Teorico

Clase 5

$T: V \longrightarrow W \quad V, W \in VS$

T1

$B = \{v_1, \dots, v_n\}$ Base de V

$B' = \{w_1, \dots, w_k\}$ " " W

$w \ni T(v_j) = \sum_{i=1}^{k} a_{ij} w_i$

$[A]_{ij} = a_{ij}$

$v \in V \quad \boxed{v = \sum_{j=1}^{n} \alpha_j v_j}$

$T(v) = T\left(\sum_{j=1}^{n} \alpha_i v_j\right) = \sum_{j=1}^{n} \alpha_j T(v_j)$

$$Tv = \underbrace{\sum_{j=1}^{n} \alpha_j \underbrace{\left(\sum_{i=1}^{k} a_{ij} w_i\right)}}_{B_i} w_i$$

$T(v) = \sum_{i=1}^{k} B_i w_i$

$A = [T]_{BB'}$

OBS $\quad I: V \longrightarrow V$

Base B $\quad$ Base B'

$C(B, B') = [I]_{BB'}$

cambio de base

$C_{BB'}$

normas (T2)

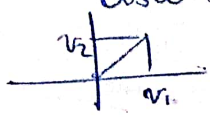
Si V es un K EV

Ej: $\mathbb{R}^n$ $\vec{v} = (v_1, \dots, v_n)$

$$d(v,0) = \|v\|_{2} = \sqrt{v_1^2 + \dots + v_n^2}$$

norma 2

distancia del v al 0



$$d(v,w) = \|v - w\|$$

Def: Una norma en un K EV V es una funcion

1) $\| \ \| : V \to \mathbb{R}_{\geq 0}$

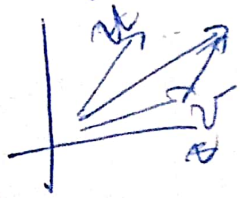
$\forall v \in V \ \|v\| \geq 0$

$\|v\| = 0 \iff v = 0$

2) $\forall \alpha \in K$ y $\forall v \in V$ $\|\alpha v\| = |\alpha| \|v\|$

3) $\forall u, v \in V$ $\|u + v\| \leq \|u\| + \|v\|$

(desigualdad TRIANGULAR)



(T3)

Propiedades: $u=(u_1,\dots,u_n)$, $v=(v_1,\dots v_n)$

$u,v\in K^n$ desigualdad Cauchy-Schwarz

$$\left|\sum_{i=1}^{n} u_i v_i\right| \leq \|u\|_2 \, \|v\|_2$$

D/ si $K=\mathbb{R}$

$$0 \leq \|u+\alpha v\|_2^2 = \left(\sqrt{\sum_{i=1}^{n}(u_i+\alpha v_i)^2}\right)^2 \quad \alpha\in\mathbb{R}$$

$$= \sum_{i=1}^{n}(u_i+\alpha v_i)^2 = \sum_{i=1}^{n} u_i^2 + 2\alpha u_i v_i + v^2\alpha^2$$

$$= \left(\sum_{i=1}^{n} u_i^2\right) + 2\alpha\sum_{i=1}^{n} u_i v_i + \alpha^2\left(\sum_{i=1}^{n} v^2\right)$$

$$= \overbrace{\|u\|_2^2}^{c} + 2\alpha \overbrace{\sum_{i=1}^{n} u_i v_i}^{2b} + \alpha^2 \overbrace{\|v\|_2^2}^{a}$$

$$0 \leq P_2(\alpha) = a\alpha^2 + b\alpha + c$$

$$= \frac{-b \pm \sqrt{\underbrace{b^2-4ac}_{\text{discriminante}}}}{2a}$$

$$b^2-4ac \leq 0$$

$$\left[2\left(\sum_{i=1}^{n} u_i v_i\right)\right]^2 - 4\|u\|_2^2\|v\|_2^2 \leq 0$$

$$\rightarrow \left[\sum_{i=1}^{n} u_i v_i\right]^{2} \leq \left(\|u\|_2^2 \, \|v\|_2^2\right)^{1/2}$$

$$\left|\sum_{i=1}^{n} u_i v_i\right| \leq \|u\|_2 \, \|v\|_2$$

$|z+w| \leq |z|+|w|$

$$||u+v||_2 \leq ||u||_2 + ||v||_2$$

normas "p" $\quad p \geq 1$

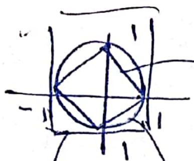
$$\forall v \in K^n, \ ||v||_p = \left(\sum_{i=1}^{n} |v_i|^p\right)^{\frac{1}{p}}$$

$\boxed{p \to \infty \ ||v||_\infty = \max_{1 \leq i \leq n} \{|v_i|\}}$

en $\mathbb{R}^2$

$p=1$ en $\quad ||(v_1,v_2)||_2 = |v_1|+|v_2|$

Circunferencia de Radio 1



$x+y=1$

$||v||_2 = 1$

$||v||_\infty = 1$

Observación:

$$v \in K^n \ ||v||_p = \left(\sum_{i=1}^{n} |v_i|^p\right)^{1/p}$$

$$||v||_p > 0$$

i) $V=0 \to ||v||_p = 0$ ✓

(ii) $\alpha \in K, v \in \underline{V}$ $\quad ||\alpha v||_p = \left(\sum_{i=1}^{n} |\alpha v_i|^p\right)^{1/p}$

$\underline{\mathbb{K}^n}$ $\quad = \sum_{i=1}^{n}$

v pertenece
E.V.
es vector
que puede
ser reales
o complejo

$$= \left(\sum_{i=1}^{n} |\alpha|^p |v_i|^p\right)^{1/p}$$

$$= \left(|\alpha|^p \left(\sum_{i=1}^{n} |v_i|^p\right)\right)^{\frac{1}{p}} = |\alpha| \left(\sum_{i=1}^{n} |v_i|^p\right)^{1/p}$$

$$= |\alpha| \ ||v||_p$$

(T5)

iii) $\|v+w\|_p \leq \|v\|_p + \|w\|_p$

OBS: $V = \mathbb{R}[x]$ $(V = C^0([a,b]))$ → continua en $[a,b]$ siempre es integrable

$$\|P\|_2 = \left(\int_a^b P(x)^2 \, dx\right)^{1/2}$$

$|P(x)|^3$ → modulo

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def

V es un KEV y $\| \ \|$ y $\| \ \|_\nu$

2 normas en V

Digo que $\| \ \|$ y $\| \ \|_\nu$

son equivalentes ⟷

$\exists \, c_1, c_2 > 0$

tales que $\forall v \in V$

$$c_1 \|v\|_\nu \leq \|v\| \leq c_2 \|v\|_\nu$$

Prop: si V es un KEV de

Dim $\cancel{\infty}$ → todas las normas son equivalentes

(T6)

ej $\mathbb{R}^n$ considero $\| \|_2$ y $\| \|_1$

D/ $v \in \mathbb{R}^n$, $\|v\|_1 = \sum_{i=1}^{n} |v_i| \cdot 1 \leq$

$$\left(\sum_{i=1}^{n} |v_i|^2\right)^{1/2} \left(\sum_{i=1}^{n} 1^2\right)^{1/2}$$

dimensión

$$\|v\|_1 \leq \|v\|_2 \underbrace{\sqrt{n}}_{c_2}$$

$$\|v\|_2 = \left(\sum_{i=1}^{n} |v_i|^2\right)^{1/2} \leq \left(\sum_{i=1}^{n} |v_m|^2\right)^{1/2}$$

$\max_{1 \leq i \leq n} |v_i| = |v_m|$ y máximo $= (n|v_m|^2)^{1/2} = \sqrt{n}\,|v_m|$

$$\leq \sqrt{n}\,\|v\|_1$$

$$|v_m| \leq \sum_{i=1}^{n} |v_i|$$

$$\underbrace{\frac{1}{\sqrt{n}}}_{c_1} \|v\|_2 \leq \|v\|_1 \leq \underbrace{\sqrt{n}}_{c_2} \|v\|_2$$