Clase 4 Practico (P1)

Matrices : $K^{n \times m}$ (n: filas, m: columnas)

$+ \quad \begin{matrix} A \in K^{n\times m} \\ B \in K^{n\times m} \end{matrix} \longrightarrow A + B \in K^{n \times m}$

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

$q \in K,\ A \in K^{n\times m} \longrightarrow q \cdot A \in K^{n\times m}$

$$(q \cdot A)_{ij} = A_{ij} \cdot q$$

$A \in K^{n\times m} \quad B \in K^{m\times r}$
$\longrightarrow A \cdot B \in K^{n \times r}$

Ejemplo

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 5 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 23 & 36 \\ & \end{pmatrix}$$

$$\begin{pmatrix} A_{i1} & A_{i2} \cdots A_{im} \end{pmatrix} \begin{pmatrix} B_{1j} \\ B_{2j} \\ \vdots \\ B_{mj} \end{pmatrix} = \begin{pmatrix} \boxed{} \end{pmatrix}$$

A B

lugar ij

$$(A\,B)_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j} + \ldots + A_{im}B_{mj}$$

$$= \sum_{k=1}^{m} A_{ik} B_{kj}$$

Propiedades

$A(B+C) = AB + AC$

$(B+A)C = AC + B \cdot C$

$AB \neq BA$ (no es conmutativo)

→ podría no $\nexists$ si no existe AB o si.

→ podría no coincidir tamaño / dimensión

dimensión = tamaño

Para Pensar

(1) Sean $A \in K^{n\times m}$ $\wedge$ $B, C \in K^{m\times r}$ Pruebe que $\underbrace{\underbrace{A}_{n\times m}\underbrace{(B+C)}_{m\times r}}_{n\times r} = \overset{n\times r}{\widetilde{AB}} + \overset{n\times r}{\widetilde{AC}}$

(2) ~~Dem~~

(1) Sea $1 \le i \le n$ $1 \le j \le r$

$$(A(B+C))_{ij} = \sum_{K=1}^{m} A_{iK} \cdot (B+C)_{Kj} = \sum_{K=1}^{m} A_{iK}(B_{Kj} + C_{Kj})$$

$$= \sum_{K=1}^{m} (A_{iK}B_{Kj} + A_{iK}C_{Kj})$$

$$\sum_{K=1}^{m} A_{iK}B_{Kj} + \sum_{J=1}^{m} A_{iK}C_{KJ}$$

$$(AB)_{ij} + (AC)_{ij} = (AB+AC)_{ij}$$

(2) Determine si las siguientes afirmaciones son V o F y justifica con dem o contraejemplo

(A) Sea V, un e.v de dim 2 y $u, v, w \in V$

$\{u, v\}$ y $\{v, w\}$ son li $\rightarrow$ $\{u, w\}$ es ld

(A) FAlso

contraejemplo

$V = \mathbb{R}^2$

$u = e_1$ $(1,0)$ $v = e_1 + e_2$ $(1,1)$ $w = (2,3)$

tenemos $\{u, v\}$ son LI, $\{w, w\}$ es LI

$\{u, w\}$ no son LD, son LI

$\left(\begin{array}{l} A \rightarrow B \\ \text{Contraej.} \end{array} \; A \wedge \neg B \right)$

Escaneado con CamScanner

(B) Sea $A \in \mathbb{R}^{n\times n}$ / $v^T A v > 0 \ \forall v \in \mathbb{R}^n - \{0\}$ ↳ v-transpuesto

Entonces $Tr(A) > 0$

(P3)

$$A \in K^{n\times m} \longrightarrow A \in K^{m\times n}$$

$$(A^T)_{ij} = A_{ji}$$

$K^n = K^{m\times 1} \longrightarrow$ Sist Ecuaciones $Ax = b$

$\underbrace{v^T A v}_{\text{es un numero}}$ (v traspuesto. por A por v)

$$v^T_{1\times n} A_{n\times m} v_{n\times 1} \in K^{1\times 1} = K \to \text{un numero}$$

Comentarios:

$A \in K^{m\times n}$. $\underbrace{e_i}_{\text{1 en el lugar i-esimo}} \in K^n \longrightarrow \underbrace{A e_i}_{} = \underbrace{A_i}_{\text{nos devuelve i-esima Columna}}$

$$\begin{pmatrix} a_{1i} \\ a_{2i} \\ \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{1i} \\ a_{2i} \\ \end{pmatrix}$$

$\underbrace{e_i^T A}_{\text{fila i de A}} = A_i$

vale $\forall v \in \mathbb{R}^n - \{0\}$ En particular con $v = e_i$

$$e_i^T A e_i = e_i^T \cdot \underbrace{A_i}_{\text{Columna}} = e_i^T \begin{pmatrix} A_{1i} \\ A_{2i} \\ \vdots \\ A_{ni} \end{pmatrix} = A_{ii} > 0 \quad \text{Por hip}$$

i esima fila

Vemos $\forall 1 \leq i \leq n \quad A_{ii} > 0$

$$Tr(A) = \sum \underbrace{A_{ii}}_{>0} > 0$$

✓ Bien probado

$\forall v \in B$
B base

Chequear

3) Calcule la inversa de

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$$

Recuerdo $A \in K^{n\times n}$ su inversa es $B \in K^{n \times n}$

$$AB \to I_d \to \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix}$$

$$AI = A$$
$$IA = A$$

Matriz Inversible $\to \exists$ su inversa

si $\exists$ decimos A es inversible (A^{-1})

$$\exists B \,/\, I = AB = A \cdot \underbrace{\left(B_1 \,|\, B_2 \,|\, \dots \,|\, B_n \right)}_{\text{columnas}}$$

$$= \left(AB_1 \,|\, AB_2 \,|\, \dots \,|\, AB_n \right)$$

$$\longleftrightarrow AB_1 = e_1, \; AB_2 = e_2, \dots, AB_n = e_n$$

admiten sol estos sistemas.

$$\left(A \,|\, e_1 \,|\, e_2 \,|\, \dots \,|\, e_n \right)$$

$$(A \,|\, I_n) \to \underbrace{(I_n \,|\, b_1 \,|\, b_2 \,|\, \dots \,|\, b_n)}_{B}$$

$Ax = b$

$(A|b) \to (\text{escalonada} \,|\, \dots)$

$(A|b) \to (I \,|\, \underset{\text{solucion}}{x^*})$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & -1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{F_1 - F_2 \to F_1} \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$F_1 + F_2 \to F_2 \quad \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$2F_1 + F_3 \to F_3 \quad \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 2 & -2 & 1 \end{array}\right)$$

$$\xrightarrow[F_3 + F_2 \to F_3]{-F_1 \to F_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & -2 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & -2 & 1 \end{pmatrix}$$

matrices cuadradas

P5

(4) calcule det $\begin{pmatrix} 102 & 1 & 0 & 2 \\ 3987 & 9 & 8 & 7 \\ 110 & 1 & 1 & 0 \\ 232 & 2 & 3 & 2 \end{pmatrix}$

Determinante $A \in K^{n \times n}$, $1 \leq j \leq n$

$$\det(A) = \begin{cases} A_{11} & n = 1 \\ \sum_{k=1}^{n} (-1)^{k+j} A_{kj} \det(A(k|j)) \end{cases}$$

$$\begin{pmatrix} A_{1j} \\ A_{2j} \\ \vdots \\ A_{nj} \end{pmatrix} \rightsquigarrow A_{1j} (-1)^{1+j} \det(A(1|j)) + A_j (-1)^{2+1} \det(A(2|j))$$

Propiedades

$\det(I_n) = 1$

$\det(A) = \prod_{i=1}^{n} A_{ii}$ si A triangular

$\det(kA) = k^n \cdot \det(A)$

$\det(AB) = \det(A)\det(B)$

$A \in K^{n \times n}$ $B \in K^{n \times n}$

$\det(A+B) \neq \det(A) + \det(B)$

$\det(A_1 | \dots | C+D | \dots A_n) = \det(A_1|\dots|C|\dots|A_n) + \det(A_1|\dots|D|\dots A_n)$

$\det(A_1 | \dots | k.C | \dots | A_n) = k.\det(A_1|\dots|C|\dots|A_n)$

$\det(A_1 | \dots | C | \dots | C | \dots | A_n) = 0$

Atención

$$\det\begin{pmatrix} a+c & g+h \\ b+d & q+j \end{pmatrix} = \det\left(\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} : \begin{pmatrix} g \\ q \end{pmatrix} + \begin{pmatrix} h \\ j \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} a & g \\ b & q \end{pmatrix} + \det\begin{pmatrix} c & h \\ d & j \end{pmatrix} ?$$

no pero ⟶

$$\det\left(\begin{matrix} a \\ b \end{matrix} \middle| \begin{pmatrix} h \\ j \end{pmatrix} + \begin{pmatrix} g \\ q \end{pmatrix}\right) + \det\left(\begin{matrix} c \\ d \end{matrix} \middle| \begin{pmatrix} h \\ j \end{pmatrix} + \begin{pmatrix} g \\ q \end{pmatrix}\right)$$

volver.

$\det(A^T) = \det(A)$

opcion Triangular, ojo con las opers antes que haga "gauss"

$C_2 = \begin{pmatrix} 1 \\ 9 \\ 1 \\ 2 \end{pmatrix}$ $C_4 = \begin{pmatrix} 2 \\ 7 \\ 0 \\ 2 \end{pmatrix}$ (P6)

$C_3 = \begin{pmatrix} 0 \\ 8 \\ 1 \\ 3 \end{pmatrix}$

(4)

$\cancel{\det\left(100C_2+10C_3+C_4 \middle| C_2 \middle| C_3 \middle| C_4 \right)}$

$$\det\left(\begin{pmatrix} 0 \\ 3000 \\ 0 \\ 0 \end{pmatrix} + 100C_2 + 10C_3 + C_4 \middle| C_2 \middle| C_3 \middle| C_4 \right)$$

$$\det\left(\begin{matrix} 0 \\ 3000 \\ 0 \\ 0 \end{matrix} \middle| C_2 \middle| C_3 \middle| C_4 \right) +$$

$$100 \cdot \det\left(\underbrace{C_2 \middle| C_2}_{=0} \middle| C_3 \middle| C_4 \right) +$$

si son l.d

$$10 \cdot \det\left(\underbrace{C_3 \middle| C_2 \middle| C_3}_{=0} \middle| C_4 \right) +$$

$$\det\left(\underbrace{C_4 \middle| C_2 \middle| C_3 \middle| C_4}_{=0} \right)$$

determine si las sig funciones son TL
$f: \mathbb{R}^{n\times n} \to \mathbb{R}$
$F(A) = \det(A)$

$$\det \begin{pmatrix} 0 & 1 & 0 & 2 \\ 3000 & 9 & 8 & 7 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 3 & 2 \end{pmatrix}$$

Fro 2 f 1 col

(-1)

$$= -3000 \cdot \det\begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 2 & 3 & 2 \end{pmatrix} = -3000\left(1 \cdot \det\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} + 2\det\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\right)$$

$$= -3000\,(1.2 - 3.0) + 2\,(1.3 - 1.2)$$

$$= -3000\,(1.(2) + 2.1)$$

$$= -3000\,(\cancel{-3+2}\ 4) = -12000$$