# Simulation of Continuous Systems

# A Pure Pursuit Problem

- We have a **target**, which moves along a pre-determined path, and there is a **pursuer**, who follows the target.

- Example of target: intruder aircraft

- Example of pursuer: defender aircraft

- The defender aircraft will fly pointing towards the intruder

- In our simulation, we have to find whether our defender is able to destroy the intruder or not.

# Path of the target

| Time | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| x(t) | 100 | 110 | 120 | 129 | 140 | 149 | 158 | 168 | 179 | 188 |
| y(t) | 0 | 3 | 6 | 10 | 15 | 20 | 26 | 32 | 37 | 34 |

| Time | 10 | 11 | 12 | 13 | 14 | 15 |
|------|-----|-----|-----|-----|-----|-----|
| x(t) | 198 | 209 | 219 | 226 | 234 | 240 |
| y(t) | 30 | 27 | 23 | 19 | 16 | 14 |

# Path of the fighter

- Suppose the speed of the defender is fixed S=20km/min

- Starting position of the defender is at (0,60)

- Suppose the defender is able to destroy the intruder if the distance is less than 10 km

- The pursuit ends if the pursuer is not able to destroy the target with in 12 minutes

# Target and the pursuer

- Distance between the target and the pursuer, dist(t)=

$$\sqrt{(xt(t) - xd(t))^2 + (yt(t) - yd(t))^2}$$

- Suppose, $\Theta$ is the angle between x axis and the line between the target and the pursuer.

- $\sin(\Theta)=(yt(t)-yd(t))/dist(t)$ and $\cos(\Theta) = (xt(t)-xd(t))/dist(t)$ --- > how?

- Then the position of the defender at time t+1 is determined as, $xd(t+1)=xd(t)+S \cos\Theta$ and $yd(t+1)=yd(t)+S \sin\Theta$

# Simulation results

| Time | xt(t) | yt(t) | xd(t) | yd(t) | dist(t) |
|---|---|---|---|---|---|
| 0 | 100 | 0 | 0 | 60 | 116.61 |
| 1 | 110 | 3 | 17.14 | 49.71 | 103.93 |
| 2 | 120 | 6 | 35.01 | 40.72 | 91.80 |
| 3 | 129 | 10 | 53.53 | 33.15 | 78.94 |
| 4 | 140 | 15 | 72.65 | 27.29 | 68.46 |
| 5 | 149 | 20 | 92.32 | 23.70 | 56.79 |
| 6 | 158 | 26 | 112.28 | 22.39 | 45.85 |
| 7 | 168 | 32 | 132.22 | 23.96 | 36.66 |
| 8 | 179 | 37 | 151.73 | 28.34 | 28.60 |
| 9 | 188 | 34 | 170.79 | 34.39 | 17.20 |
| 10 | 198 | 30 | 190.79 | 33.93 | 8.21 |
| 11 | **BOMB!** | | | | |

# Simulation of a Chemical Reaction

# A chemical reaction

- Suppose there are two reagents ch1 and ch2 that react together to produce another reagent ch3

- So there will be one forward reaction and a backward reaction

- Forward Reaction: ch1+ch2 → ch3 (rate kf)

- Backward Reaction: ch3 →ch1+ch3 (rate kb)

# Chemical Equilibria

- Initially the reaction starts with only ch1 and ch2 and produces ch3

- As soon as the reaction makes progress, the backward reaction also produces ch1 and ch2

- In equilibrium, the rate of changes for all the reagents becomes close to zero.

- We have to simulate until the reaction reaches the equilibrium condition

# Rate of changes

- We need three equations for the rate of changes for each of the reagents.

- We suppose c1, c2 and c3 are the mounts of three reagents at any time instance t.

$$\frac{dc1}{dt} = kb * c3 - kf * c1 * c2$$

$$\frac{dc2}{dt} = kb * c3 - kf * c1 * c2$$

$$\frac{dc3}{dt} = 2\text{kf} * c1 * c2 - 2\text{kb} * c3$$

# Amount of Reagent

- We need three equations for the amount of any reagent at any time instance t+Δt

$$c1(t+\Delta t)=c1(t)+\frac{dc1(t)}{dt}*\Delta t$$

$$c2(t+\Delta t)=c2(t)+\frac{dc2(t)}{dt}*\Delta t$$

$$c3(t+\Delta t)=c3(t)+\frac{dc3(t)}{dt}*\Delta t$$

# Starting the simulation

- We set initial values, c1(0)=50, c2(0)=25 and c3(0)=0, and rates, kf=0.025 and kb=0.01

- Now if we set Δt=0.1 s,

$$c1(t+\Delta t)=c1(t)+[kb*c3(t)-kf*c1(t)*c2(t)]*\Delta t$$

$$c2(t+\Delta t)=c2(t)+[kb*c3(t)-kf*c1(t)*c2(t)]*\Delta t$$

$$c3(t+\Delta t)=c3(t)+[2*kf*c1(t)*c2(t)-kb*c3(t)]*\Delta t$$

# Simulation Results

| Time | c1 | c2 | c3 |
|------|-------|-------|-------|
| 0.000 | 50.00 | 25.00 | 0.00 |
| 0.1 | 46.88 | 21.88 | 6.25 |
| 0.2 | 44.32 | 19.32 | 11.36 |
| 0.3 | 42.19 | 17.19 | 15.62 |
| 0.4 | 38.86 | 15.39 | 19.22 |
| 0.5 | 37.53 | 13.86 | 22.29 |
| 0.6 | 36.38 | 12.53 | 24.93 |
| 0.7 | 35.37 | 11.38 | 27.24 |
| 0.8 | 34.49 | 10.37 | 29.25 |
| 0.9 | 33.80 | 9.49 | 31.03 |
| 1.0 | 32.37 | 8.70 | 32.60 |
| 1.1 | 31.81 | 8.00 | 34.00 |
| And so on | ... | ... | ... |

# A serial Chase Problem

# A serial chase problem

- Similar to the pure pursuit problem
- But now we have a number of moving objects
- Suppose we have 4 objects A,B,C and D
- We have the following scenerio:
  - D moves freely
  - D is chased by C
  - C is chased by B and
  - B is chased by A
  -

# A serial chase problem

- Each of them have their own speed and starting locations

- The objects do not have any idea of whether they are being chased or not

- At each time instance, a chaser or follower adjusts the direction of its chase towards the moving target (similar to the pure pursuit problem)

- Simulation ends as soon as a hit occurs

# Initial conditions

- Speed of each object
  - Va=35km/hr
  - Vb=25km/hr
  - Vc=15km/hr
  - Vd=10km/hr
- Starting locations
  - xa(0)=10, ya(0)=10
  - xb(0)=30, yb(0)=10
  - xc(0)=30, yc(0)=30
  - xd(0)=10, yd(0)=30
- Suppose D moves according to a straight line parallel to x axis
- A hit occurs if distance is less than 0.005
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Simulation results
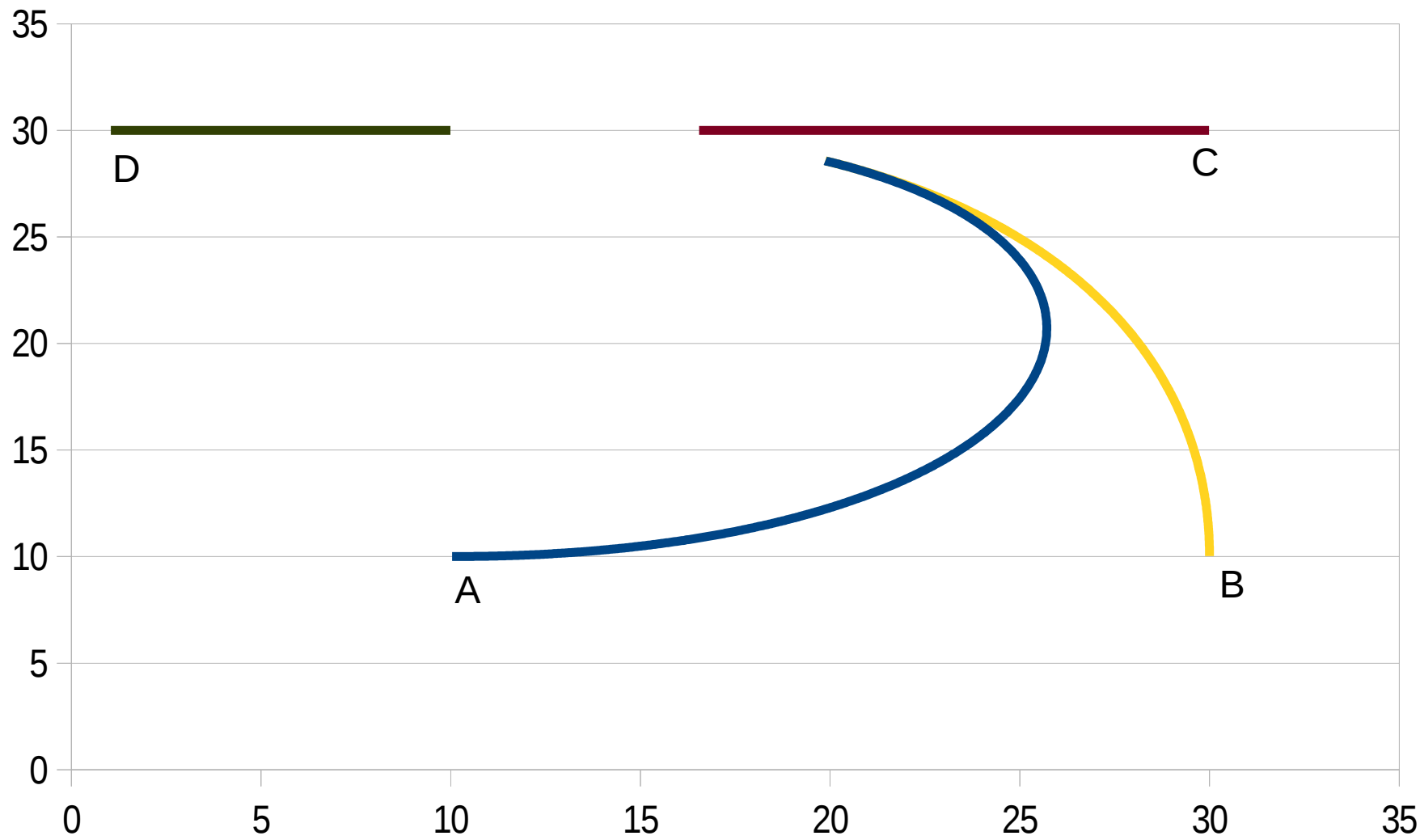
# Another variant

- What happens if, A is chased by D?