高数

习题 2-2.

2.(2) $y = 5x^3 - 2^x + 3e^x$

$y' = 15x^2 - 2^x \ln 2 + 3e^x$

(6) $y = 3e^x \cos x$

$y' = 3e^x(\cos x - \sin x)$

(9) $y = x^2 \ln x \cos x$

$y' = 2x \ln x \cos x + x \cos x - x^2 \ln x \sin x$

(10) $s = \frac{1+\sin t}{1+\cos t}$, $s' = \frac{1+\cos t+\sin t}{(1+\cos t)^2}$

3.(2) $\rho = \theta \sin\theta + \frac{1}{2}\cos\theta$, $\frac{d\rho}{d\theta} = \sin\theta + \theta\cos\theta - \frac{1}{2}\sin\theta$

$= \frac{1}{2}\sin\theta + \theta\cos\theta$

$\therefore \left.\frac{d\rho}{d\theta}\right|_{\theta=\frac{\pi}{4}} = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}\pi}{8}$

5. $y = 2\sin x + x^2$, $y_0 = 0$

$y' = 2\cos x + 2x$ $y_0' = 2$

设切线 l_α, 法线 l_τ

$l_\alpha = 2x$ $k_{l_\tau} \cdot k_{l_\alpha} = -1$

$\therefore k_{l_\tau} = -\frac{1}{2}$

$\therefore l_\tau = -\frac{1}{2}x$

6 (6) $y = \sqrt{a^2-x^2}$. $\frac{dy}{dx} = \frac{dy}{d(a^2-x^2)} \cdot \frac{d(a^2-x^2)}{dx} = \frac{-2x}{2\sqrt{a^2-x^2}} = -\frac{x}{\sqrt{a^2-x^2}}$

(7) $y = \tan u$, $u = x^2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{2x}{\cos^2 x^2}$.

(8)

$y = \arctan u$, $u = e^x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^x \cdot \frac{1}{\sqrt{1+e^{2x}}}$

7.(3) $y = e^{-\frac{x}{2}}\cos 3x = -\frac{1}{2}e^{-\frac{x}{2}}\cos 3x + 3e^{-\frac{x}{2}}\sin 3x$

$= -\frac{1}{2}e^{-\frac{x}{2}}\cos 3x - 3e^{-\frac{x}{2}}\sin 3x$

(7) $y = \arcsin\sqrt{x} = \frac{1}{2\sqrt{1-x}\cdot\sqrt{x}}$

(10) $y = -\csc x \cot x - \csc^2 x$ $-\cot x - \csc x$ $= \frac{\cos x + 1}{\ldots \sin x}$

11 (4) $y=\frac{\ln x}{x^n}$, $y'=\frac{x^{n-1}-x^n\ln x\ln n}{x^{2n}}$

(8) $y=\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot(1+\frac{1}{2\sqrt{x}})$

12. (3) $\operatorname{th}\ln x=\frac{e^{\ln x}-e^{-\ln x}}{e^{\ln x}+e^{-\ln x}}=\frac{x-\frac{1}{x}}{x+\frac{1}{x}}=\frac{x^2-1}{x^2+1}$

$y'=\frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2}$

(8) $y=\arctan(\operatorname{th}x)$

$y'=\frac{1}{1+\operatorname{th}^2x}\cdot\frac{1}{\operatorname{ch}^2x}$

(10) $y=\operatorname{ch}^2(\frac{x-1}{x+1})$

$y'=2\operatorname{sh}(\frac{x-1}{x+1})\operatorname{ch}(\frac{x-1}{x+1})\frac{2}{(x+1)^2}$

高数习题 2-3.

1-(5) $y = \sqrt{a^2-x^2}$

$$y' = \frac{-x}{\sqrt{a^2-x^2}}$$

$$y'' = \frac{-\sqrt{a^2-x^2} + x \cdot \frac{-x}{\sqrt{a^2-x^2}}}{a^2-x^2} = \frac{-a^2 \cancel{+x^2}}{(a^2-x^2)\sqrt{a^2-x^2}}$$

(9) $y = (1+x^2)\arctan x$

$$y' = 2x\arctan x + (1+x^2)\cdot\frac{1}{1+x^2}$$

$$y'' = 2\arctan x + \frac{2x}{1+x^2}$$

(12) $y = \ln(x+\sqrt{1+x^2})$

$$y' = \frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{2x}{2\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$$

$$y'' = \frac{-1}{2(1+x^2)\sqrt{1+x^2}}$$

3. $y = f(x^2)$ $\quad \frac{dy}{dx} = f'(x^2)\cdot 2x$

$$\frac{d^2y}{dx^2} = f''(x^2)\cdot 4x^2 + 2f'(x^2)$$

$y = \ln[f(x)]$ $\quad \frac{dy}{dx} = \frac{f'(x)}{f(x)}$

$$\frac{d^2y}{dx^2} = \frac{f''(x)f(x) - f'^2(x)}{f^2(x)}$$

4. $\frac{dx}{dy} = \frac{1}{y'}$ $\quad \frac{dy}{dx} = y'$ $\quad \frac{d^2y}{dx^2} = y''$

$$\frac{d^2x}{dy^2} = \frac{d\frac{dx}{dy}}{dy} = \frac{d\frac{dx}{dy}}{dx}\cdot\frac{dx}{dy} = \frac{-y''}{(y')^2}\cdot\frac{1}{y'} = \frac{-y''}{(y')^3}$$

$$\frac{d\frac{d^2x}{dy^2}}{dy} = \frac{d\frac{d^2x}{dy^2}}{dx}\cdot\frac{dx}{dy} = \frac{d^3x}{dy^3} = \frac{-y'''(y')^3 + 3\cdot y''^2(y')^2}{(y')^6}\cdot\frac{1}{y'}$$

$$= \frac{-y'''y' + 3y''^2}{(y')^5}$$

10.

(2) $y=\sin^2 x$ ，令 $u=\sin x$ ，$v=\sin x$

$y^{(n)}=(uv)^n$ ，$y'=2\sin x\cos x$ $\cancel{y''=2}=\sin 2x$

$y''=2\cos 2x$

由莱布尼茨公式

$y^{(n)}=\sum_{i=0}^{n} C_n^k u^{(n-k)} v^{(k)}= \cdots$

$$\begin{cases} y^{(n)}=2^{n-1}\sin 2x & n=4k+1 \\ y^{(n)}=2^{n-1}\cos 2x & n=4k+2 \\ y^{(n)}=-2^{n-1}\sin x & n=4k+3 \\ y^{(n)}=-2^{n-1}\cos x & n=4k+4 \end{cases}$$

$k=0,1,2,\cdots$

(3) $y=x\ln x$ $y'=\ln x+1$ $y''=\frac{1}{x}$ $y'''=\frac{-1}{x^2}$

$$\begin{cases} y'=\ln x+1 \\ y''=\frac{1}{x} \\ y^{(n)}=\frac{(-1)^n(n-2)!}{x^{n-1}} & n\geqslant 2 \end{cases}$$

习题 2-4

1.

(1) $y^2 - 2xy + 9 = 0$

$$2y\frac{dy}{dx} - 2y - 2x\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y}{y-x}$$

(4) $y = 1 - xe^y$. $\frac{dy}{dx} = -e^y - x\cdot e^y\cdot\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-e^y}{1+xe^y}$$

2. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. l_α 为切线 . l_τ 为法线

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

在点 $(\frac{\sqrt{2}}{4}a, \frac{\sqrt{2}}{4}a)$ 处

$\frac{dy}{dx} = -1$, $l_\alpha : y = -x + \frac{\sqrt{2}}{2}a$

$l_\tau : y = x$

4. (4)

① $\frac{dy}{dx} - 2 = (1-\frac{dy}{dx})\ln(x-y) + (x-y)\cdot\frac{1}{(x-y)}\cdot(1-\frac{dy}{dx})$

② $\frac{d^2y}{dx^2} = -\frac{d^2y}{dx^2}(1+\ln(x-y)) + (1-\frac{dy}{dx})\cdot\frac{1}{x-y}\cdot(1-\frac{dy}{dx})$

由① $\frac{dy}{dx} - 2 = 1 + \ln(x-y) - \frac{dy}{dx}(1+\ln(x-y))$

$$\frac{dy}{dx} = \frac{3+\ln(x-y)}{2+\ln(x-y)} = 1 + \frac{1}{2+\ln(x-y)}$$

由②) $\frac{d^2y}{dx^2} = \frac{(1-\frac{dy}{dx})^2}{x-y}\cdot\frac{1}{2+\ln(x-y)} = \frac{1}{(x-y)(2+\ln(x-y))^3}$

5. $\ln y = \frac{1}{2}(\ln x + \ln\sin x + \frac{1}{2}\ln(1-e^x))$

$$\frac{y'}{y} = \frac{1}{2}(\frac{1}{x} + \frac{\cos x}{\sin x} + \frac{1}{2}\frac{-e^x}{1-e^x})$$

$y' = \frac{1}{2}(\frac{1}{[illegible]} + [illegible] + \frac{1}{2}\frac{e^x}{[illegible]})\cdot\sqrt{x\sin x\sqrt{1-e^x}}$

6. (2) $\begin{cases} x = \theta(1-\sin\theta) \\ y = \theta\cos\theta \end{cases}$

$$\frac{dy}{d\theta} = \cos\theta - \theta\sin\theta$$

$$\frac{dx}{d\theta} = 1 - \sin\theta - \theta\cos\theta$$

$$\frac{dy}{dx} = \frac{\cos\theta - \theta\sin\theta}{1-\sin\theta - \theta\cos\theta}$$

8. $\begin{cases} x = \dfrac{3at}{1+t^2} \\ y = \dfrac{3at^2}{1+t^2} = tx \end{cases}$ $\cancel{\Rightarrow \frac{x}{t} + y = 3a}$.

~~$t = 2$ 时~~ $\cancel{x = \frac{6}{5}a \quad y = \frac{12}{5}a}$

$$\frac{dx}{dt} = \frac{3a(1+t^2) - 2t(3at)}{(1+t^2)^2} = \frac{3a - 3at^2}{(1+t^2)^2}$$

$$\frac{dy}{dt} = x + \frac{dx}{dt} = \frac{3at}{1+t^2} + \frac{3a-3at^2}{(1+t^2)^2}$$

$$= \frac{3at + 3at^3 + 3a - 3at^2}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{t+t^3+1-t^2}{1-t^2}$$

9. (2) $\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$ $\quad \dfrac{dx}{dt} = -a\sin t \quad \dfrac{dy}{dt} = b\cos t \quad \dfrac{dy}{dx} = \dfrac{b\cos t}{-a\sin t}$

(3) $\begin{cases} x = 3e^{-t} \\ y = 2e^{t} \end{cases}$ $\quad \begin{cases} \dfrac{dx}{dt} = -3e^{-t} \\ \dfrac{dy}{dt} = 2e^{t} \end{cases}$ $\quad \dfrac{dy}{dx} = -\dfrac{2e^{2t}}{3}$

11. $\dfrac{dr}{dt} = 6 \quad \dfrac{ds}{dt} = ?$

$s = \pi r^2$, $\dfrac{ds}{dr} = 2\pi r$, $\dfrac{ds}{dt} = 12\pi r$

当 $t = 2$ 时, $r = 12$

$\therefore \dfrac{ds}{dt} = 144\pi$