习题2-5

1. $y=x^3-x$

$x=2$ 时 $y=6$

$\Delta x=1$, $\Delta y = 3^3-3-6=18$

$\Delta x=0.1$ $\Delta y = 2.1^3-2.1-6=1.161$

$\Delta x=0.01$ $\Delta y = 2.01^3-2.01-6=0.110601$

2. $dy = 3x^2dx-dx$

$\because x=2$

$dy=11dx$

$dx=1$ 时, $dy=11$

$dx=0.1$ 时 $dy=1.1$

$dx=0.01$ 时 $dy=0.11$

3. (3) $y^2=\frac{x^2}{x^2+1}=1-\frac{1}{x^2+1}$. $y=\frac{x}{\sqrt{x^2+1}}$

$2ydy=\frac{2xdx}{(x^2+1)^2}$

$dy=\frac{1}{(x^2+1)^{\frac{3}{2}}}dx$

(4) $dy=2\ln(1-x)\cdot\frac{1}{1-x}\cdot(-1)dx$

(7) $y=\arcsin\sqrt{1-x^2}$

$\sin y=\sqrt{1-x^2}$

$\sin^2 y=1-x^2$

$-2xdx=\sin 2y\,dy=2\sin y\cos y\,dy$

$dy=\frac{1}{|x|}\cdot d\sqrt{1-x^2}=\frac{1}{|x|}\cdot\frac{1}{2\sqrt{1-x^2}}\cdot d(1-x^2)$

$=\frac{1}{|x|}\cdot\frac{1}{2\sqrt{1-x^2}}\cdot(-2x)dx$

$=\frac{x}{|x|}\cdot\frac{1}{\sqrt{1-x^2}}dx$

(8) $dy = 2\tan(1+2x^2)\cdot d\tan(1+2x^2)$

$= 2\tan(1+2x^2)\cdot \frac{1}{\cos^2(1+2x^2)}\, d(1+2x^2)$

$= 2\tan(1+2x^2)\cdot \frac{1}{\cos^2(1+2x^2)}\cdot 4x\,dx$

(9) $dy = \frac{1}{1+\left(\frac{1-x^2}{1+x^2}\right)^2}\, d\left(\frac{1-x^2}{1+x^2}\right)$

$dy = \frac{1}{1+\left(\frac{1-x^2}{1+x^2}\right)^2}\cdot \frac{-2}{(1+x^2)^2}\, d(1+x^2)$

$= \frac{-2}{(1+x^2)^2+(1-x^2)^2}\cdot 2x\cdot dx$

$= \frac{-2x}{x^4+1}\, dx$

(10) $ds = A\cos(wt+\varphi)\, d(wt+\varphi)$

$ds = Aw\cos(wt+\varphi)\, dt$

4. (1) $d2x = 2dx$ (2) $d\,\frac{3}{2}x^2$ (3) $d\sin t$

(4) $d\,-\frac{\cos wx}{w}$ (5) $d\ln(1+x)$ (6) $d\,-\frac{e^{-2x}}{2}$

(7) $d\,2\sqrt{x}$ (8) $d\,\frac{1}{3}\tan 3x$

5. $\Delta s = \cancel{\frac{4l}{3l^2}}\ \frac{4}{3l}\left[(f+\Delta f)^2 - f^2\right]$

$= \frac{4}{3l}\left[2\Delta f\cdot f + \Delta f^2\right]$

总习题二

6. (1) $f(x)=\begin{cases}\sin x & x<0\\ \ln(1+x) & x\geq 0\end{cases}$ $\qquad \frac{2x+h}{2} \quad \frac{h}{2}$

$$f'_-(0)=\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0^-}\frac{\sin(x+h)-\sin x}{h}$$

$$=\lim_{h\to 0^-}\frac{2\sin\frac{h}{2}\cos\frac{2x+h}{2}}{h}=1$$

$$f'_+(0)=\lim_{h\to 0^+}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0^+}\frac{\ln(x+h+1)-\ln(x+1)}{h}$$

$$=\lim_{h\to 0^+}\frac{\ln\left(\frac{h}{x+1}+1\right)}{h}=\lim_{h\to 0^+}\frac{\ln(h+1)}{h}=1$$

$f'_-(0)=f'_+(0)$ $\therefore$ $f'(0)$ 存在且 $f'(0)=1$

(2) $$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{f(h)}{h}$$

$$f'_+(0)=\lim_{h\to 0^+}\frac{f(h)}{h}=\lim_{h\to 0^+}\frac{\frac{h}{1+e^{\frac{1}{h}}}}{h}=\lim_{h\to 0^+}\frac{1}{1+e^{\frac{1}{h}}}=0$$

$$f'_-(0)=\lim_{h\to 0^-}\frac{1}{1+e^{\frac{1}{h}}}=1$$

$f'_+(0)\neq f'_-(0)$ $\therefore$ $f'(0)$ 不存在.

8. (3) $y=\ln\tan\frac{x}{2}-\cos x\cdot\ln\tan x$

$$y'=\frac{1}{\tan\frac{x}{2}}\cdot\frac{1}{\cos^2\frac{x}{2}}\cdot\frac{1}{2}+\sin x\ln\tan x-\cos x\cdot\frac{1}{\tan x}\cdot\frac{1}{\cos^2 x}$$

$$=\frac{1}{\sin x}+\sin x\ln\tan x-\frac{1}{\sin x}$$

$$=\sin x\ln\tan x$$

(4)
$y=\ln(e^x+\sqrt{1+e^{2x}})$

$\because$ 当 $f(x)=\ln(x+\sqrt{1+x^2})$ $f'(x)=\dfrac{1}{\sqrt{1+x^2}}$

$\therefore y'=\dfrac{e^x}{\sqrt{1+e^{2x}}}$

9(2) $y^2=\dfrac{x^2}{\cancel{1+x}\ 1-x^2}=\dfrac{1}{1-x^2}-1$

$2y\,dy=\dfrac{\cancel{-4}\ 2x}{(1-x^2)^2}dx$

$y'=\dfrac{x}{(1-x^2)^2}\cdot\dfrac{\sqrt{1-x^2}}{x}=\dfrac{1}{(1-x^2)^{\frac{3}{2}}}=(1-x^2)^{-\frac{3}{2}}$

$y''=+\frac{3}{2}(1-x^2)^{-\frac{5}{2}}\cdot 2x=3x\cdot(1-x^2)^{-\frac{5}{2}}$

13(2)

$\dfrac{dx}{dt}=\dfrac{1}{\sqrt{1+t^2}}\cdot\dfrac{1}{2\sqrt{1+t^2}}\cdot 2t=\dfrac{t}{1+t^2}$

$\dfrac{dy}{dt}=\dfrac{1}{1+t^2}$

$\dfrac{dy}{dx}=\dfrac{1}{t}$ $(\sqrt{e^{2x}-1}=t)$

$\dfrac{d^2y}{dx^2}=\dfrac{d\frac{dy}{dx}}{dt}\cdot\dfrac{dt}{dx}=-\dfrac{1}{t^2}\cdot\dfrac{1+t^2}{t}=-\dfrac{1+t^2}{t^3}$

16. $d=0$, $\therefore y=x\cdot(ax^2+bx+c)$

$y'=3ax^2+2bx+c$, $y'|_{x=-L}=0$, $y'|_{x=0}=0$

$\begin{cases}(-L)(aL^2-bL+c)=H\\ 3aL^2-2bL+c=0\\ c=0\end{cases}\Rightarrow\begin{cases}(-L^2)(aL-b)=H\\ L(3aL-2b)=0\end{cases}$

$3aL = 2b$, $b = \frac{3a}{2}L$

$\therefore$ $\frac{1}{2}aL^3 = H$, $a = \frac{2H}{L^3}$

$b = \frac{3H}{L^2}$

$\therefore y = \frac{2H}{L^3}x^3 + \frac{3H}{L^2}x^2$

(18) 令 $y = \sqrt[3]{x}$，当 $x = 1$ 时，$\Delta x = 0.02$.

$y^3 = x$

$3y^2 dy = dx$

$dy = \frac{dx}{3y^2} = \frac{1}{3} \cdot x^{-\frac{2}{3}} dx$

$\Delta x = 0.02$ 时，$dy = \frac{1}{3}(\cancel{0.02}\,1)^{-\frac{2}{3}} \cdot 0.02 = \frac{1}{150}$

7. 令$f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x=0$

$f(0)=0$，$f(x_0)=0$

∵ $f(x)$ 为多项式

∴ $f(x)$ 在 $[0,x_0]$ 上连续

且在 $(0,x_0)$ 上可导

那么 由罗尔中值定理

$\exists x_1\in(0,x_0)$

$f'(x_1)=0$

又 $f'(x_0)=a_0nx^{n-1}+a_1(n-1)x^{n-2}+\cdots+a_{n-1}$

∴ 原命题成立

8. $F(2)=0$，$F(1)=0$

$F'(x)=2(x-1)f(x)+(x-1)^2f'(x)$

$F'(1)=0$

10 ~~原式~~

$\cancel{\frac{1}{b}-\frac{1}{a}<\frac{\ln a-\ln b}{b}}$

原式 $\Leftrightarrow 1-\frac{b}{a}<\ln\frac{a}{b}<\frac{a-b}{b}$

令 $\frac{b}{a}=t$　$t\in(0,1)$

原式 $\Leftrightarrow 1-t<-\ln t<\frac{1}{t}-1$

∵ 当 $t\in(0,1)$，$\ln t<0$，$t-1>\ln t$

$\cancel{\frac{t-1}{\ln t}<1}$　∴ $\frac{1}{t}-1>\ln\frac{1}{t}$

∴ 原式成立

11. (1) ~~设~~ $\forall a, b$

$\exists t \in R$

$\left|\frac{\arctan a - \arctan b}{a-b}\right| = \left|\frac{1}{1+t^2}\right| \le 1$ (由拉格朗日中值定理

12. $f(x) = x^5 + x - 1$, $f(0) = -1 < 0$

$f'(x) = 5x^4 + 1 > 0$

$\therefore f(x)$ 单增

又 ~~lim~~ $f(2) = 33 > 0$

$\therefore f(x)$ 有且仅有1个正根

14. $f(x)$ 在 R 上可导且连续, $f(0)=1$, $f'(0)=1$

$\therefore$ ~~$t \in R$~~, $\forall a, b \in R$ $\exists t \in R$

$\frac{f(a)-f(b)}{a-b} = f'(t) = f(t)$