira et al. (2012) report typical rates of 60–80%, of which this paper assumes a value of 70%

$$\sum_k\sum_l\sum_p a_{i,klp}x_{klp} \geq 1 \quad \forall\ i \in I \tag{26}$$

$$\sum_k\sum_p x_{klp} \leq 1 \quad \forall\ l \in L \tag{27}$$

$$\sum_i a_{i,klp}x_{klp} \cdot (2 \times \min_j T_{ijkl} + \text{Loading}_k + \text{Unloading}_k) \leq \mu \cdot p_{\text{Dur}} \quad \forall\ p \in P, \forall k \in K, \forall l \in L \tag{28}$$

The 4D-SCP model formulated herein was solved using *Gurobi Optimizer*. Applying the model on the illustrative case confirms the intuitive solution suggested earlier where two k4 cranes are deployed at L5 and L7 to support Zones B and A, respectively. The total cost of employing the 4D-SCP approach is \$304,000, which translates into savings of 14.8 and 13.4% over Approaches 1 and 2.

Application and Discussion of 4D-SCP on a Real Case Study

The 4D-SCP approach was used in the construction of an 11-story academic building with a 7-story adjoining workshop to determine the optimal crane deployment in terms of crane type and location. This was then used to study the impact of considering construction methods and work sequences involving multiple stages on the crane layout.

The project had a footprint of 72×63 m, and was flanked by two access routes, with 29 potential supply points identified during site planning. A further 69 possible crane locations were identified along the building perimeter, as shown in Fig. 5.

The scope of lifting operations was confined to standard structural precast construction elements including columns, beams, walls, and hollow core planks, with an average weight of 1.5 t. The available cranes are the same as those detailed in Table 2. The project was divided into three zones: the main academic block was decomposed into Zones A and C, while Zone B comprised the adjoining workshop block. The maximum height of the elements in Zones A and C was 47.85 m, while the workshop building height in Zone B was 29.8 m. Zones A, B, and C contained 708, 468, and 2,166 distinct construction elements, respectively. The expected cycle time in Zone A and B was 1 week per floor, while Zone C was expected to take 3 weeks per floor. The total project time was estimated to take 51 weeks to complete. In formulating the 4D-SCP for this case study, each element was designated as a demand point.

Eight construction scenarios were identified with different construction methods and varying sequences of work. Scenario A made no distinction between the stages of the project. Scenario B considered a vertical bottom-up construction method, where all three zones started simultaneously in Stage 1 until the completion of Zone B. Subsequently in Stage 2, Zones A and C continued until project completion. Scenarios C–H were horizontal construction methods, where each stage corresponded to the construction of individual zones; each scenario comprised a different work sequence as given in Table 4. For every scenario, it is assumed the scope of work in each stage could be carried out independently of the work in other stages.

Table 4 provides the various optimal solutions obtained under the different scenarios. Each scenario has a specific sequence of

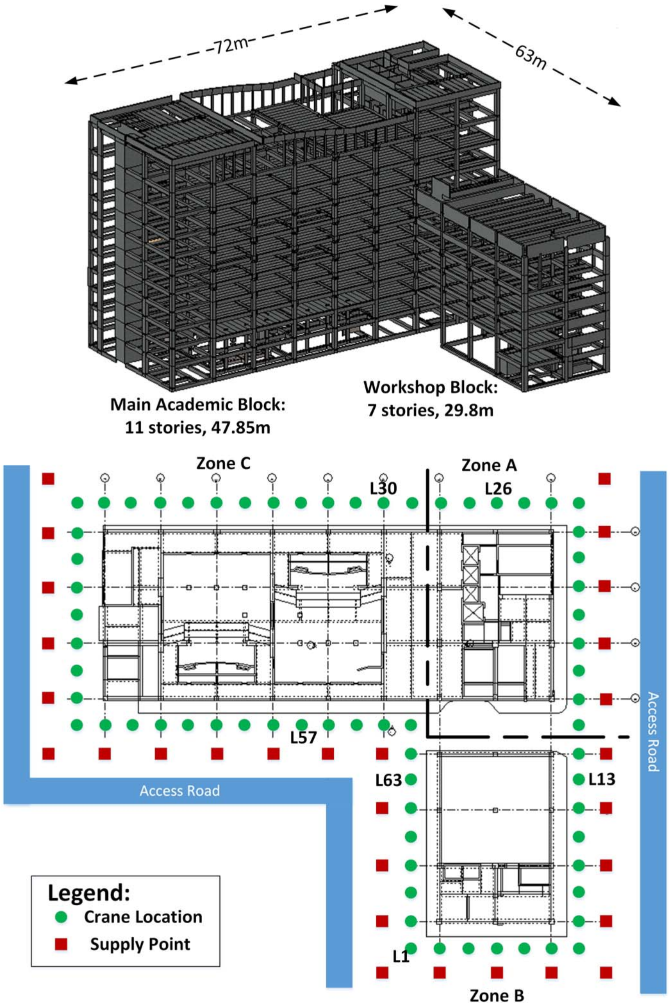

Fig. 5. Site layout of case study

work which is described in the Description column. The optimal cost and deployment in each individual stage for each scenario is also presented in the table, with the weekly duration of each stage marked in parenthesis. The locations are marked in Fig. 5.

The results presented several interesting observations and conclusions. Firstly, the choice of horizontal or vertical construction methods affected the optimal total crane deployment cost in this project. Scenarios C–H, which employed horizontal construction methods, produced better total costs as compared to Scenario B.

Secondly, embedding time information (in the form of stages) enhanced the crane-selection and location-decision problem, and consequently improved total crane deployment cost. This could be observed by the improvement in cost between Scenario A and the other scenarios. Scenarios A and B exhibited no difference in total cost because the building height in each zone was not significantly different. Despite the smaller building footprint in Stage 2 requiring smaller and cheaper cranes, the cost savings was marginal compared to the fixed cost of a new crane deployment in the second stage. Hence, it was cheaper to deploy the same crane throughout both stages. This may not hold for other projects with different building configurations.

Lastly, the sequence of work also affected the total crane deployment cost. Deciding the sequence of zones to construct is hence an important consideration that has an impact on crane deployment and consequently crane cost. The results from Scenarios C, E, G and H showed that deploying a k3 crane at Location L63 as a shared resource to support Zones A and B with another k2 crane at Location L30 to facilitate the construction in Zone C has the best outcome in terms of total crane deployment cost ($505,000). To the construction manager, sequencing the scopes of work in Zones A and B together provides a more-cost-effective crane deployment, as a single crane may be deployed to service both zones together.

The presented case study thus demonstrates how 4D-SCP could be employed on a real construction project and how it could be further used to study the impact of multiple construction stages on crane selection and deployment. In this case study, the difference between the optimal scenarios and the base scenario (Scenario A) amounted to 18.7% deployment cost savings for the contractor. The main conclusion from this case study is that staging information has a significant impact on total crane deployment cost. The 4D-SCP model allowed the contractor to analyze the most cost-effective division of scope as well as the optimal sequence of work.

Conclusion, Discussion of Limitations, and Future Work

Cranes typically represent the single biggest equipment investment on a construction site. Proper crane selection and location is a major component of the decision-making process for construction managers. However, this paper has shown through an illustrative case that the current model of using minimum cost-weighted hook traversal time may give suboptimal deployment cost solutions when complex projects involving multiple stages are considered. The reason is because the current crane-selection and layout-planning models are unable to leverage on the schedule information to derive better solutions.

To address the aforementioned issue, the problem of crane selection and layout planning is recast as a 4D set cover problem. This formulation has several advantages over the current models. First, it is able to provide better solutions when multiple stages are considered. The 4D-SCP incorporates time information and may provide better solutions because it identifies opportunities in which to share resources. Second, it provides a tool to analyze construction methods and work sequences from the perspective of total crane deployment costs. A case study of an academic building with

Table 4. Results of Scenarios with Different Construction Methods and Work Sequences

Scenario	Description	Cost	Stage 1	Stage 2	Stage 3
A	No construction method considered	$620,900	(51) k10 at L26	—	—
B	Vertical construction sequence: two stages	$620,900	(35) k10 at L26	(16) k10 at L26	—
C	Horizontal construction sequence: Zone A to B to C	$505,000	(11) k3 at L63	(7) k3 at L63	(33) k2 at L30
D	Horizontal construction sequence: Zone A to C to B	$547,800	(11) k2 at L57	(33) k2 at L30	(7) k2 at L13
E	Horizontal construction sequence: Zone B to A to C	$505,000	(7) k3 at L63	(11) k3 at L63	(33) k2 at L30
F	Horizontal construction sequence: Zone B to C to A	$547,800	(7) k2 at L13	(33) k2 at L30	(11) k2 at L57
G	Horizontal construction sequence: Zone C to A to B	$505,000	(33) k2 at L30	(11) k3 at L63	(7) k3 at L63
H	Horizontal construction sequence: Zone C to B to A	$505,000	(33) k2 at L30	(7) k3 at L63	(11) k3 at L63

an adjoining workshop is used to demonstrate these advantages. It was found that incorporating construction staging information led to savings of 18.7% for the contractor.

Several assumptions have been made in this model, which could serve as limitations. First, this paper has only addressed the use of static tower cranes in the model formulation. Incorporating other crane types into the proposed 4D-SCP is possible. This entails amending the rules for determining the values of the coverage element $a_{i,lkp}$ (Fig. 4). Future work is being conducted to identify these rules, and encode them into the 4D-SCP framework.

The current solution strategy for 4D-SCP employs an exact solver that returns one possible optimal solution, where multiple optimal solutions may exist. Future work in the form of metaheuristic and heuristic algorithms can be explored to solve larger problem instances. Multiobjective formulations may also be introduced to refine the set of multiple optima.

The findings of this paper highlight the impact of construction methods and work sequences on crane selection and deployment. It is hoped that the concepts covered in this paper would help construction managers make informed decisions with regards to proper crane deployment on site.

Notation

The following symbols are used in this paper:

$a_{i,lkp}$ = coverage element such that it takes a value of 1 if demand i is covered by crane k at location l in period p;
Cr_l^x, Cr_l^y, Cr_l^z = x, y, and z coordinates of crane location l;
$\overline{c_k}$ = average operating cost of crane k per minute;
c_k = operating cost of crane k per week;
c_{klp} = deployment cost of a crane k at location l spanning period p;
D_i^x, D_i^y, D_i^z = x, y, and z coordinates of demand point i;
fc_k = fixed cost of crane k;
i = demand point in the set of all demand points I;
j = supply point in the set of all supply points J;
k = crane in the set of all available cranes K;
k^{st} = cranes deployed in stage st;
k_{TL}, k_{Cap}, k_R, k_{LM}, k_{Ht} = tipload, capacity, radius, maximum load moment, and height-under-hook of crane k, respectively;
Loading_k, Unloading_k = loading and unloading times of crane k (assumed to be 5 min per operation);
l = crane location in the set of all possible crane locations L;
l_{ij} = horizontal distance between demand i and supply j;
M = arbitrarily large value;
N_{cranes} = upper limit to the allowable number of cranes deployed;
P = powerset, of cardinality $|P|$, representing possible combinations of stages st;
p = period of time which is an element of the powerset P, with duration p_{Dur};
q_{ijlk} = binary assignment variable indicating that a demand point i to a supply point j is served by a crane k at location l;
S_i = set that covers some elements in U, with associated cost $Cost_{S_i}$;
S_j^x, S_j^y, S_j^z = x, y, and z coordinates of supply point j;
st = construction stage with duration, Dur_{st};
T_{ijlk} = hook traversal time from demand i to supply j using crane k at location l;
T_{lk}^h, T_{lk}^z, T_{lk}^r, T_{lk}^ω = respective horizontal hook traversal time, hoisting time, trolleying time, and slewing time for given crane k at location l;
U = universe of n elements;
V_ω, V_r, V_z = slewing velocity, trolleying velocity, and hoisting velocity, respectively;
Weight_{D_i} = weight of element i;
x_{klp} = binary decision variable which is true if crane k at location l spans period p;
y = number of times a crane is installed and dismantled;
α = degree of coordination of hook movement in radial and tangential directions in the horizontal plane (assumed to be 0);
β = degree of coordination of hook movement in vertical and horizontal planes (assumed to be 0);
$\delta 1_{lk}$, $\delta 2_{lk}$ = auxiliary binary variables for crane k at location l;
μ = crane utilization rate (assumed to be 70%);
$\rho(D_i)$, $\rho(S_j)$ = linear distances of demand i and supply j from crane location l; and
χ_{lk} = binary variable indicating that a crane k is located at l is assigned.

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