Separation Logic

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Outline

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Brief recap: reasoning about code

- Program semantics described by logical conditions satisfied by language constructs
- Classical model, first put forward by Robert W. Floyd and Tony Hoare

Floyd-Hoare Logic in 1 slide

$$\{P\}S\{Q\}$$

- P : pre-conditions
- S : statement
- Q : post conditions

Partial correctness: **If the inital state fullfils pre-conditions and the statement terminates**, the final state satisfies the post conditions.

Total correctness: **If the initial state fullfils the pre-conditions** then the statement terminates and the final state satisfies the post-conditions.

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Global view of state becomes a burden when introducing pointers(think of pointer aliasing..)

Motivating example

```
void deletetree(struct node *root){
    if(root != NULL){
    struct node *left = root->l;
    struct node *right = root->r;
    deletetree(left);
    deletetree(right);
    free(root);
    }
}
```

How can we prove memory safety?

First attempt

Outline

The model

Ints $\triangleq \{\cdots, -1, 0, 1, \cdots\}$

Atoms, Locations $\subseteq Ints$

Stores $\triangleq$ Variables $\rightharpoonup_{fin}$ Ints

Variables $\triangleq \{x, y, \cdots\}$

Locations $\cap$ Atoms $= \{\}$, nil $\in$ Atoms

Heaps $\triangleq$ Locations $\rightharpoonup_{fin}$ Ints

States $\triangleq$ Stores $\times$ Heaps

$[\![E]\!]_s \in Ints$, $[\![B]\!]_s \in \{\text{true}, \text{false}\}$

$h \in Heaps$, $h[E] \in Ints$

The language

Expressions:

$$
\begin{aligned}
E, F, G &:= x, y, \cdots \mid 0 \mid 1 \mid E + F \mid E \times F \mid E - F \\
B &:= false \mid B \Rightarrow B \mid E = F \mid E < F \mid isatom?(E) \mid isloc?(E)
\end{aligned}
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Assertions:

$$\begin{aligned} P, Q, R ::= {} & B \mid E \mapsto F && \textit{Atomic Formulae} \\ & \mid false \mid P \Rightarrow Q \mid \forall x.P && \textit{Classical Logic} \\ & \mid emp \mid P * Q \mid P -\!\!* \; Q && \textit{Spatial Connectives} \end{aligned}$$

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$\neg P = P \Rightarrow False$
$true = \neg(false)$
$P \vee Q = \neg(P) \Rightarrow Q$

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$$true = \neg(false)$$
$$P \vee Q = \neg(P) \Rightarrow Q$$

$$P \wedge Q = \neg(\neg P \vee \neg Q)$$
$$\exists x.P = \neg\forall x.\neg P$$

Some notation

1. $dom(h)$ and $dom(s)$ denote the domain of definition for $h \in Heaps$ and $s \in Stores$, respectively
2. $h \# h' \rightarrow dom(h) \cap dom(h') = \emptyset$
3. $h * h'$ is the union of disjoint heaps
4. $(f | i \mapsto j)$ represent the partial function that behaves like f except that i goes to j.

Semantics

For store s and heap h

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$$s, h \models B \; iff \; [\![B]\!]_s = true$$

$$s, h \models E \mapsto F \; iff \; \{[\![E]\!]_s\} = dom(h) \; and \; h([\![E]\!]_s) = [\![F]\!]_s$$

$$s, h \models false \quad never$$

$$s, h \models P \Rightarrow Q \; iff \; if \; s, h \models P \; then \; s, h \models Q$$

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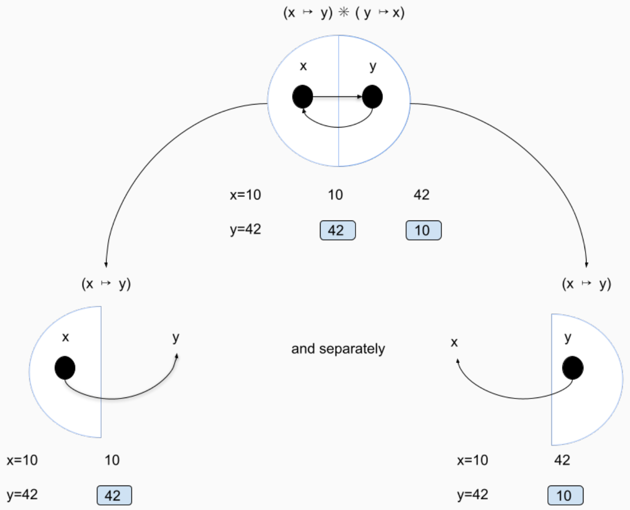
$$s, h \models emp \; iff \; h = [] \; is \; the \; empty \; heap$$

$$s, h \models P * Q \; iff \; \exists h_0, h_1.h_0 \# h_1, \; h_0 * h_1 = h, \; s, h_0 \models P \; and \; s, h_1 \models Q$$

$$s, h \models P -\!\!* \; Q \; iff \; \forall h'. \; if \; h' \# h \; and \; s, h' \models P \; then \; s, h * h' \models Q$$

Visual example



Outline

1. Introduction
2. Theoretical Foundations
3. Reasoning with separation logic
4. Tools

Outline

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